

Variational inference for stochastic differential equations driven by fractional Brownian motion

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Abstract

Stochastic differential equations (SDE) driven by white noise are important models for stochastic dynamical systems in natural science and engineering. The statistical inference of the parameters of such models based on noisy observations has also attracted considerable interest in the machine learning community. Using Girsanov's change of measure approach one can apply powerful variational techniques to solve the inference problem.

A limitation of standard SDE models is the fact that they show typically a fast, exponential decay of correlation functions. If one is interested in stochastic processes with a long time memory, a well known possibility is to replace the Brownian motion in the SDE by the so called fractional Brownian motion (fBM) which is no longer a Markov process. Unfortunately, variational inference for this case is much less straightforward.

Our approach to this problem utilises a somewhat overlooked idea by Carmona and Coutin (1998) who showed that fBM can be exactly represented as an infinite dimensional linear combination of Ornstein-Uhlenbeck processes with different time constants.

Using an appropriate discretisation, we arrive at a finite dimensional approximation which is an 'ordinary' SDE model in an augmented space. For this new model we can apply (more or less) off-the shelf variational inference approaches. We also discuss application of this approach to generative diffusion models.

The talk is based on the paper: "Variational Inference for {SDE}s Driven by Fractional Noise", Rembert Daems, Manfred Opper, Guillaume Crevecoeur and Tolga Birdal, ICLR (2024).