Bocconi
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12:00 pm **Room 3-E4-SR03** Via Roentgen 1 Milano

On the behavior of posterior probabilities with additional data: monotonicity and nonmonotonicity, asymptotic rates, log-concavity, and Turán's inequality

Abstract

Given a parametric model, a prior, and data, Bayesian statisticians quantify their belief that the true parameter is $\vartheta 0$ by its posterior probability. The starting question of this paper was whether the posterior at $\vartheta 0$ increases when the data are generated under $\vartheta 0$, and how it behaves when the data come from $\vartheta \neq \vartheta 0$. Can it decrease and then increase, and thus additional data may mislead Bayesian statisticians?

For data arriving sequentially, we consider monotonicity properties of the posterior probabilities as a function of the sample size for certain stochastic orders, starting with likelihood ratio dominance.

We compute the asymptotic rate of convergence of the expectation of the posterior for observations from an exponential family and show that the expectation of the $\vartheta 0$ -posterior under $\vartheta \neq \vartheta 0$ is eventually strictly decreasing. Finally, we show that in a number of interesting cases this expectation is a log-concave function of the sample size, and thus unimodal. In the Bernoulli case, we obtain this result by developing an inequality that is related to Turán's inequality for Legendre polynomials.

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