English Auctions with Resale:

An Experimental Study

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February 2009

Abstract

I study in a series of experiments how the possibility of resale alters the common results in the literature on English auctions. I designed and tested a simple English auction and two English auctions with resale, but with different informational backgrounds. All three treatments theoretically have the same equilibrium. I find, however, that the possibility of resale alters behavior significantly. In the two treatments with resale, subjects deviated from both the Nash prediction and the common results about bidding behavior in English auctions. Subjects tend to overbid, when they are certain they can reap the whole surplus in the resale market. I employ different models like QRE and levels of reasoning and conclude that overbidding can be explained as a rational response to the noisy environment in markets with human participants, that is, as rational decision making when anticipating others to make errors. When the outcome of the resale market is not certain, there is significant signaling behavior and auction prices tend to be lower than the the Nash prediction.

JEL classification: D44, C90

*Bonn Graduate School of Economics, University of Bonn. My thanks go to Reinhard Selten, Rosemarie Nagel, John Kagel, Benny Moldovanu and Joerg Oechssler for their comments and support. I would like to acknowledge the financial support of Spain's Ministry of Education under grant nr. BEC 2000-1029, of the Chair of Economic Theory II and Bonn Econ Lab at the University of Bonn. Additionally I want to thank three anonymous referees, Steffen Altmann for his very thorough proofreading, Nagore Iriberri for very fruitful discussions, Thomas Palfrey, Vincent Crawford and Charles Holt for useful comments, Michal Paluch and Lars Koch for their technical support, Noemi Sanchez for translating the instructions and many participants at the Econometric Society NA Meeting at Duke, the ES European Meeting in Budapest, the ESA World Meeting in Amsterdam, the EDP Jamboree in Louvain and the Bonn-Cologne Experimental Workshop. The experiment was programmed and conducted with the software z-Tree (Fischbacher 1999).

1 Introduction

Auctions are very often followed by a resale opportunity. For instance, after virtually every durable good auction, the winner can choose to resell the good to the competing bidders or other third parties. Even when resale is explicitly prohibited, ways can be found to get around the prohibition. Consider the case of mobile phone and wireless spectrum licences where often "use-it-or-lose-it" conditions to prevent resale are imposed. Still these restrictions can be circumvented. The company holding the licence can be bought and there have even been cases where special companies were used to buy and resell the licence.

Until recently resale was not considered in auction models. However the work of Haile (1999, 2000, 2001, 2003) has shown that the existence of resale opportunities is important and can change many results of auction theory. In order for resale to be meaningful, the outcome of the initial auction must be inefficient with a positive probability. In Haile's paper (2003) the highest value player does not necessarily win in the initial auction because he does not perfectly know his value or because he is not participating. Other ways to induce resale in equilibrium can be asymmetries in the values of the bidders (Hafalir and Krishna 2008, Garratt and Troeger 2006) or new participants arriving in the resale stage (Haile 1999).

In this paper I test auctions with resale in the laboratory based on a simple model from Haile (1999) and find an alternative reason for resale that has not yet been considered in the abstract theoretical models but is plausible when human players are involved, namely noisy decision making. People make mistakes and anticipate others to make mistakes. This can lead subjects to deviate from theoretical predictions in systematic ways. To see how it can induce resale, consider following example.

Suppose you are participating in an English auction for works of art, regarded often to be a textbook example of private values, in the absence of resale possibilities. Suppose that the current price for the Picasso painting under sale is 10 euros. Even if you do not have a taste for cubism and thus your private value is zero you might still want to participate in the auction, expecting to resell the painting for a higher price. Thus resale introduces a common value element to the valuations of bidders and can induce overbidding. We could use a similar argument in the markets for real estate, bonds, operating licences and more.

Thinking in line with standard models one could note that in such a simple setting, bidding one's value is still a symmetric equilibrium. A strategy of overbidding expecting to resell is not consistent with this equilibrium. If others bid their values, no profitable resale is possible. Thus winning with a bid higher than your value can only result in zero or negative payoffs. Crucially however, this is only true if bidders never make mistakes, as is usually assumed.

From our experience in the lab and the field, we know humans are prone to making mistakes. Expecting high value bidders to make mistakes can make it optimal for low value bidders to bid more than their values. This in response can lead to high value bidders bidding less and expecting to buy cheaper in the resale stage. Thus resale opportunities can be exploited even if standard theory predicts they will not and they can give rise to richer bidding strategies than theoretically expected. Let it be emphasized that this deviation from standard models is quite natural. There is no need for restrictive assumptions on the structure of markets or the private information of bidders to induce resale in real life situations. A sufficient condition, as we shall show, is just the presence of a small amount of noise. Such noise exists in many markets, even in financial markets where stakes are very high (see Shleifer and Summers 1990). It can stem from experimentation, lack of experience or misunderstanding of the rules, false transmission of information and mistakes in the execution of orders, liquidity constraints or other exogenous reasons that are not adequately modeled in theory but whose presence in real markets cannot be easily dismissed.

To examine the importance of resale opportunities and the effect of noise in a controlled environment, I designed and ran two experimental treatments of English auctions with resale, with different informational backgrounds but with the same equilibrium bidding functions prescribing that players bid their values. Subjects exhibited a significantly different behavior with respect to both the theory and previous auction experiments without resale. Instead of bidding their values in both treatments, they overbid relative to equilibrium when they can be certain they can reap all the rents in the resale markets, and they tend to underbid when the resale outcomes are uncertain. Moreover this result should not be attributed simply to irrational behavior in the laboratory, but seems to have a reasonable explanation. Subjects do try to maximize their profits. But while doing so, they anticipate the possibility of others making mistakes and they use this knowledge more or less optimally. In that sense, this paper presents a previously unstudied example of a more general class of games where the anticipation of noise drastically changes players' best responses[1]. In such cases, standard game theory loses much of its predictive power and concepts of bounded rationality, such as a Quantal Response Equilibrium (McKelvey and Palfrey 1995) and levels of reasoning (Nagel 1995, Stahl 1995, Camerer 2004, Crawford and Iriberri 2008), perform much better.

The experimental economics literature has not focused on auctions with resale yet, for the same reasons that there were precious few theoretical models of resale until recently, as we shall see below. To my knowledge there exist two other experimental papers on auctions with resale, Georganas (2003) on which part of the present paper is based and recent independent work by Lange, List and Price (2004). Their experimental treatments are similar to the ones in this paper, but they differ in important ways: first they used first-price sealed-bid auctions and secondly they gave players noisy signals about their private values. They found a deviation from the equilibrium predictions, which they attribute to risk aversion. However risk aversion alone does not change the equilibrium in our games, so it cannot explain the data in the present study.

Even though the possibility of resale and its potential importance has been recognized in the theoretical literature (Milgrom and Weber, 1982 and Milgrom 1987 with the first models of auctions with resale) there has been a striking absence of formal models featuring resale until recently. A frequent argument has been that resale is covered by the assumption of common values. However, as shown in Haile (2003) players in the initial auction have common values when there is a possibility of resale but, importantly, valuations are endogenously determined and equilibrium strategies are not the same as in the simple common value model. Moreover Revenue Equivalence holds under some assumptions although it does not in the CV case. In this paper bidders have noisy signals about their values in the initial auction. Noisy signals work in a similar way as the noisy bids in this paper, as they lead to inefficient outcomes and profitable resale.

There are of course other possibilities to make resale potentially profitable. Haile (1999) assumes that an a priori known number of bidders is added to the bidder pool in the second

[1] Games with this property include the guessing game, the centipede game and the traveler's dilemma.

period. These new subjects arriving in the resale auction can have higher private values than the winner of the first auction, opening thus a resale possibility.

Alternatively, one can construct models with asymmetric equilibria. Intuitively, resale seems plausible in such an equilibrium, as the players will have asymmetric strategies and thus the highest value player will not necessarily be the highest bidder in the initial auction. This option is explored in Garratt, Troeger (2006). In a setup similar to ours they include speculators with zero valuations and find asymmetric equilibria where the speculator wins with positive probability. Hafalir and Krishna (forthcoming) and Gupta and Lebrun (1999) have bidders with potentially positive use values, which however are asymmetrically distributed. This setup also gives rise to inefficient outcomes and subsequent resale.

Finally some other models including some flavor of resale are Ausubel and Cramton (1999), McAfee (1998) and Jehiel and Moldovanu (1999), although their setups are not directly related to ours.

This paper is structured as follows. The experimental procedure is introduced in section 2. Section 3 presents the equilibrium predictions and the results are presented in section 4. Models of bounded rationality involving some flavor of noisy decision making are presented in section 5. Ideas for future work are discussed in section 6 and section 7 concludes.

2 Experimental design

There are two stages in the game. In the first stage four bidders $i = 1, 2, 3, 4$ bid in an English auction[2] for one unit of an indivisible object. Each bidder has a use value v_i, which is identically independently drawn from a discrete uniform distribution with support [0,100]. The distribution of the use values is common knowledge, but the actual use values are private knowledge. We have to emphasize the distinction between a bidder's *use value*, i.e. the value a bidder places on owning the object ignoring any resale possibilities and the bidder's *valuation*, which is the value she places on winning the auction and which is determined endogenously, taking account of the resale opportunity.

[2]To be more specific I use the Japanese variant of the English auction (see Milgrom and Weber 1982). There exist of course other, quite different variants which are not so practical to implement in a laboratory experiment.

For the auction we use an ascending clock design (see Kagel et al. 1987). There is a clock on each computer screen, starting simultaneously from zero and synchronously rising every second in steps of one unit. Each subject can exit the auction at any time by pressing a button. Once out of the auction no reentry is possible. The other bidders can observe the price at which one exits. After three bidders have left the auction, the last remaining bidder obtains the good and has to pay the price p_1 at which the last one left. This concludes the first stage.

In the second stage there is the possibility of resale. This is done through an English auction, where the seller can choose a reservation price. The difference between the two resale treatments, lies in the informational background of the second stage. As discussed there are many ways to model the resale stage. I chose two extremes with a big span between them, to test for a wide range of possibilities. In the first, incomplete information treatment (hence INC), the only information the bidders get about the others' values is through the bids in the initial auction. The seller decides about the reservation price r and then the remaining bidders can see the reserve price and decide simultaneously if they want to participate in the resale auction or not[3]. If no bidder chooses to participate then the ownership of the good and the payoffs remain the same as in the first stage. If only one bidder participates, then she obtains the good and pays the reservation price to the owner. Thus the final payoffs are $r - p_1$ for the first stage winner and $v_i - r$ for the second stage winner. If more than one bidder decides to participate we have an English auction like in the first stage, with the difference that this time the clock starts at the reserve price. Again when only one bidder remains, she obtains the good and has to pay the price p_2 where the last bidder left the auction. The following payoffs are then communicated to the subjects: $p_2 - p_1$ for the first stage winner, $v_i - p_2$ for the winner in the second stage and zero for the others. In the same screen they can see the price of the initial auction, the reserve price, the number of participants and the price in the resale auction (zero if there was no resale auction), the highest private value and information about past periods. The information feedback was so rich in order to facilitate learning, as otherwise bidders would be getting too few experiences of winning and thus

[3] I did not give the seller the explicit choice not to put the good up for resale, however sellers were advised to set a reservation price of 100 if they did not want to resell.

learning chances. Note that there are 30 periods in each experiment and subjects win on average only 1/4 of the time, which means that they get to win 7 or 8 times.

In the second treatment with complete information (COMP), after the first stage bidders get to know the use values of the others as in Gupta and Lebrun (1999). Thus in equilibrium they will ask for a reservation price equal to the highest private value. This amounts to a take-it-or-leave it offer to the person with the highest private value equal to his private value. We know however that subjects in experiments very often deviate from the equilibrium in the direction of a 50-50 split of the surplus, probably because of fairness considerations. As it is not the subject of this paper to treat bargaining games[4] I decided to avoid this problem by forcing the winner of the first auction to automatically resell the object in the second stage to the bidder with the highest value. She then received as payoff the highest private value minus the price she paid in the first auction. The rest of the players, including the person who obtains the good after resale, have a payoff of zero. After each auction players can see their payoffs, private values, the auction price, the private value of the winner, the highest private value and information about past periods.

The resale treatments are to be compared with the bidding behavior in simple English auctions. Such auctions have been extensively discussed in the literature (see for example Kagel et al. 1987), however it is important to make sure that the unexpected results in COMP and INC are not due to some kind of framing effect or an unusual subject pool. Thus, I also ran a standard English auction (ENG), with IPV drawn from a uniform distribution [0,100]. The experimental mechanism was in all other respects equal to the one used in COMP and INC, so ENG can be directly compared to them. As the results in ENG are very similar to previous studies, this treatment will not be discussed on its own, but only in comparison with the other treatments.

Observe that the use of the English auction in all treatments does not allow us to observe the intended bid of the winner, but only a lower bound. One could possibly argue that a second-price sealed-bid auction in the first stage would suit our purpose better. With this configuration the unobservable final bid problem is avoided. However behavior in sealed bid

[4]For a good review of this problem in bargaining games see Chapter 4, specially pages 258-274 in the Handbook of Experimental Economics by Kagel and Roth (1995).

Session	Treatment	Exchange rate	Paying Periods	Players	Location
1	COMP	20	30	16	UPF
2	COMP	20	30	16	UPF
3	INC	25	30	16	UPF
4	INC	25	35	16	UPF
5	ENG	20	30	16	UPF
6	COMP	20	30	16	Bonn
7	COMP	20	30	16	Bonn
8	INC	25	30	16	Bonn
9	INC	25	30	16	Bonn
10	ENG	20	30	16	Bonn

Table 1: Summary of sessions

auctions usually presents large deviations from equilibrium, even without resale. Thus it is not sure one can separate the effect of resale from the other factors which are pushing behavior away from equilibrium[5]. On the other hand the English auction is widely studied and subjects seem to understand the Nash equilibrium and follow the predicted strategies quite closely.

We conducted 10 experimental sessions, with 16 participants in each. For the first 5 sessions, subjects were undergraduate students, mainly from the faculties of law and economics, at the Universitat Pompeu Fabra in Barcelona. The next five sessions were conducted at the University of Bonn, with subjects from many faculties. The analysis finds no consistent differences[6] between the two groups, so the data are pooled together.

At the beginning of the experiment the participants were divided in two subgroups[7] of 8 and then the players in every subgroup were randomly rematched every period in groups of 4. Subjects did not know what group they have been assigned to or who are the other members of the group. There were 31 periods in almost[8] every experimental session. The first period was a practice period that did not count for the players' payoffs and was not used

[5]For results in sealed bid experiments see for example Kagel et al. (1987). Despite the mentioned problems, testing a second price sealed bid auction remains an interesting idea for the future. Additionally these experiments would allow a test of the theoretical result of revenue equivalence of the English and sealed-bid auctions under complete information in the resale stage (for a proof of revenue equivalence see Haile 2003).

[6]A Mann Whitney U test could not reject the hypothesis that behaviour was the same at the 0.05 level.

[7]This was done for statistical purposes, in order to have two independent observations in every session. Still the subjects did not know this and they thought they were being rematched with another 15 players. So the probability they will try to induce cooperating behaviour and the interperiod effect should remain small.

[8]There was one session with two practice periods, but they did not seem necessary so subsequent sessions had only one. It does not matter for the analysis, as we always use observations after the 9th paying period.

in the statistical analysis of the data. After this period subjects received an initial capital of 150 units of our experimental currency, the drachma. In the following periods subjects were rewarded according to their success and their profits or losses were added to the initial capital. Despite the sometimes quite aggressive bidding, there were no bankruptcies, although two subjects came close. After the end of the game the experimental currency was transformed to euros in a ratio of 25 drachmas per euro in COMP and 20 drachmas per euro in INC. The reason for this difference is that INC is more complicated. Sessions lasted about thirty minutes longer than COMP and we wished to keep average profits per hour constant. Thus average profit in COMP was 10.56 euro and in INC 15.5 euro. Naturally, this difference is not only due to the different exchange rate but due to the different bidding behavior too.

3 Equilibrium predictions

3.1 Complete information - COMP

In this section I compute the symmetric equilibrium of COMP. An important difference with respect to usual auction models, is that in the presence of resale, players have a value v_i (their exogenous private value) for the good and a valuation u_i which is something else, the value she places on winning the auction. The valuation is determined endogenously, as it depends on the outcome in the resale market too. Use values v_i are drawn from a discrete uniform distribution with support $[0, 100]$. Consider the two-stage game COMP played by 4 risk-neutral players for a single indivisible object as described above. Let $y_1^i = \max\{v_j | j \neq i\}$, $i, j \in \{1, 2, 3, 4\}$ denote the highest use value among a given bidder's opponents and let v_{-i} denote the vector of the use values of all players, except i. Let f denote the final price of the game, which is equal to

$$f = \begin{cases} p_2 & \text{, if there was a resale auction} \\ r & \text{, if exactly one bidder participates in the resale auction} \\ p_1 & \text{, if no bidder participates in the resale auction} \end{cases}$$

Following proposition, similar to Theorem 1 in Haile (1999), describes the equilibrium in the first stage.

Proposition 1 *The symmetric bid your value equilibrium for an English auction without re-*

sale is also a Perfect Bayesian Equilibrium bidding strategy when the same auction is followed by a resale opportunity, where the private values of the bidders are publicly announced.

Proof. Suppose bidder i with use value v_i deviates to a bid $\tilde{v} > v_i$, while all other bidders follow the proposed equilibrium strategy and bid their use values. This would change i's payoff only in the event that $\tilde{v} > y_1^i > v_i$. However if this is the case, i would have to pay y_1^i for the object but could only resell it for some price p_2 in the interval $[r, y_1^i]$. In equilibrium the reseller will set $r = y_1^i$ under complete information in the resale market, but this still leaves him with nonpositive expected profit. By bidding v_i, i would have received zero profit with certainty. A similar argument shows that bidder i would not profit by bidding less than v_i. ■

This proposition provides the risk-neutral symmetric Nash equilibrium[9] under complete information in the resale market, but as we shall see later the theorem remains valid under risk aversion and incomplete information. Therefore I will refer to this equilibrium as symmetric[10] Nash equilibrium (SNE). Also, this equilibrium covers the special case of the automated resale market that was actually used in the lab.

A characteristic of the equilibrium that should be noted is, that unlike the simple English auction, bidding your value is not a weakly dominant strategy in the presence of a resale possibility. If the person with the highest use value were to deviate from equilibrium and bid less than his value, it is clear that the best response for the others would be to bid up to this highest value (see Section 5 for an extensive discussion).

3.2 Incomplete Information - INC

In treatment INC the theoretical prediction is the same as in COMP. The argumentation is similar to the one above. The only difference is that since private values do not become common knowledge in the resale stage, the reserve price has to be calculated in a more

[9] Note that we treat the game as a second price sealed bid auction. The equilibrium we find is the equilibrium for the last stage of the English auction too, where only two bidders remain, as the previous 2 exits do not carry any important information that alters the players' strategies.

[10] It is worth noting here that in discrete value English auctions there exists an asymmetric equilibrium too where one player bids his value and the other bids her value minus one increment. In the non payoff relevant questions we asked after our experiments, some subjects actually reported playing such a strategy! However a few players employing such a strategy do not significantly influence our analysis.

complicated way using the information that the signals (bids) in the initial auction have given us. However, independently of these signals the reserve price has to be higher than the private value of the first stage winner. This makes sure that in equilibrium we do not have resale and thus bidding one's value remains an equilibrium strategy in the initial auction. More formally:

Proposition 2 *Let bidders in INC have following pure strategy:*

i) In the first stage player i bids her value, ie $b_i = v_i$

ii) if i *wins in first stage she sets a reserve price* $r_i \in [v_i, 100)$

iii) if i *loses in first stage she participates in the second iff* $v_i \geq r$

iv) bidders in second stage bid their values, ie $b_i = v_i$

All strategies of this kind are equilibrium strategies in INC.

Proof. The proof is similar to Proposition 1 and is omitted. ■

4 Experimental Results

In the following I present the general results and in the subsequent sections I offer explanations for the data. When making the statistical analysis of the results I will start with period 10 unless otherwise stated. The reason for this is that in almost all sessions there was a small number of subjects who indicated problems understanding the game, up to period 9 in the worst of these cases[11]. Additionally, this assumption of learning taking place before the 10th period is confirmed by the data.

The main question I was posing, is if resale possibilities alters behavior in auctions. The answer from the data seems to be a definite *yes*. In Figure 1 I compare bidding in the first stage of the three treatments, using simple boxplots[12] which include all but the winning bids[13].

[11] See also Kagel et al (1987), p. 1286 where the authors claim "subjects' adjusting to experimental conditions argue for throwing out the first three auction periods" or Fehr/Schmidt (1999) who only use last period values.

[12] More detailed ones follow in the individual analysis of each treatment.

[13] Keep in mind that the exit price of the last bidder is not equal to the maximum bid he was prepared to make, because he exits automatically once the last-but-one bidder exits the auction. As a consequence we

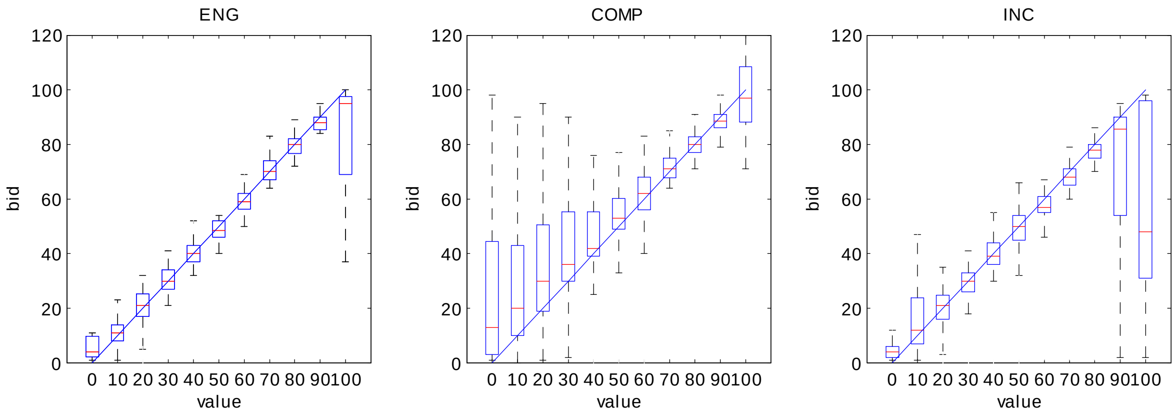

Figure 1: Series of boxplots of private values vs exits in the various treatments. Each box drawn represents the distribution of the bids for a block of values. The length of the box represents the interquartile range (IQR). The whisker extends from the box to the most extreme data value whithin 1,5 times the IQR.

We can see that although the three games have equilibria with the same bidding functions in the first stage, actual bidding behavior is quite different. The mere presence of a resale market makes subjects deviate much more from the equilibrium than in the simple English auction. The deviation is confirmed in Table 2, where we see that bidding in the simple English auction is significantly different to behavior in the treatments with resale (COMP, INC)[14]. This result shows that studies of auctions should take resale possibilities explicitly into account.

This is not the only interesting result. The specific structure of the resale market makes a difference for the bidding strategies. We see in Figure 1 that in COMP, when subjects have common knowledge of the private values before the second stage, resale gives the low value

only have a lower bound on the actual bidding strategy of these players. For the graphs and other statistics we exclude the winning bids. Although it leads to some bias, including them leads to an even higher bias. Techniques such as censored regressions do not completely eliminate this problem and they introduce new ones, e.g. they would rely on the restrictive assumptions that bidding is symmetric and follows some particular functional form.

[14]Bidding in COMP is significantly different from the bidding in the treatment ENG (without resale). Comparing INC with ENG we do not always have statistic significance. This can be attributed to some reasons. First, we do not have many observations for the simple English auction. Most previous experiments however, have found bidding which is very close to the bid-your-value equlibrium, and including these experiments we would get a significant difference between ENG and INC. The second problem is that for high values, where bidding differs most from the equilibrium , in INC, we have a strong problem of unobservable bids. We tried to control for this by running a censored regression of bids on values including data from ENG and INC. The dummy and interaction terms were highly significant, which supports the hypothesis that bidding in ENG and INC was indeed different.

Treatments	Values 0-20	21-40	41-60	61-80	81-100
COMP - SNE	-20.23	-13.80	–5.69	-1.12	1.91
	(0.000)	(0.000)	(0.006)	(0.083)	(1)
INC - SNE	-4.47	-0.99	2.16	7.97	20.25
	(0.000)	(0.404)	(0.0404)	(0.000)	(0.000)
ENG - SNE	-8.03	-2.70	-0.27	-0.40	5.73
	(0.004)	(0.004)	(0.150)	(1)	(0.000)
COMP - INC	-15.75	-12.81	-7.85	-9.09	-18.33
	(0.003)	(0.001)	(0.015)	(0.003)	(0.008)
COMP - ENG	-12.20	-11.10	-5.42	–0.71	–3.81
	(0.072)	(0.008)	(0.072)	(0.808)	(0.153)
INC - ENG	3.55	1.71	2.43	8,38	14.52
	(0.682)	(0.153)	(0.808)	(0.004)	(0.153)

Table 2: Differences in average deviations (private values minus bids), calculated excluding the censored observations. The numbers in parentheses are the p values of a Mann Whitney U test.

types an incentive to overbid.[15] On the other hand, low value types bid close to their values in INC and ENG. High value types bid close to their values in COMP and ENG but not in INC. In general, bids in COMP are highly significantly different from bids in INC for all possible values.

Note that the underbidding of the high types in INC is not a spurious phenomenon due to the censoring of winning bids or to a presence of extreme observations driving the average down. A Mann Whitney U test shows that the percentage of bidders with use values greater 50 who bid less than 20 in INC are with 5.4% highly significantly more than the 1.2% in COMP. Comparing high value bidders who bid less than 50 we get an even higher difference with 17.97% versus 6.23% respectively.

Theoretically the only difference between the two treatments was in the informational background of the resale stage. Naturally it is possible that differences in the bidding strategies of the subjects are not only due to the theoretical difference, but also due to the different mechanisms used in practice. In particular there is evidence from bargaining games where subjects do not behave "rationally" and split the surplus in ways that do not follow the Nash prediction. In COMP I did not allow subjects to deviate, enforcing on them exogenously the predicted outcome of the second stage. In INC this was not possible and as a consequence subjects were allowed to play the resale game themselves. This difference in the mechanisms

[15] In this paper we use the term overbidding/underbidding loosely, to describe bids higher/lower than a subject's use value, even when such bids are not necessarily irrational.

	COMP	INC	ENG
Average Observed Price	67.04	56.85	57.97
Average Equilibrium Price	60.41	60.99	60.68
Average Pr. Value	50.37	51.34	50.68

Table 3: Average Prices, Equilibrium Predictions and Private Values. The Difference between COMP/INC and COMP/ENG are highly significant

used could be a problem, however the data about the rationality in the choice of reserve prices and in the choice of participation presented in Section 4.2 indicates that subject's behavior was fairly competitive. Any deviations from optimal behavior in the resale stage are probably not due to fairness concerns but can be attributed to other effects[16].

The difference in strategies between treatments translates into different prices in the auction. As we can see in Table 3, average prices in COMP were almost 18% higher than in INC and 15% higher than in ENG, whereas the average private values were very similar, as happened with the average equilibrium prices too. This difference is not only highly significant[17] but also quite large and economically important. Observe that the average highest value in every auction was about 80 so revenues in COMP were almost halfway between the Nash prediction and the maximum rents the seller could possibly appropriate. Prices in ENG were slightly higher than in INC but the difference is not so big and not significant. It should be noted that prices in both are a bit lower than the predicted ones though[18].

In the following I analyze these results in more depth individually for every treatment and I will compare a variety of models of bounded rationality that could explain them.

[16]For instance, there is evidence that subjects cannot calculate difficult equilibria. As a matter of fact setting an optimal reserve price given your beliefs is a fairly complex task even for theorists.

[17]P values of a Mann Whitney U test using the 8 independent observations of COMP and INC and the four of ENG are 0.002 for the INC-COMP comparison and 0.008 in the case of COMP vs ENG.

[18]The results regarding the simple English auction should be received a bit carefully as we do not have many observations. I did not run many experiments, as there exists already a very large literature on simple English auctions. Thus, for comparison purposes I refer to these older results too.

4.1 Complete Information - COMP

Figure 2 graphs average[19] prices in the initial auction, average resale prices[20] p_2 and SNE predictions - which as shown in Proposition 1 are equal to second highest values in the group- over time, for the pooled data of all sessions of COMP. There were differences between individual sessions but the general tendency to overbid was the same in all of them, so it is not necessary to present individual session data. Table 2 reports mean deviations from the

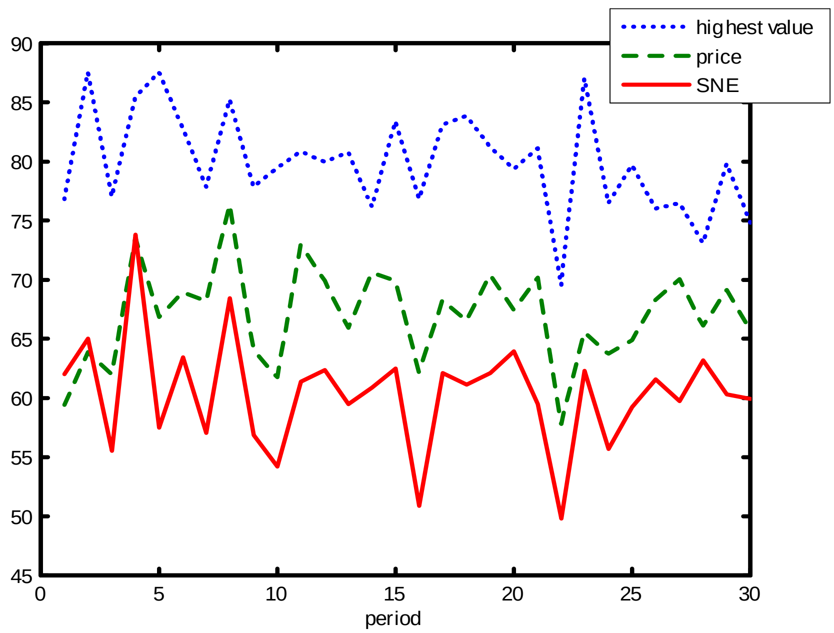

Figure 2: Mean prices, highest values and SNE predictions over time in first stage of treatment COMP.

SNE predictions pooled over all sessions of treatment COMP.[21]

In treatment COMP some underbidding is observed in the first few periods. As explained before, we can view these periods as adaptation periods. In the next periods mean prices in the initial auction lie always over the theoretical prediction, sometimes substantially so.

[19]In every period we take the average over the four groups that were formed in every experiment.

[20]Recall the resale price of the good is automatically equal to the highest value in group. Thus, the profit of the initial auction winner is just the difference between the highest value and the initial auction price.

[21]Following Harrison (1989) it seemed important to use two metrics to measure the deviation. Metric 1 is the usual metric, measuring the deviation in the message space of the auction, ie the deviation in the bids. Metric 2 measures the deviation in the payoff space. This measures the incentives the subjects have to play the equilibrium strategy, or alternatively how high is the cost of deviation. In this table only the results for Metric 1 are presented, as the statistical significances are quite similar using Metric 2.

Nonetheless, resale occurred in about 25.6% of the cases and mean resale prices are still higher than the initial auction prices, so the winners in the initial auction realize positive profits on average.

A closer view of individual bidding behavior, the box plot of values versus exits in Figure 3 can be very informative. Note that if the SNE prediction were valid, all bids should lie on the 45-degree line through the origin. In this plot the overbidding is even clearer than if we only look at auction prices, especially if we compare bids in this plot with the bidding in ENG or INC. We also see that the high auction prices come almost entirely from the overbidding of the low value players. In fact low value players overbid 40% of the time and in about 83% of the cases where they do so, the highest value bidder does not win the auction. But since there were 4 bidders in every group, overbidding did not necessarily lead to winning. Thus, players who did not have the highest value could keep overbidding without obvious punishment, as they did not win the auction very often and when they won their profits were not very low[22].

The persistent excess of bids and prices above the equilibrium predictions has to be compared with the results of ENG and the previous results in the literature, like the English auction experiment in Kagel et al. (1987). In this study the fast learning and eventual convergence to the equilibrium predictions was attributed partly to the negative profits of subjects who started by overbidding in the first periods. This effect, pushing subjects towards equilibrium behavior does not exist in sessions 1 and 3 and was very weak in sessions 2 and 4.

Thus subjects were not always best responding to the other bidders, but they were still choosing strategies that yielded payoffs close to their best response payoffs. In Section 5 I present models that allow for this kind of behavior, by assuming that subjects do not play pure best responses but have a mixed strategy, choosing a probability for an action depending on its expected payoff. I show that such a model rationalizes overbidding as a response to high value players underbidding with a positive probability and explains behavior better than the SNE.

[22] Mean profit of bidders who did not have the highest value but won was 0.77 over all periods and -0.20 in the last 15.

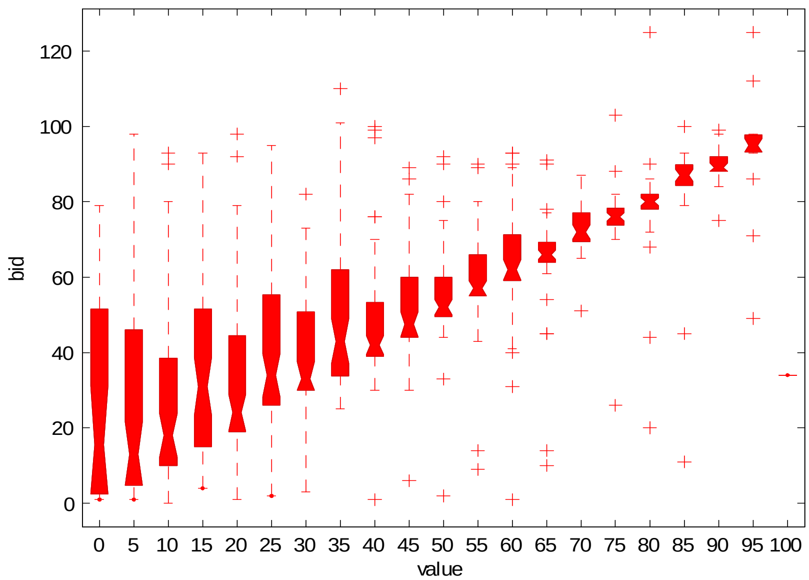

Figure 3: Box plots of values (x axis) versus exits (y axis) in all sessions of treatment COMP. The length of the box represents the interquartile range (IQR). The whisker extends from the box to the most extreme data value whithin 1,5 times the IQR. The crosses represent outliers outside this range. The middle of the notches is the mean, and the extent of the notches graphs a robust estimate of the uncertainty about the means for a box to box comparison.

4.2 Incomplete Information - INC

Figure 4 graphs average prices in the initial auction, average highest values and SNE predictions over time, for treatment INC. Table 3 reports mean deviations from the SNE model's predictions pooled for all sessions of treatment INC.

As in treatment COMP we observe some learning in the first periods and then behavior tends to stabilize. We do not observe any big differences from session to session of treatment INC. It has to be noted that in this treatment the asymmetric information in the resale stage makes a richer strategic behavior possible. In particular signaling can be expected to play a significant role, so that looking just at prices or at aggregate values is less informative and a closer look on individual bidding behavior should be more revealing. Still there are some important facts to notice in Figure 4. The most obvious is that the overbidding from COMP has virtually disappeared. In most periods we even have underbidding. As we can see in Tables 2 and 3 this underbidding is small, but statistically significant. This underbidding

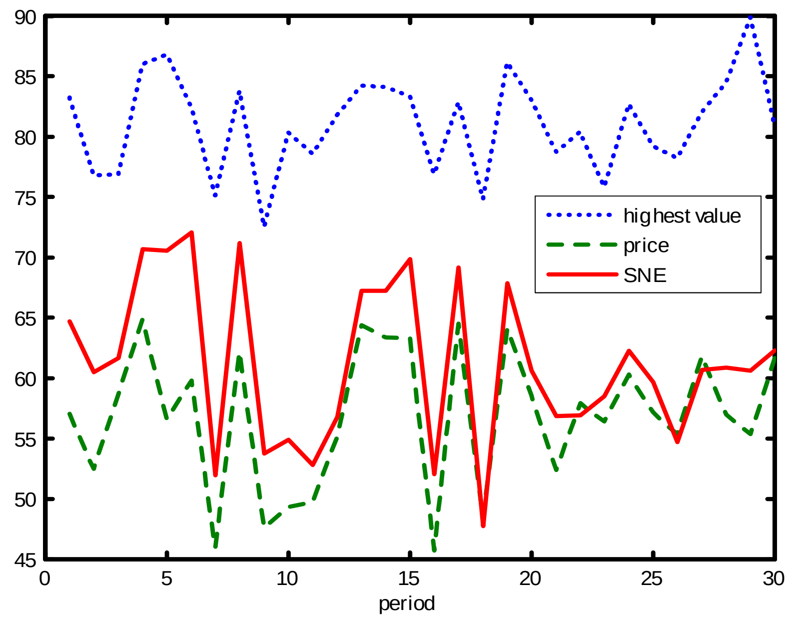

Figure 4: Mean prices, highest values and SNE predictions over time in first stafe of treatment INC.

for the high types means the highest value player loses the initial auction in about 13.4% of the cases. In the resale stage, prices are always higher than the prices in the initial auction, but still sometimes lower than the second highest value. In these cases it has to be that the subject with the second highest value does not participate in the resale auction or that she exits this auction before her use value has been reached. Measuring the rationality of subjects' behavior in the second stage is warranted.

To this end I have prepared two rationality indices. The first, RatR, is a measure of the optimality of the reserve prices in the resale auction. The optimal reserve price depends on the beliefs of the subjects and the beliefs depend on the signals from the initial auction, so it is impossible for us to calculate the optimal reserve price and deviations from it without knowing the subjects' beliefs. However we can expect that when all subjects are rational, the seller has to set a reserve price that is weakly higher than her use value for the good[23].

[23]Note that if one of the subjects is not rational setting a reserve price becomes even more complex. The reasons for such irrationality vary. For instance it is possible and it was observed in some extreme cases that subjects do not participate in the auction when the reserve price is 10 units lower, or less, than their use value. This can be due to fairness considerations. Subjects may find an offer unfair if it gives them a small part of the available rent.

RatR measures the percentage of sellers who choose a reserve price $r > v_i - 3$. This index varied a bit in the four sessions. In the first session of INC it ranged from 0.5 to 1 with no trend to disappear over time. In session 2, RatR was higher and time had an effect. While in the first 10 periods it mostly ranges from 0.5 to 0.75 in the next periods it is always between 0.75 and 1 and the average is 0.88. In sessions 3 and 4 RatR was quite high, at 0.97 after the 10th period in both treatments. It is not clear why some subjects set reserve prices with such errors. The seller has her use value as an outside option and she should ask for this value at least, if others are rational. However as noted above there are subjects who think it is not interesting to participate in an auction where the starting price is below their value but very close to it, so maybe setting these reserve prices was a rational response to this behavior. A second explanation is that subjects just did not understand that when calculating the optimal reserve price they should think about their use values[24]. In an experiment such as this one, which was arguably more complex than usual auction experiments, such mistakes could occur, so one might think more learning periods are necessary. However this argument is contradicted by the second index I calculated.

RatC, measures the percentage of the subjects who chose not to participate in the resale auction, despite the fact, that their use value was higher than the reserve price. Apart from very few mistakes in the early periods, subjects' behavior according to this index was 100% rational. This result is encouraging and suggests that probably the low RatR figures are also not due to miscalculation of the profits, but deliberate choices.

Figure 5 shows boxplots of values versus bids. The stark contrast to COMP becomes clear. Low value players bid close to their values with a small tendency to overbid, while high value players greatly underbid. Furthermore there are some cases, many more than in COMP, where subjects bid 0 or very close to it. These characteristics of the bids can also be explained with the anticipation of noisy bidding or signalling, as we shall see in the next section.

[24]In the questions subjects had to answer following the reading of the instructions, a number of subjects had answered the questions about second stage profits wrongly. Despite the efforts of the instructors to make these points clear after observing these mistakes, it could be the case that some players mistakenly thought the profit from the first period to be their outside option and set a reserve price that was just higher than this number but possibly lower than their value.

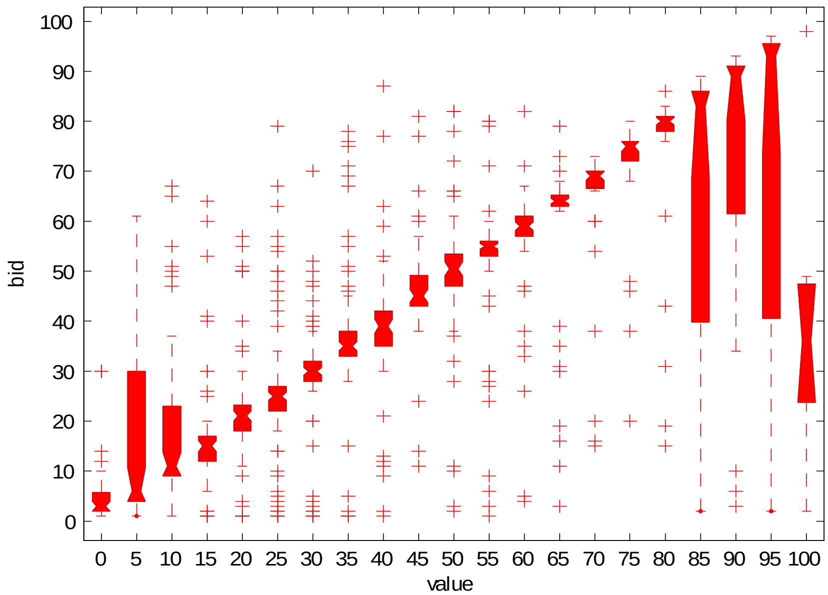

Figure 5: Box plot of values (x axis) versus exits (y axis) in all sessions of treatment INC. The length of the box represents the interquartile range (IQR). The whisker extends from the box to the most extreme data value whithin 1,5 times the IQr. The crosses represent outliers outside this range. The middle of the nothces is the median, and the extent graphs a robust estimate of the uncertainty about the means for a box to box comparison.

5 Bounded rationality and noisy decision making

In this section I present a variety of models of bounded rationality which are prominent in the literature, to explain subjects' behavior. Before proceeding I will first show that common explanations for overbidding in other auction experiments, like first-price auctions (see Cox et al. 1992), cannot explain the data. Consider "joy of winning", meaning that a player's utility is increased by a fixed amount if they manage to get the object and realize profits in the auction. A pure joy-of-winning model predicts the same absolute value of overbidding, for all private values, unless the joy of winning is somehow correlated with use values. However, as we saw in the previous section low value bidders bid much higher than their values, whereas high value players' bids are very close to their values. More evidence against this hypothesis is that in the simple English auction, no overbidding is observed after the initial learning periods.

A second explanation, used for example in Kagel et al. (1987), is risk aversion. If subjects are risk averse they could value the higher probability of winning, when bidding above their values, more than the loss in their expected profit. In English auctions with no resale the equilibrium is, as noted before, in dominant strategies, so risk aversion does not induce different behavior. In COMP risk aversion *alone* does not change the equilibrium predictions. However risk aversion *combined* with some noise in the bidding (explained in the following section) could be a factor influencing the results.

Another motive for low value players overbidding is spite, as has been found for example in Andreoni et al. (2007). The authors gave subjects information about other bidders' values in second price private value auctions. When players have a low chance of winning due to a low value, but know that some other player has a high value they sometimes tend to overbid in order to lower the winner's earnings. The authors observe that when subjects get more information about others' value, this behavior becomes less risky and overbidding is more frequent. Note that if we model the auction as a series of stages (see Milgrom, Weber 1982), entering a new stage every time a bidder leaves the auction, this behavior is compatible with individual rationality. The SNE equilibrium described in Section 3 is unique only in the last stage of the auction when there are only two bidders left, while in the earlier stages other equilibria are possible. Of course any equilibrium arrives to the same outcome regarding the identity of the winner and the auction price.

While this explanation seems plausible it cannot account for the entire amount of overbidding observed. The first reason is that in simple English auctions with no resale, the extent of overbidding in early stages is much lower although the risk from overbidding is exactly the same as in INC and COMP. Secondly, in the last stage of the initial auction in COMP when only two bidders are left, overbidding is indeed less risky than in ENG, but unlike the previous stages it is still not part of any symmetric equilibrium strategy and can lead to negative profits[25]. It seems there is a need for more complex explanations, and in the following we propose some that consider noise.

[25] In fact it can be part of an asymmetric equilibrium, where everyone bids his value except for one player who overbids. This strategy however is very risky. If for example there are two spiteful players with low values and they both overbid, the winner of the two will suffer a serious loss.

5.1 Complete information in the resale market - COMP

Let us start from the basic observation that the Nash equilibrium of the games we tested is not robust to noisy behavior, that is, it is not robust to small perturbations of the bidding strategies. Human players make mistakes and anticipate *others* to make mistakes. In general, as has been shown for example in Goeree et al. (2002), adding noise to the equilibrium bids can shift subjects' best responses quite radically. It remains to be seen if the same argumentation can be used in the present experiment. In the complete information treatment, if the other players use the SNE strategy, the expected payoff functions of a subject contemplating a deviation from this strategy are, as we shall see, broadly the same as in a simple English auction with no resale. I will show however that if there exists some kind of noisy behavior, which means that subjects make errors when choosing their bids, the payoff functions are quite different.

The following graphs in Figure 6 plot expected profits, depending on one's bid, in the case of a simple English auction (ENG) and an auction with resale (COMP). There are 4 curves plotted in every figure, representing expected profits calculated for use values of, 20, 30, 40 and 50. The upper left graph represents expected profits in ENG when three opponents bid their values without any noise, averaged over every possible value of the opponents. Notice that a bid is a best response given a use value, if it lies at the point where the expected profits reach their maximum value. In this case, payoff is maximized when a player bids her value. For example, the curve drawn for a signal of 50 reaches its highest value exactly for a bid of 50. In the upper right figure I calculate the expected profits, again given that other subjects bid according to the Nash equilibrium but adding a normally distributed noise to these bids. This means that an opponent with a value of v is assumed to bid $v + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$ and $\sigma = 9$. I proceed to calculate by numerical simulation the expected profit functions of a player facing three opponents employing this noisy Nash bidding[26]. Bid-your-value is still

[26] I calculate this by independently drawing 2 million sextuples of private values v and errors ε for the three opponents. For every opponent I obtain the noisy bid $\tilde{b} = v + \varepsilon$. I then calculate for every possible bid b_i of player i the winning frequency given this bid and the mean highest bid and highest value of her opponents, conditional on the highest bid $\max\{\tilde{b}_{-i}\}$ being lower than b_i. A player's expected profit is then caclulated for any given private value v_i as

$\Pi_i = prob\{b_i > max\{\tilde{b}_{-i}\}\} E[\max\{v_i, v_{-i}\} - \max\{b_{-i}\} | b_i > max\{\tilde{b}_{-i}\}]$

The numerical simulation is helpful because we can calculate these functions for any noise specification a best response. This is to be expected, as bid-your-value is a weakly dominant strategy in English auctions, ie. an optimal strategy regardless what others do.

The lower graphs depict the same for COMP. The utility functions without noise look quite similar to the ones of ENG. Expected utility has now a lower bound, equal to zero[27], but is maximized at exactly the same points as in ENG. This corresponds to proposition 1, which says that bidding-your-value is the equilibrium strategy in COMP. However when we add noise in the way described above, we find that the curves change dramatically, as is evident in the lower right figure. For every signal we have two local maxima and the position of the global maximum depends on one's value. For use values 20, 30 and 40 it is optimal to bid very high (approximately 83) while for a use value of 50, it is optimal to bid your value.

The existence of two local maxima[28], when opponents bid with some small errors, brings forth a kink in the players' best response functions, as the global maximum jumps at some point from the one local maximum to the other. Note that this discontinuity exists for many different specifications regarding the functional form of the noise distribution (e.g. triangle, logistic, uniform, Laplace) and even when the standard deviation is very small (a standard deviation of $\sigma = 3$ is enough in the case of the normal distribution). The intuition is that the second hump rises higher the more noisy the bids. Even when there is not much noise and the hump is very low, it can still be higher than the first hump, e.g. when private values are low and thus expected utility without resale is very low. This means that low value players should overbid massively even with small amounts of noise.

To test in a systematic way if the characteristics of the game discussed above are indeed

and the accuracy of the method is very high, as I have verified in the cases where an algebraic solution is straightforward to obtain.

[27] If I bid more than my value and I win, given that others bid their values, I can never pay more than the highest value among the other bidders. But this is exactly equal to what I receive in the second stage.

[28] It is interesting to note that the multiplicity of the maxima is the result of the resale opportunity. The expected profit of a first stage bidder is a maximum of two values, expected utility if she consumes the good now and expected profit if she resells it in the second stage. The utility functions graphed are thus the upper envelope of these two utilities. In the right part of these curves, which is common for all use values, the resale effect dominates. In the left part which changes with players' use values, the usual utility enjoyed when she consumes the good herself is dominant. Without noise the utility from resale is zero, as the expected revenue in the second stage equals the expected price in the first auction (both equal the highest value among the other bidders). With noise however this not true anymore, as the expected price in the first stage becomes smaller than expected revenue in the second. This difference is maximized for a bid of around 83 (which is actually higher than the unconditional expected highest value, 75), depending on the size of the errors.

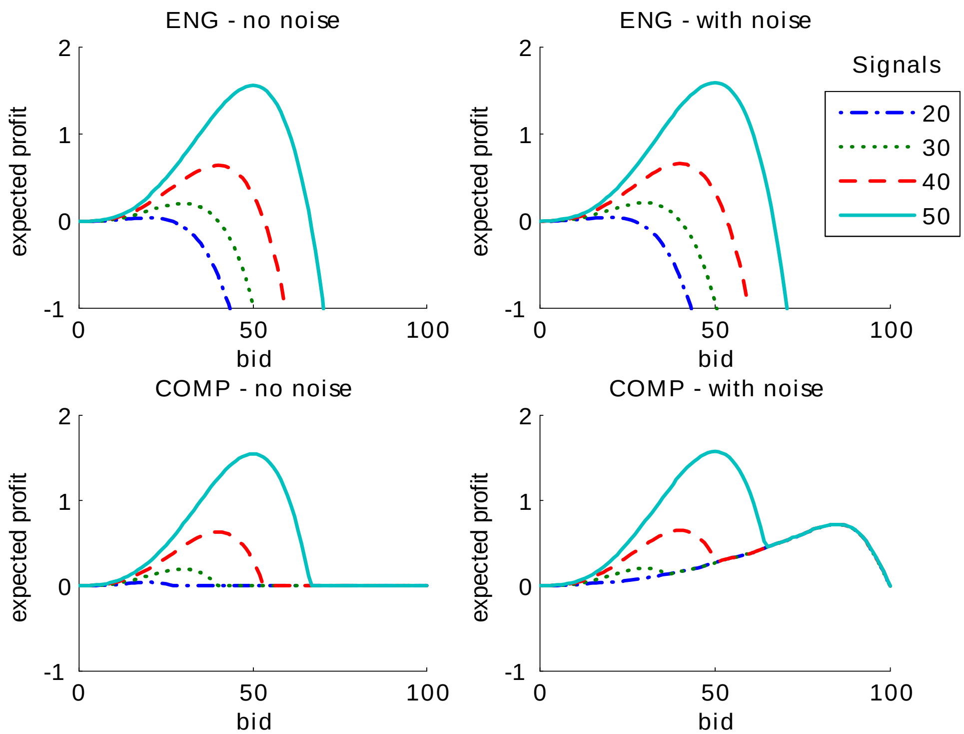

Figure 6: Expected utilites in ENG (upper two figures) and COMP (lower two) without and with noise (normally distriubuted with a σ of 9). The curves are drawn for private use value signals equal to 20, 30, 40 and 50. In the lower two panels all 4 curves overlap for bids higher than ca. 65.

influencing the bidders' behavior we can consider following model. Suppose a player believes her 3 opponents want to bid their values but make small errors, distributed normally[29] with $\sigma = 15$. Then this player's best response[30] given such beliefs is given by:

$$BR_{naive} = \begin{cases} 78, v \leq 47 \\ v, v > 47 \end{cases}$$

An alternative to this model is to calculate the best response to the *actual bidding distri-*

[29]I choose here a σ that is lower than the minimum of the actual estimated standard deviation σ of players' bids in the various sessions of COMP, assuming errors are distributed normally, as can be seen in Table 4. I also tried other distributions and the result was robust to these variations.

[30]The BR and other alternative models we present will be under the assumption that bidders do not update their beliefs after they observe the exits of other players. It does not change the results by much but it greatly simplifies the calculations. Additionally it is confirmed by the data, the main determinant of a player's bid was her use value and the unconditional distribution of the other player's values. Actual observed exits were not a significant factor.

bution[31] (and not to the one predicted by the theory). I find it predicts serious overbidding of the following form:

$$BR_{act} = \begin{cases} 50, v \leq 40 \\ v, v > 40 \end{cases}$$

Using the BR_{act} model I can test the hypothesis that subjects were responding optimally to the actual play of the others. The fit of both these models is presented in table 4

Levels of reasoning A model that has been found to explain many anomalies in experiments is a level of reasoning model (see for example Nagel 1995, Stahl 1995, Camerer 2004). In specific I will use the level-k version (Crawford and Iriberri, 2008). The idea is simple and rather intuitive. There exist k types of players, varying in their degree of sophistication. Level 0 (L0) players bid randomly with a uniform distribution. Their bids can be interpreted as pure noise, given that values do not correlate with bids at all. In this way this version of the levels of reasoning model used in the literature has built-in the idea of noisy behavior. Level 1 players believe they are playing against L0 players and play a best response, Level 2 players play a best response to Level 1 and so on. I first derive the strategy for a Level 1 player best responding to N players who bid randomly. Her expected profit will equal the maximum of her value and the expected highest value among the opponents minus the expected highest bid, given that the latter is lower than her own bid.

$$\Pi_i = \int_0^{b_i} (\max\{v_i, E[\max\{v_{-i}\}]\} - x) N F(x)^{N-1} dx$$

Note that opponents' values and bids are not correlated. Rearranging and taking first order conditions (see appendix) leads us to following strategy for a level 1 player when we have 4 bidders in total and values are uniform in [0,100]

$$b_{L1} = \begin{cases} 75, \text{ for } v_i < 75 \\ v_i, \text{ for } v_i \geqslant 75 \end{cases}$$

[31] To calculate this best response I first estimated a joint bid-value distribution using the data from the experiments. Then I find by numerical simulation the bidding function that maximises a player's expected payoff when playing against 3 opponents who are employing this empirical bidding strategy.

Level 2 types best respond to Level 1. It is simple to show that this results in a bid your value strategy. Given that the opponents all bid at least equal to their values there is no opportunity for profitable resale, thus the game reduces to a simple English auction with no resale and bid your value is an equilibrium[32]. However the expected profit curves are not the same as in the Nash equilibrium, as the probability of winning with a bid lower than 75 is zero[33]. L3 is then exactly equal to the Nash equilibrium, with the same expected payoff functions.

I fit the model assuming that each bid we observe is a draw from a common distribution over the three types. The frequency of L1 players in the population is χ_1, χ_2 is the frequency of L2 players and the remaining $1-\chi_1-\chi_2$ is the frequency of L3. L0 just exists in the mind of L1, as has been found in Crawford and Iriberri's work[34]. For each type we can calculate expected utility for every possible action, given the beliefs of this type. I assume that a player of a certain type makes errors with a frequency that depends on the expected utility of each action, according to a logit specification. This means the probability for a subject i playing a particular action j out of all actions J is calculated in the following way:

$$p_{ij}(\lambda) = \frac{e^{\lambda U_{ij}}}{\sum_{k=1}^{J} e^{\lambda U_{ik}}}$$

The numerator is the utility from each action transformed by an exponential function, in the denominator we have the sum of all these exponential weights as a scaling factor, so the probabilities add up to one. The parameter λ determines how sensitive errors are to payoff differences. Bids become uniform as $\lambda \to 0$ and errors are eliminated when $\lambda \to \infty$.

QRE The last model I calculate is a Quantal Response Equilibrium which captures the idea of noisy behavior but predicts that players' deviations will be systematic. Similar to the level

[32] The best response to a level one player is actually a correspondence and not a function, for private values under 75. Any bid is in principle part of the best response, however I think that bid your value is the obvious focal part of this best response. I accordingly expect L2 players to bid their values.

[33] This will be important when fitting the model to the data using logistic errors, as they depend on the exact structure of the expected payoff curves.

[34] In this paper there is also an alternative specification of the model with truthful bidding as a L0 starting point, but it is not useful here, as it would obviously lead to the L1 type bidding her value and thus no difference with the SNE.

k model, I use a logit specification that has been found to give intuitive theoretical predictions in auctions (see Anderson et al. 1998) and to fit experimental data well (see Goeree et al. 2002). Bidders with a given use value have a probability distribution over every possible bid which depends on their payoff sensitivity parameter λ and the actual payoffs.

Players correctly anticipate the bidding distributions of their opponents and all choose the probabilities according to the rule above. Thus, a best response will be played with a higher probability, but not with certainty. The equilibrium is a fixed point of a mapping from choice probabilities to choice probabilities. Note, that although a QRE approaches a Nash equilibrium in the limit when the noise parameter tends to infinity, it can be far away from it for intermediate values of the parameter.

Calculating a QRE with such a large strategy space is a daunting task. With the usual differential equations approach (used for example by Goeree et al. 2002) it is even considered numerically impossible, to the best of my knowledge, as it involves solving a system of 101 simultaneous non-linear differential equations. Therefore I used a different method to calculate the QRE, namely a Cournot process. Starting with a random bidding function[35] I calculate the expected utility of a player facing three bidders employing such a bidding function. I proceed to calculate a quantal response by weighing the utility to get choice probabilities according to the formula above. This process is then iterated until the quantal responses converge to a stable state. Convergence is usually reached after about 15 periods and does not depend much on the initial bid function.

A question when calculating the QRE is the choice of the parameter λ, which is a measure of sensitivity to payoff differences. A different λ can lead to radically different predictions. Because of this I decided to restrict λ to values found in earlier research with auction data[36], that is to values around 1 and not to include any risk aversion parameter[37]. It so happens

[35]Note that the way we calculate the QRE reveals an interesting relationship to the levels of reasoning model. Every iteration of the Cournot process to find the QRE, corresponds to a level of reasoning.

[36]Note that λ depends on the payoff space and we adjust it accordinlgy. For example, in the Goeree et al (2002) paper where the private values have a support of [0,11] λ is found to be on average 10 (actually they use a parameter μ which is equivalent to $1/\lambda$ and they find $\mu = 0.1$). Then for our auction where values are in $[0, 100]$, we restricted the values of λ to be close to 1.

[37]There has been critique by Haile et. al (2006) that a QRE with two parameters, suitably chosen, can be used to fit any data. This is an additional reason why I did not want to include a risk aversion parameter. Actually however, as Thomas Palfrey and Charles Holt have brought to my attention, this critique does not apply to this analysis, as with the logit structure of the errors I have assumed, the payoff perturbations are

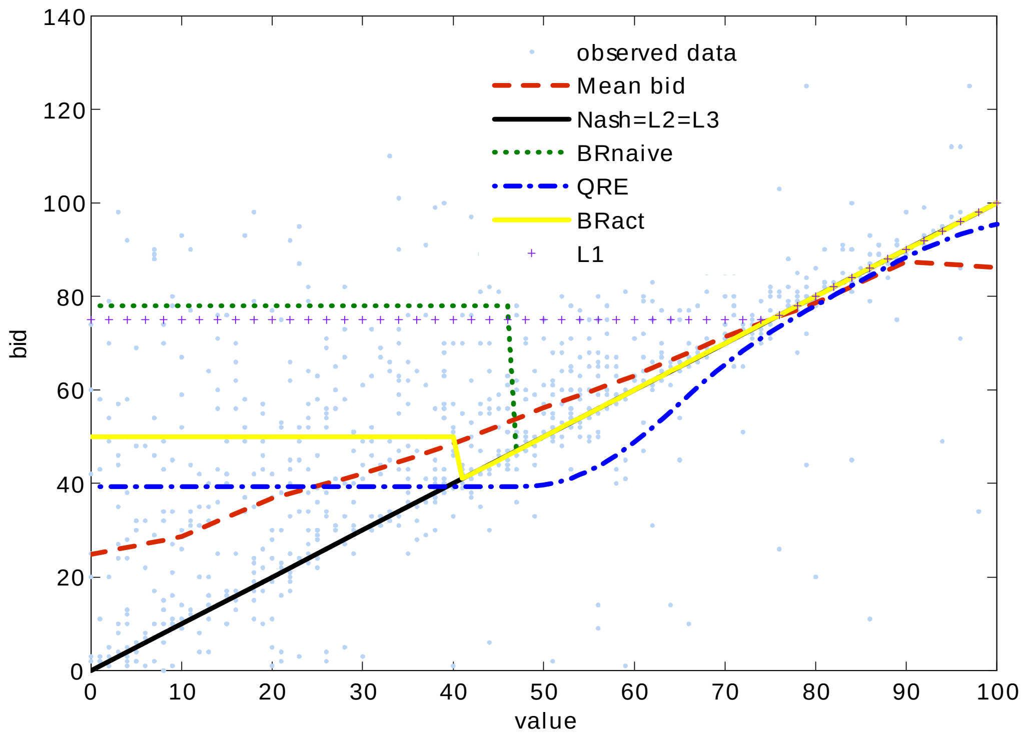

Figure 7: Comparison of the different models for treatment COMP. The QRE predicts a distribution of bids for every use value, so the mean of these bids is presented. Keep in mind however, the model with the best fit is not the one closest to the mean actual behaviour but the one where the whole predicted distribution is closest to the actual one.

that the restriction of λ was not binding and the values estimated in other experiments gave a very good fit for our data too, which indicates that the QRE is an appropriate model to explain behavior in a wide range of auction experiments.

Comparison of models The different models calculated above for treatment COMP are fitted to the data in this section and compared with the same models for treatment ENG. The predictions of the various models in the simple English auction are straightforward. The strategy is actually equal to bid your value for all models except QRE, as this is the weakly dominant strategy in simple English auctions with IPV. Note that unlike in treatment COMP, L0 is now exactly equivalent to bid your value, as the L1 type's payoff is not influenced any

i.i.d. See the aforementioned paper for a discussion.

more by the values of his opponents but just by their bids, which are uniform in L0 and Nash. The QRE predictions are found by simulation, as in the case of COMP.

Results are shown in Figure 7 and the goodness of fit can be found in Table 4. In the first row I present maximized log likelihood values for each model. In the case of the pure strategy models (Nash and BR), where no dispersion is predicted by theory, I allowed for normally distributed errors and the estimated σ is shown in brackets. The QRE predicts a dispersion according to the logit formula presented above, so there was no need for additional errors. For the levels of reasoning models I posit logistic errors as described above and calculate them numerically. I assume that λ is independent of subject or type.[38] Thus in total the model has three parameters, the common precision λ, χ_1 and χ_2.

Lastly I estimate a Nash model but with logistic instead of normal errors, calculated in the same way[39]. This gives us a fairer comparison to the QRE and level k. Since the models are not nested I use the Bayesian Info Criterion (BIC) for model selection. Recall that we have more observations for treatment COMP than ENG so the respective likelihoods are not directly comparable. Note that all models except the level k have just one free parameter.

Model	**Nash**	**BR**$_{naive}$	**BR**$_{act}$	**L1**	**L2**	**L1+L2+L3**	**QRE**	**Nash+logit**
LL COMP	-4469.2	-4988.2	-4457.1	-4564.9	-4479.5	-4204.1	-4225.2	-4243.4
BIC	8945.4	9983.2	8921.2	9136.7	8965.9	8429.0	8457.3	8493.7
Est. λ or σ	20.9	35.1	20.68	0.1	0.23	1.79	1.1	1.14
LL ENG	-2706.2	-2706.2	-2706.2	-2732.4	-2732.4	-2732.4	-2715.1	-2732.4
BIC	5419	5419	5419	5471.4	5471.4	5471.4	5436.8	5471.4
Est. λ or σ	10.38	10.38	10.38	1.13	1.13	1.13	1.1	1.13

Table 4: Goodness of fit of different models for treatments COMP and ENG. LL is the maximised log likelihood.

The model that performs best in explaining the results in COMP is the mixed levels of reasoning model. Nash with normal errors does not fit the data well and BR$_{act}$ (the best response to actual behavior model) performs only slightly better. There is a leap in the likelihood when we estimate the Nash model with logistic errors instead of normal. This

[38] In Crawford and Iriberri's study they compared such a model to models where precisions can be type-specific or subject-specific. Forcing subjects to be of a single type in COMP leads to a very large increase in the number of parameters, without adding much to the fit. Also the estimated population frequencies do not change much. If we assume type-specific precisions we get a LL of 3882.2, BIC=7782.6, a frequency of 25.6% L1 types with precision $\lambda = 0.2483$, 18.22% L2 with $\lambda = 4.07$ and 56.18% L3 with $\lambda > 35$.

[39] Actually note that Nash+logit is exactly the same as the L3 model fitted with logistic errors.

means that the assumption of symmetric errors is not plausible. Still, the QRE with logistic errors performs even better than Nash+logit which shows that logistic errors are not enough to explain the subjects' behavior. Players not only make errors in a systematic way as is modelled through the logistic distribution, but anticipate others to make errors too, which leads them away from the Nash prediction and towards a quantal response equilibrium. These results are reinforced by the fact that the estimated error parameter λ was quite similar for both treatments and for both models, Nash and QRE. However the mixed levels of reasoning model has a slightly lower BIC, despite the fact that it is punished for the higher number of parameters. The estimated frequencies of the types was 4.82% for L1, 11.05% for L2 and 84.13% for L3.

All the models except Nash predict overbidding in the resale treatment COMP. Thus there is no way to separate them based on the qualitative predictions. They differ however in their predictions when we vary the number of bidders in the auction. The levels of reasoning model clearly predicts a monotonic rise in the total amount of overbidding while the QRE predicts an initial rise up to 4 bidders and then a slight fall in parts of the bidding function[40]. In specific, for middle-of-the-range use values, the QRE prediction falls when there are many bidders. Thus an experiment with COMP and 5 or more bidders would help to separate the models.

In ENG all models exhibit a similar performance, which is not surprising given that their predictions are very similar and the greatest difference stems from the different distributions of the errors (logit vs normal). The Nash and BR models have the lowest log likelihood with the QRE a bit worse and the mixed LOR and Nash+logit with still a bit higher LL. Overall however the great improvement in fit given by the last three models in COMP means that the total predictive power of these models is higher. If we use the average performance in the two treatments as a selection criterion, the levels of reasoning model and QRE emerge

[40] The reason for the fall has to do with the feature of the QRE, where strategies with a payoff of zero are played with a positive probability. As the number of players grows the probability of winning with a low to middle bid falls dramatically. Thus the part of the bidding function that gives an expected payoff of zero grows and the bidding distribution for a given value comes close to uniform. Thus while for $n = 3$ the QRE predicts a bidder with a value of 70 to bid on average close to 60, for $n = 8$ she will bid close to the average of the uniform distribution in [0,100] which equals 50.

as clear winners[41].

5.2 Incomplete information in the resale market - INC

As in treatment COMP, the anticipation of noisy behavior can be used to explain the data in INC. Very high value players know they will win with a very high probability. But if they try to win in the first stage they would possibly have to pay a price higher than the second highest value in the group because some low value players can be (relatively costlessly) overbidding. They prefer to signal low values[42] and wait for the second stage auction where they know that overbidding for the low value players is exactly as costly as in a simple English auction and will thus be avoided. Given actual behavior such a strategy would be more profitable than the Nash prediction.

Note that this logic is exactly captured by the logistic errors which allow for the fact that players do not always best respond, but still try to avoid the most costly mistakes. Low value types can costlessly overbid in the first stage but avoid overbidding in the second stage. High value types will not avoid underbidding in INC as much as in COMP or ENG, since in case of losing in the initial auction they can still make some profit in the second stage. The question that arises is which of the previously discussed models employing logistic errors will fit the actual behavior better. The QRE assumes that subjects correctly anticipate the logistic errors of their opponents and arrive at an equilibrium where subjects play noisy best responses to each other. On the other hand the level k model assumes bidders do not think past a limited number of iterated best responses. They intend to play a best response to their opponents, given the beliefs that correspond to their level of reasoning, but make logistic errors. Finally the Nash+logit model just assumes players intend to play the Nash strategies, but make logistic errors.

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[41] Nash+logit is actually equivalent to L3, thus it is nested in the level k model. A likelihood ratio test rejects the hypothesis that the two models are equivalent at the 0.001 level.

[42] Assume there are two biders. Imagine a bidder with value 50 believes the other player is playing the bid your value equilibrium with small symmetric mistakes of maximum magnitude 10, as described in the previous section. He will then bid more than his value, say 60, expecting to resell. If the opponent has a high value, say 90, he will then have an incentive to bid much less. For example if he bids 40 he will lead the winner to believe that he has a value of maximally 50 and he will thus get the good in the second stage for a price of 50. See example 1 in Hafalir and Krishna (forthcoming) for a similar argumentation.

Due to the interdependence of the two stages and the additional complexity, it is not straightforward to calculate the logistic errors and the QRE and level k models for treatment INC. One has to use a shortcut, building a reduced form of the game to make it tractable. I therefore assume that the reserve price in the second stage is equal to the use value of the seller and that players who have a higher value than the reserve, do participate in the auction. I also assume that players who have decided to participate in the second stage auction proceed to play exactly as in a simple English auction, bidding their values. These assumptions are largely consistent with actual behavior and partly with theoretical arguments too[43]. I then plug the expected continuation payoffs from the second stage subgame in the first stage payoffs and this results in a game where the various models' predictions can be calculated[44] as described in treatment COMP. The main difference of this reduced version of INC with COMP is that the winner of the first stage can only expect to get the second highest value among the other bidders in the second, but the losers now have a chance to win in the second stage and appropriate a part of the rent (for example in case they have the highest value, they will get the difference between this value and the second highest). Note that in this reduced game, bid your value is still a Nash equilibrium.

Model	**Nash**	**L1+L2+L3**	**QRE**	**Nash+logit**
LL INC	- 4659.8	-4370.9	-4373.2	-4328.8
BIC	9320.7	8762.8	8753.5	8664.7
Est. λ or σ	18.1	0.98	0.75	1.03

Table 5: Goodness of fit of different models for treatment INC. LL is the maximised log likelihood, BIC is the Bayesian Information Criterion for model selection.

The estimated frequencies for the mixed levels of reasoning mode are 0.0035 for L1, 0.2088 for L2 and 0.7877 for L3, quite close to the values estimated in the previous section for COMP. I find again that the simple Nash model with symmetric normal errors performs very badly. All the models using logistic errors are better by a large factor. As in treatment COMP,

[43] The bidding behaviour I prescribe for the second stage bidders is rational. On the other hand, for the second stage seller setting a reserve price equal to her use value is not an optimal choice. However, when the number of bidders is high enough, the reserve price becomes irrelevant. For example when selling to 3 bidders as in our experiments with values uniformly distributed in [0,1] the expected revenue under the optimal reserve price is around 0.53 and the second highest value (which is the expected revenue without a reserve price) is 1/2. This means the reserve price enhances revenues by not more than 6%.

[44] $\mathbf{BR}_{act}$ is not caclulated as we did not have enough observations in the second stage to estimate the empirical distribution of reserve prices, participation strategies etc.

the QRE and the level k models perform similarly well, with the level k model having a very slight advantage which is however reversed when I calculate the BIC which punishes the levels of reasoning model for its additional free parameters.

These models improve upon Nash mainly by predicting some underbidding for the high types. An alternative reasoning for very low bids is reported in Kamecke (1994). In this study it has been found that some subjects tended to bid very low when they thought they did not have a good chance of winning in order to raise the profits of the winner. In Cox et al. (1982) this tendency for low value holders to throw away bids was argued to have economic sense, once one accounts for subjective costs of calculating a more meaningful bid under the circumstances. However all these arguments can not explain the amount of very low bids in INC. After all in the similar COMP the magnitude of low bids was significantly lower as noted in the beginning of this section.

6 Discussion and ideas for future work

The preceding results show, that when we are interested in real bidding behavior, Nash equilibrium analysis is not adequate. It does not suffice to study the best responses *in* equilibrium to arrive at an equilibrium bidding strategy and work with this as a prediction, as is commonly done. The exact shape of the expected payoff functions is important, as it will influence the errors of players. We find these errors to be asymmetric and systematically depending on the expected payoff of each action. The stability of the equilibrium is important too, that is, we have to study what happens to expected payoffs and best responses when some player *deviates a bit* from equilibrium. In games such as the present, where as we have seen the best responses change dramatically when opponents tremble a bit around the equilibrium, we should not expect subjects to play according to the Nash prediction.

As an extension of this work, in order to test our hypothesis of subjects anticipating mistakes, one could run an experiment where human players face computerised opponents. As computerised opponents do not make mistakes, we should expect very similar behavior in all three treatments. On the other hand it is questionable whether players' behavior when playing against machines allows us useful predictions of how they will play against real humans.

A promising idea for future research is the explicit inclusion of a speculator in the game as in the Garatt and Troeger (2006) paper. This experiment will be very useful to compare with INC and will give us valuable insights to the source of the asymmetric behavior in our data. Other models where resale happens in equilibrium are also interesting, in particular the model of Hafalir and Krishna with asymmetric values seems to be promising and our results hint that the weak players will indeed bid much higher with resale than without, if the resale market is appropriately designed.

Finally, as discussed in the design section, experiments with sealed bid auctions can also be interesting. It would be additionally useful to design these experiments in a way that makes the results comparable with the results of the empirical study in Haile (2001), which has found evidence of the effect of resale markets on US Forest Service timber auctions. As already mentioned, independent work of List et al. (2004) has run first-price sealed bid experiments and compared them with these timber auctions. They seem to have found a significant presence of risk aversion in the data. While this seems like a plausible explanation, it is very likely that the combination of risk aversion with noisy behavior can enhance their results.

7 Conclusions

In the resale treatment under complete information it seems we have a case similar to the "ten little treasures" in Goeree and Holt (2001). The simple English auction represents the "treasure treatment", the case where Nash theory seems to work perfectly, predicting subjects' behavior with a very high accuracy. When we change the game a bit, adding the resale opportunity, the Nash equilibrium remains the same, prescribing that players should bid their values. Nonetheless, subjects seem to see a difference where theory does not see one. Players significantly overbid in the presence of a resale opportunity, under complete information in the resale market, and that this overbidding does not tend to fade away with the passage of time and the effect of learning.

However when there is no complete information in the resale market, the results are quite different. Subjects with low values tend to bid a bit more than their values, whereas high value bidders bid much less than their values. This indicates that instead of the usual separating equilibria there is pooling[45], high value players pretend to have smaller values and expect to get a better offer in the resale market.

In both cases the addition of the resale opportunity alters the strategic behavior of the subjects significantly in comparison to common results in simple English auctions. In most cases these changes in the bidding behavior lead to substantial differences in the revenues that accrue to the initial seller. For policy prescription purposes these findings should be taken carefully into account. While some features of a laboratory experiment will probably not apply in real markets (for instance we do not expect real-life investors to have fairness concerns or to display altruistic behavior), others like noisy behavior and the anticipation thereof are surely present and of significant importance. Thus we believe our results to have some external validity.

The second and more general result of this paper is the importance of thinking about noisy decision making and the exact form of expected payoff functions. The three treatments I tested had exactly the same Nash equilibrium, but subjects' behavior was quite different in each one of them. I argued that the reason for this is the presence of errors (even small ones suffice) on behalf of some players. These errors can be attributed to experimentation with different strategies, trembling, idiosyncratic preferences and moreover, not adequately modelled liquidity constraints in the case of auctions in the field. Errors and noise are present even in the most important financial markets where the stakes are very high (see Shleifer and Summers 1990). In cases where the anticipation of such errors on behalf of some players does not alter best responses by much, the Nash prediction can be valid. However, in cases as the present experiments where best responses are sensitive even to small amounts of noise, we should not expect rational human subjects to follow the Nash equilibrium strategies. Additionally, I find that whatever the reason for subjects making errors, they systematically try to avoid the most costly ones, thus the shape of the payoff functions is a good indicator for the empirical distribution of players' errors.

[45] For a similar pooling effect see Haile (2000).

A Appendix

Derivation of level 1 bids for treatment COMP (all values in the interval [0,1]).

$$\Pi_i = \int_0^{b_i} (\max\{v_i, E[\max\{v_{-i}\}]\} - x) N F(x)^{N-1} dx$$
$$= \int_0^{b_i} (\max\{v_i, \tfrac{N}{N+1}\} - x) N x^{N-1} dx = \int_0^{b_i} N(\max\{v_i, \tfrac{N}{N+1}\} x^{N-1} - x^N) dx$$
$$= [N(\max\{v_i, \tfrac{N}{N+1}\} \tfrac{1}{N} x^N - \tfrac{1}{N+1} x^{N+1})]_0^{b_i} = N(\max\{v_i, \tfrac{N}{N+1}\} \tfrac{1}{N} b_i^N - \tfrac{1}{N+1} b_i^{N+1})$$
$$\Pi_i = \begin{cases} v_i b_i^N - \frac{N}{N+1} b_i^{N+1}, \text{ for } \frac{N}{N+1} < v_i \\ \frac{N}{N+1} b_i^N - \frac{N}{N+1} b_i^{N+1}, \text{ for } \frac{N}{N+1} \geqslant v_i \end{cases}$$

Taking first order conditions:

$$\text{for } \tfrac{N}{N+1} \geqslant v_i \text{ FOC: } N \tfrac{N}{N+1} b_i^{N-1} - (N+1) \tfrac{N}{N+1} b_i^N = 0 \rightarrow b = \tfrac{N}{N+1}$$
$$\text{for } \tfrac{N}{N+1} < v_i \text{ FOC : } N v_i b_i^{N-1} - (N+1) \tfrac{N}{N+1} b_i^N = 0 \rightarrow b = v_i$$

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