An Economic Theory of Fidelity in Network Formation[1]

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Abstract: We study the stability and welfare properties of fidelity networks. These are networks that form in a mating economy with two types of agents (e.g., men and women), where each enjoys having relationships with the opposite type. Having multiple partners is viewed as infidelity, which is punished if detected by the cheated partner. There is female discrimination in that infidelity is punished more severely for women than for men. We obtain a full characterization of pairwise stable or equilibrium networks, which is sensitive to the size of the market. Women, but not men, typically obtain their optimal number of partners. Also, most equilibrium networks are Pareto-efficient, and maximize the aggregate welfare of women, but not men.

We subsequently introduce a new approach to studying information transmission in a network, and use it to fully identify *female-information-biased economies*, which are economies in which women concentrate more information than men in any equilibrium network. We show that female-information-biased economies are segmented, and we obtain an upper bound for the size of each segment. Our results generalize to economies characterized by female-to-male subjugation, and economies of class societies. In the former class of economies, within each couple, the woman is always available to the man whenever he needs her. In the latter, each agent has a distinct social rank and higher-ranked agents are more preferred as partners. We find in particular that each economy of class societies has a unique equilibrium network, and that both classes of economies are female-information-biased. Our findings apply to several two-sided markets in the real world. In particular, we document for the first time in a unified framework the role of female discrimination, market segmentation and economic inequality in gender difference in HIV/AIDS prevalence.

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1 Introduction

We study network formation in a mating economy with two types of agents (e.g., men and women). Each agent enjoys having direct relationships with the opposite type. However, having multiple partners is viewed as infidelity or disloyalty, which is punished if detected by the cheated partner. Infidelity after detection is punished more severely for women than for men. This assumption of female discrimination results in each woman desiring fewer partners than each man. We call *fidelity networks* those networks that arise in this environment.

Our goal in this paper is threefold. First, we study the static predictions of the fidelity model using the concept of pairwise stability. In particular, we characterize pairwise stable fidelity networks, which also makes it possible to describe their possible structures and configurations.[3] Second, we analyze the efficiency of these networks, and examine the relationship between stability and efficiency.[4] Third, we introduce a new approach to studying information transmission in a network, which we subsequently use to identify female-information-biased economies, which are economies in which women concentrate more information than men in any pairwise stable or equilibrium network.

We extend the fidelity model to two natural classes of mating economies. The first is the class of economies characterized by *female-to-male subjugation*. These are economies in which within each male-female relationship, the woman is subjugated to the man in the sense that she is always available to him whenever he needs her. As we will see later, this constraint of female-to-male subjugation on network formation turns out to be a generalization of the principle that underlies the formation of monogamous and polygynous networks. The second is the class of *economies of class societies*, wherein each agent has a distinct social rank, and higher-ranked agents are more desired as partners. We characterize equilibrium networks and study gender difference in information concentration. For the latter class, we examine a number of relationships, using a comparative statics approach in some cases. In particular, we study the effect of female discrimination on the outcomes of women. We also examine the connection between social rank and the likelihood of concentrating information, and analyze the relationship between inequality and information concentration in the overall economy.

The mechanism driving network formation within our framework- that is fidelity- is of practical relevance to the study of a wide range of mutually beneficial relationships in the real world, including business, social, and intimate relationships. Indeed fidelity lies at the heart of the relationship between a wife and her husband, a soldier and his army, a citizen and his country, or a worker and his employer. But despite its appeal, the promise of fidelity may be broken. For instance, a married man might enjoy having a mistress, just as an employee might sell his employer's information (e.g., R&D programs, etc.) to a competitor for personal gains. Even an exclusive contract may be violated. This occurs for instance when a military firm secretly supplies

[3] We extend our analysis to a dynamic setting in Pongou and Serrano (2009), fully characterizing networks that arise and persist in the very long run. The static and the dynamic approaches turn out to be complementary in theory and in applications.

[4] We consider efficiency for the overall market, and optimality for each side of the market.

identical military equipment to two rival countries, or when a scholar submits the same paper to different journals simultaneously to maximize the chance of it being accepted somewhere. While we assume that such acts of disloyalty are punished, sanctions follow only when the cheated party becomes aware of them. Thus, the cheater has an incentive to engage in them, especially when detection is unlikely. In such an environment where fidelity to partners is important, the analysis of the fidelity model reveals the structure of networks that are likely to arise. An interesting feature of these networks is that *a priori*, an agent does not know his/her partners' other partners, and therefore derives no utility from these indirect links.[5] These networks are also bipartite graphs in nature, with a very specific distribution of links.[6]

Because the sphere of intimate relationships is one in which fidelity is most often invoked, we apply our theoretical findings to shed new light on the possible configurations of sexual networks and on the mechanisms of HIV/AIDS transmission. In this respect, the identification of female-information-biased economies corresponds to a complete characterization of sexual markets in which HIV/AIDS is always more prevalent among women than men.[7] Our findings also highlight the distinct role of sexual market segmentation and economic inequality in gender differences in HIV/AIDS prevalence. We also derive conditions under which economic inequality leads to overall higher HIV prevalence in a society. Further, we examine the connection between social rank and the likelihood of HIV infection, providing the first theoretical foundation to some empirical studies on this topic (Mishra et al. (2007), Fortson (2008)).

The remaining of this introduction is organized as follows. Section 1.1 discusses the relevance of fidelity in social and economic relationships, providing further motivation for our general framework. In Section 1.2, we give an overview of our results on the stability and efficiency of fidelity networks. Section 1.3 summarizes our results on the characterization of female-information-biased economies, and in Section 1.4, we show how these results extend to economies of female-to-male subjugation and economies of class societies. Section 1.5 exposes the applications of these results to HIV/AIDS, and Section 1.6 discusses the contribution of our analysis to the literature. Finally, section 1.7 announces the plan of the paper.

1.1 Fidelity: Relevance and Reality

Fidelity is at the foundation of most types of social and economic relationships. In most traditions, couples exchange vows of love and fidelity to each other during wedding celebration. Citizens pledge allegiance to their country. Soldiers bear faith and allegiance to the constitution of their country and their army. Most work

[5] The extension of our analysis to the case in which an agent's well-being may be affected by indirect links is important, but beyond the scope of this paper.

[6] The use of bipartite graphs in the literature has helped to address a variety of matching problems, and has significantly advanced our understanding of the structure of relationships in many two-sided markets. These markets include for instance the marriage market, the hospital-intern market, the college admissions market, the buyer-seller market, and the employee-employer market (see, e.g., Hall (1935), Gale and Shapley (1962), Roth and Sotomayor (1989), Echnique and Oviedo (2006), Kojima and Pathak (2008), Sotomayor (2003), Kranton and Minehart (2001)).

Note that our findings may also apply to non-fidelity networks, such as those between buyers and sellers, lenders and borrowers, doctoral students and their advisors, or faculty and their departments. In each of these cases, the optimal number of partners for each agent on a given side of the market is usually lower than that for each agent on the opposite side.

[7] As it turns out, this finding is likely to inform the current debate on the role of female discrimination in the higher vulnerability of women to the AIDS epidemic in several societies. In this regard, our analysis yields testable predictions.

contracts stipulate a *prima facie* duty of loyalty of employees to their employers, and the protection of the former by the latter. Finally, even when it is not written explicitly, there is an understanding of fidelity and loyalty between parties in most types of relationships.

Fidelity is also a prime requisite of the good functioning of the above-mentioned relationships. For instance, it is essential in sustaining a marriage relationship. Also, citizens of some countries may not have a dual citizenship.[8] In general, in a competitive economic and political environment, employees or members' loyalty is needed in securing confidential information in such various organizations as firms, governments, intelligence services, military, political parties, research labs, pharmaceutical industries, or financial institutions. In all these cases, improper leaks of key information (regarding technology, R&D programs, marketing strategy, military strategy or tactics, political secrets, and so on) to competitors or to the media by disloyal employees or members are generally harmful.[9]

The promise of fidelity is however often violated. In numerous instances, the media obtains key information on an organization's program from some of its members speaking on "condition of anonymity". In the sphere of intimate relationships, Psychiatrist Frank Pittman argues that 90% of first time divorces in the United States find their root in the infidelity of one or both partners (Pitman (1990, 1999)). A study of a large sample of DNA tests in Australia revealed that 10-15% of children were conceived as a result of an affair (ALRC (2003)), and in the United States, the father was not the true biological parent in 30% of paternity tests conducted by the American Association of Blood Banks (AABB (2003)). Globally, 33 million people are infected with the AIDS virus, and infidelity in sexual relationships is the main driver of this epidemic (UNAIDS (2008)).

Despite playing an essential role in the determination of important social, epidemiological and economic outcomes, the notion of fidelity and how it affects the formation of partnerships among self-interested agents who otherwise have a *prima facie* duty of loyalty to each other have not been studied in the economic literature.[10] The goal of the current study is to begin to fill this gap.

1.2 Overview of the Fidelity Model and Theoretical Results

Our economic environment consists of a finite population of two equal-size exogenously determined sets of individuals, say men and women. Each individual derives utility from the number of direct links with agents of the opposite sex, while engaging in multiple links is considered an act of infidelity, and is punished if detected by the cheated partner. Detection occurs with positive probability, and it is assumed that a woman whose infidelity is detected is punished more severely than a man in a similar situation. These considerations result

[8] Section 31 (a) of the Cameroon nationality code of 1968 states that "Cameroon nationality is lost by any Cameroon adult national who wilfully acquires or keeps a foreign nationality [...]"

[9] The ability to secure employees' loyalty has been identified as a key determinant of a firm's growth and prosperity (Reichheld (1996)); leakage of technological information and its various economic consequences also have been documented (see, e.g., Mansfield et al. (1982), Mansfield (1985), Helpman (1993), Aghion et al. (2001)).

[10] Networks have been used to study a wide variety of topics including job search through contact information (Boorman (1975), Montgomery (1991), Calvó-Armengol (2004), Ioannides and Loury (2004)), purchasing behavior (Frenzen and Davis (1990)), interaction and learning (Kandori, Mailath and Rob (1993), Ellison (1993)), technology diffusion and adoption (Coleman (1966)), HIV/AIDS risk perceptions (Kohler, Behrman and Watkins (2007)), friendship (Jackson and Rogers (2007a)), and community insurance (Fafchamps and Lund (2000)).

in each agent having a single-peaked utility function, which implies that each agent has an optimal number of partners that is finite. Due to gender asymmetry in the punishment of infidelity, this number is strictly greater for each man than for each woman. Let n be the size of the population, and s_m^* and s_w^* the optimal number of partners for each man and for each woman, respectively.

Our first exercise consists of characterizing the *pairwise stable* or *equilibrium networks* of this mating economy. The notion of pairwise stability is based on the assumption that individuals form new links or sever existing ones based on the net reward that the resulting network offers them relative to the current network. In particular, we say that a network is pairwise stable or in equilibrium if (i) no individual has an incentive to sever an existing link he or she is involved in, and (ii) no pair of a man and a woman have an incentive to form a new link while perhaps at the same time severing some of the existing links they are involved in.[11]

We find that the characterization of stable networks depends on the size of the economy as well as the optimal number of partners for men and women. In particular, we distinguish three cases. The first case corresponds to very small economies, the second case to small economies, and the third case to large economies.[12]

In the first case, there exists a unique pairwise stable network in which all men are matched with all women. This case corresponds to a situation in which there is a short supply of partners on both sides of the market, which makes it optimal for each agent to match with all agents on the opposite side. In small economies, we find that a network is pairwise stable if and only if each woman has her optimal number of male partners (s_w^*), while each man is matched with anywhere from no woman to all the women in the economy. In the third case, the characterization result is a bit more complex. We find that a network is pairwise stable if and only if all women obtain their optimal number of partners, except at most $s_w^* - 2$ women, and each man is matched with no more than his optimal number of partners.[13]

We notice that in general, female discrimination in infidelity punishment results in women supplying fewer links than the ones demanded by men, which leads to men competing for women. This competition guarantees that each woman is matched in equilibrium, even if some do not obtain their optimal number of partners. But it may lead to some men being unmatched, especially in small and large economies.

We now turn to the study of the efficiency of fidelity networks, and its relationship to stability. We consider two notions of efficiency, namely *strong efficiency* and *Pareto-efficiency*. A network is said to be strongly efficient if its total value, given by the sum of individual utilities in the network, is maximal. For each network, we also consider optimality for each side of the market.[14] A network is said to be Pareto-efficient if one cannot increase the utility of one agent (by adding and/or deleting links, and/or by redistributing partners) without decreasing

[11] As mentioned earlier, the analysis in the current paper is static. It does not get into the dynamic process of network formation. For its dynamic counterpart, see Pongou and Serrano (2009).

[12] Formally, these three cases correspond respectively to $(1) : n \leq 2s_w^*$; $(2) : 2s_w^* < n \leq 2s_m^*$; and $(3) : n > 2s_m^*$.

[13] Given that $s_w^* - 2$ is at least equal to zero, this result implies that all women obtain their optimal number of partners when $s_w^* = 1$ or $s_w^* = 2$. It is when s_w^* exceeds 2 that at most $s_w^* - 2$ may have fewer than s_w^* partners. When this happens, it is because of an absence of coordination in network formation.

[14] This is defined in terms of male-optimality and female-optimality. More precisely, we say that a network is male-optimal if its total value for men (the sum of men's utilities) is maximal. Female-optimality is similarly defined.

the utility of another agent.

We find that in very small economies, the unique pairwise stable network that exists is strongly efficient and Pareto-efficient. However, in small and large economies, it is possible that no equilibrium network be strongly efficient. This tension however does not exist between stability and Pareto-efficiency. In fact, we find that all pairwise stable networks are Pareto-efficient, except those in which some women are matched with fewer than their optimal number of partners.

With respect to gender-based optimality, we find that in very small economies, the unique equilibrium network that exists is male-optimal and female-optimal. This is uniquely attributed to the fact each agent obtains the maximal possible number of partners in this network. In small and large economies, all equilibrium networks are female-optimal, except those in which there exists at least one woman who is matched to fewer than s_w^* partners. No pairwise stable network is male-optimal. In fact, male-optimality to be achieved requires that each man obtain his optimal number of partners, which would imply that the number of links coming from the women's side strictly exceed the number of links that these women can optimally supply, something that is impossible in equilibrium. One therefore notes a tension between male-optimality and network stability, and an equivalent tension between female-optimality and male-optimality, the unique underlying factor being female discrimination. The effect of female discrimination on women is thus paradoxically positive as it generally guarantees optimality on their side.

1.3 Communication Potential and Female-information-biased Economies

We now turn to the third goal of our paper. We introduce an approach to studying information concentration in a network, and use it to provide a characterization of female-information-biased economies, which are mating economies in which women concentrate more information than men in any equilibrium network. This model of information diffusion answers the following questions:

- If a random agent in a network receives from an exogenous source a piece of information that he/she communicates to his/her neighbors who in turn communicate it to their other neighbors and so on, what proportion of the population will end up receiving the information?
- What is the male-female difference in the proportion of such people?

The answers to these two questions define respectively what we call *communication or contagion potential* of a network, and *gender difference in the communication potential* of a network. Note that information could be of any nature. It could be a piece of news, a financial shock, a power transmission shock, or a random infection shock (such as becoming infected with the AIDS virus due to a random event). This model therefore naturally lends itself to the study of a wide range of phenomena including for instance information diffusion in social networks, "systemic risk" in finance, "cascading failure" in power grids or computer networks, or, in the case of fidelity sexual networks, the spread of diseases such as HIV/AIDS.

We provide a complete characterization of female-information-biased economies. We find that information is more prevalent among women in any equilibrium network of a mating economy if and only if that economy is segmented such that no segment has more than $4s_w^* + 2$ men and women whenever s_w^* exceeds 1.[15] Empirically, a segmented economy may correspond to an economy in which the supply and demand for partners obey to rules that partition the population into pairwise disjoint groups of agents.[16]

An implication of our characterization result is that in a sufficiently large and homogeneous (or non-segmented) economy, female discrimination in infidelity punishment does not necessarily lead to information being more concentrated among women in statically stable networks, except when the level of discrimination is sufficiently high, discouraging any attempt by any woman to cheat.

1.4 Two Extensions

We extend the fidelity model to two classes of mating economies. The first extension is to economies characterized by female-to-male subjugation, and the second is to economies of class societies.

An economy of female-to-male subjugation is an economy in which within each couple, the woman is always available to the man whenever he needs her.[17] Under the assumption that each agent is endowed with one unit of time that he/she splits equally among all his/her partners, female subjugation is formally equivalent to saying that within each couple, the woman invests at least as much time in the relationship as the man. This imposes a constraint on the formation of networks that we take into account in the characterization of pairwise stable networks in this environment. This characterization is similar to the one obtained for economies without constraint, with the only difference that in all pairwise stable networks, no woman has more partners than any of her partners. We find that all economies of female-to-male subjugation are female-information-biased. This is because the number of women who receive a piece of information initially given to a randomly chosen man exceeds the number of men who receive the information when it is initially given to a randomly chosen woman.

A few remarks regarding the predictions of economies of female-to-male subjugation are in order. First, notice that if the optimal number of partners for each woman is 1, an economy under the constraint of female subjugation admits the exact same set of equilibrium networks as the corresponding economy without constraint. This set consists solely of monogamous and polygynous networks in which each man is matched to no more than his optimal number of partners (that is, $s_w = s_w^* = 1$ and $0 \leq s_m \leq s_m^*$). Second, we note that in a mating

[15] A segmented mating economy is here defined as a collection of mutually disjoint mating economies, that is, a collection of economies that operate separately. Each of these economies is called a segment.

Note that in a segmented economy, s_w^* need not be the same in all segments. Our characterization result thus implies that if $s_w^* = 1$ in a segment, that segment can be of any size, but if $s_w^* > 1$, its size must not exceed $4s_w^* + 2$, for the economy to be female-information-biased.

[16] Examples of such economies abound in real life. The sexual market for instance can be viewed as partitioned by factors that govern partners choice such as anti-miscegenation law, religion, geographical distance, family background, biological distance, etc.

In a non-fidelity context, an example of a segmented economy is the academic job market in which job candidates and departments are matched based on academic specialization. In such a market, a holder of a PhD in English Literature cannot be matched with a department of Economics. One therefore can think of such a market as a collection of disjoint small markets.

A mating economy may also be segmented along a discrete time space in which, in each period, an equal number of men and women are on the market, and they are replaced by a new generation of agents in the following period. Each time-segment will be relatively small (because not everybody is on the market at the same time), even though the overall economy will look large.

[17] An implicit assumption here is that if a woman has two partners, both will not need her at the same time.

economy of female-to-male subjugation, women always concentrate more information than men in any network that arises. In a sufficiently large and non-segmented economy without constraint, this is the case if and only if $s_w^* = 1$. These two remarks suggest that the constraint of female subjugation can be viewed as a generalization of the normative principle that governs the formation of relationships in monogamous and polygynous societies, and is therefore far from being restrictive.

Our second extension is to economies of class societies. In these economies, each agent has a distinct social rank, and higher-ranked agents are more desired as partners. We find that each such economies has a unique equilibrium network. This network presents a pattern of matching in which lower class women match with men who are of higher ranks, even though men on the top do not generally match with women on the bottom. We find that all economies of class societies are female-information-biased. An interesting feature of these economies is that an increase in the level of discrimination against women also increases the quality of their matches, as well as their amount of information. We also show that economic inequality leads to higher concentration of information in the overall population if and only if female discrimination is sufficiently severe. Further, we find that there is a positive relationship between an individual's rank and his/her likelihood of concentrating information, and that a woman is more likely to concentrate information than a man of the same rank.

1.5 HIV as Biological Information: Gender, Discrimination and HIV/AIDS

We apply our theoretical findings to understand the role of female discrimination in the gender gap in HIV/AIDS prevalence. Globally, the proportion of women among HIV infected adults was 43% in 1990, but increased to stabilize at 50% in 2001 (UNAIDS (2008)). In sub-Saharan Africa where the AIDS epidemic has caused the most damage, this proportion has grown from 53% in 1990 to 60% in 2007 (UNAIDS (2008)). A recent analysis of Demographic and Health Surveys and AIDS Indicator Surveys, which are household surveys commissioned by the United States Agency for International Development through MEASURE DHS, confirms the higher concentration of HIV/AIDS among women in most developing countries (Mishra et al. (2009)).

Two main hypotheses have been advanced to explain the greater vulnerability of women to HIV/AIDS. The first is the assumption that the male-to-female transmission rate of the AIDS virus is greater than the female-to-male transmission rate (WHO (2003)). The second hypothesis posits anti-female discrimination as a key underlying factor (WHO (2003)).

The argument behind the first hypothesis is mainly speculative, and rests on the claim that women have larger exposed surface area of mucous membrane during sexual intercourse, as well as a larger quantity of potentially infectious fluids than men (WHO (2003)). Furthermore, this hypothesis was tested and invalidated in the African context in two influential studies from Uganda (Quin et al. (2000), Gray et al. (2001)). These studies use samples of monogamous heterosexual, HIV-discordant couples.[18] These couples were identified retrospectively from a population cohort in Rakai, Uganda. Frequency of intercourse within couples and HIV-1 seroconversion in the

[18] An HIV-discordant couple is a couple in which one partner is infected with the AIDS virus and the other partner is not.

uninfected partners were assessed prospectively. Men and women independently reported similar frequencies of sexual intercourse.[19] In the first study, the male-to-female transmission rate of the AIDS virus (12.0 per 100 person-years) was not found to be significantly different from the female-to-male transmission rate (11.6 per 100 person-years). The second study yielded a similar conclusion. The probability of the virus transmission per coital act from infected women to their initially uninfected male partners (0.0013) was not significantly different from the transmission probability per act from infected men to their initially uninfected female partners (0.0009). These findings invalidate the early hypothesis and explanation for gender differences in HIV/AIDS prevalence.[20] It is also the case that in several Western regions with low HIV/AIDS prevalence, women are not significantly more infected than men (UNAIDS (2008)). For these reasons, the focus is being shifted to the second hypothesis which is that of societal discrimination against females.

Female discrimination has been documented in almost every society.[21] There is an increasing interest in its role in the greater vulnerability of women to HIV/AIDS (WHO (2003)), but how discrimination really plays out is still not well understood. India for instance has a long history of pronounced discrimination against women (see, e.g., Sen (1999)), but HIV/AIDS is more concentrated among men than women in this country (Mishra et al. 2009). Perhaps confirming this observation, our study shows that female discrimination does not necessarily lead to HIV/AIDS being more prevalent among women in statically stable networks.[22]

The theoretical identification of female-information-biased economies given earlier basically corresponds to a complete characterization of sexual markets in which HIV/AIDS is always more prevalent among women than men due to female discrimination. An important implication of that result is that in a sufficiently large and homogeneous market, HIV/AIDS will not necessarily be more prevalent among women if the level of female discrimination is not high enough. But as social segmentation emerges, women become more vulnerable. Other markets in which women bear a greater share of the HIV/AIDS burden are economies of female-to-male subjugation[23], and economies of class societies.[24] This means that social and economic inequality is a key factor in the higher prevalence of HIV/AIDS among women. Interestingly, in the latter class of economies, while increasing female discrimination leads to higher HIV/AIDS prevalence among women, it improves the quality of their matches. Economic inequality leads to higher overall HIV prevalence if and only if female

[19] This is an important feature of these data that is generally absent in most data with information on sexual behavior. It seems to reflect that partners were faithful to each other, and thus infected individuals who were initially uninfected contracted the AIDS virus through intercourse with their initially infected partners. This makes it possible to assess gender differential transmission rates.

[20] See also Powers et al. (2008) for a recent review of literature.

[21] See, e.g., Wollstonecraft (1792), Nussbaum and Glover (1995), Sen (1999)). There are several manifestations of the unfavorable treatment of women, whether it is in the household or on the labor market. One of these manifestations, known as the desirability bias, often appears in household surveys where women generally underreport their sexual activity (Fenton et al (2001), Zaba et al (2004), Mensch, Hewett, and Erulkar (2003), Jaya et al (2008)), consistent with the notion that women find it more difficult to admit having experienced sex outside a socially sanctioned relationship (Dare and Cleland (1994)).

[22] Note that Pongou and Serrano (2009) examine long-run predictions, and find that female discrimination results in HIV/AIDS being at least as prevalent among women as among men in the long run. How these findings and our findings complement each other, and combine to produce a more efficient solution to the issue of gender inequality in HIV/AIDS prevalence, are discussed later.

[23] As we have seen earlier, a good example of an economy of female-to-male subjugation is a society in which polygyny is part of the cultural norms.

[24] In an economy of class societies, it is assumed that all factors that determine partners choice such as education, income, age, etc., are aggregated into a single quantifiable factor that determines the rank of each agent. Alternatively, there could be a predominant factor, which constitutes the only thing that people care about. It could also be the case that the market is segmented along religious lines for instance, and within each religious group, there is a class economy.

discrimination is sufficiently severe. Our findings also imply that higher ranked agents are more vulnerable to HIV/AIDS, confirming empirical findings that HIV/AIDS prevalence is higher among richer and more educated men and women (Mishra et al. (2007), Fortson (2008)). In sum, the identification of markets that are prone to greater female vulnerability to HIV/AIDS is likely to inform policies that seek to address this very crucial issue of gender equity.

1.6 Related Literature

A general framework for the study of the stability and efficiency of social and economic networks was introduced by Jackson and Wolinsky (1996). The basic assumption underlying the analysis in this study is that agents form and sever links with each other based on the reward from doing so. A notion of pairwise stability is developed, which allows to predict networks that are likely to form. The benefit to a stable network accruing to each agent depends on the productive value of this network and the allocation rule. A natural and important question answered by the authors is whether there exists an allocation rule that ensures that agents form an efficient network. They find that in general, there exists a tension between stability and efficiency.

Following the seminal paper by Jackson and Wolinsky (1996), Dutta and Mutuswami (1997) further study the relationship between stability and efficiency. Their analysis is based on a strategic form game in which each agent announces a set of other agents with whom he/she wants to form a link, and a link between two agents is formed if both announce it. They define two stability notions, strong stability and weak stability, which they use to predict networks that are likely to form. As in Jackson and Wolinsky (1996), the study of the relationship between stability and efficiency also reveals a tension between the two notions.[25]

Our study is related to these works in that we analyze network formation among agents that trade off the cost and benefit of forming links. We define a notion of pairwise stability that allows for simultaneous link formation and severance, and which differs from the ones proposed in these studies.[26] Our analysis of the relationship between stability and strong efficiency confirms the tension between the two notions as in Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997). A distinctive contribution of our study however is in taking advantage of its underlying bipartite environment to investigate welfare on each side of the economy. We note a tension between male-optimality and stability, but stability agrees with female-optimality in general. We also examine the relationship between stability and Pareto-efficiency, and provide a full characterization of pairwise stable

[25] The basic framework in Dutta and Mutuswami (1997) is similar to the one introduced by Aumann and Myerson (1988). Aumann and Myerson (1988) study a two-stage game in which in the first stage, players form bilateral links, yielding a cooperative structure to which the Myerson value (Myerson (1977)) is applied to determine the payoff to each player in the second stage. Extensions and variants of this game have been considered in Dutta, van den Nouweland and Tijs (1996), Slikker and van den Nouweland (2001a), and Slikker and van den Nouweland (2001b).

It is also important to note that the pioneering works of Aumann and Myerson (1988) and Jackson and Wolinsky (1996) have spawn a number of studies on strategic network formation (see, e.g., Dutta and Mutuswami (1997), Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Jackson and van den Nouweland (2005), Page, Wooders and Kamat (2005), Dutta, Ghosal and Ray (2005), Bloch and Jackson (2007), etc.). Some of these papers use a dynamic framework (see, e.g., Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Ghosal and Ray (2005), etc.), and the focus in some of them is on directed networks, which are networks in which an individual can link to another one without the consent of the latter, such as sending an email or inviting someone to a party (see, e.g., Bala and Goyal (2000)).

[26] Our notion of pairwise stability is however close to the one underlying the analysis of the marriage problem in Jackson and Watts (2002). Also, one can show that the set of stable networks is the same under the definitions of pairwise stability proposed in both our paper and Jackson and Wolinsky (1996).

networks that are Pareto-efficient. We find that such networks always exist, perhaps showing that the tension between stability and efficiency may be sensitive to the notion of efficiency investigated. Another distinctive feature of our study is our focus on fidelity networks, which are networks in which *a priori*, agents do not know the other partners of their partners, and do not gain anything from being indirectly linked to them. These networks therefore have different incentive properties and different applications.

Our study is also related to the relatively new literature on many-to-many matching markets. In general, this literature provides conditions for the existence of equilibrium matchings, assuming a notion of stability (see, e.g., Echnique and Oviedo (2006) and some of the references therein). Our contribution here lies in the fact that within our framework, we do not only prove the existence of equilibrium networks (here using the notion of pairwise stability), but we also provide a complete characterization of these networks. Interestingly, this characterization highlights a feature often noted in the general literature on matching markets- that is a conflict of interest between the two sides of the market in some equilibrium matchings. In our case however, this conflict only favors women, as they typically obtain their optimal number of partners in equilibrium.

The fidelity model has been extended in two ways. Pongou (2009b) generalizes it to multi-ethnic societies, deriving testable predictions for the effects of ethnic diversity on sexual behavior and the spread of HIV/AIDS that he tests empirically. Pongou and Serrano (2009) focus on its long-run predictions, based on a dynamic and stochastic approach. They characterize networks that are visited a positive amount of time in the very long run, and find that in these networks, women always concentrate more information than men. In addition to their methodological differences, our paper and Pongou and Serrano (2009) differ in their scope, as issues pertaining to efficiency or its relationship to stability are not covered in the latter study. Another distinctive feature of our study is in extending the fidelity model to other economies such as the economies of female-to-male subjugation, and the economies of class societies. When it comes to understanding the effect of female discrimination on gender differences in HIV/AIDS prevalence, the findings of the two papers turn out to complement each other. To be more precise, while Pongou and Serrano (2009) show that discrimination does not favor women in the very long run, our study fully identifies the characteristics of mating economies that produce the same outcome in the early stage of network formation. In a context of limited resources to address gender inequity in HIV/AIDS prevalence and its detrimental social and economic consequences, such an identification calls attention to markets that should be prioritized in initial interventions. When considered together, findings from both studies establish priorities as to where to invest these scarce resources, and with what timing and intensity.

Finally, we introduce a simple model to study information transmission and concentration in a network. This model assumes that information travels the network via word-of-mouth or neighbors' contagion, and so does not spread if received by an isolated agent. Some studies on the effect of network structure on the diffusion of information or the spread of certain behaviors make a similar assumption (see, e.g., Pastor-Satorras and Vespignani (2000, 2001), Jackson and Rogers (2007b), Jackson and Yariv (2007), and Lopez-Pintado (2008)). Our study however differs from this literature in at least two important respects. First, our modeling of information diffusion is conceptually different. Existing models generally assume a distribution of connections in the population, and a payoff function whose arguments include an individual's and her neighbors' choice of a certain behavior. Thus each individual faces the choice of adopting a certain behavior, such as buying a new product or not, and this behavior spreads as it is adopted. Our model differs in that it studies "information transmission", not "information adoption." Distinguishing between the two notions is important. Within our framework, an agent who receives information about, say a new product, communicates it to her friends, but we are not interested in whether the latter purchase the product or not. In the same way, an agent who is infected with the AIDS virus infects his/her sexual partners; the latter do not make the choice of becoming infected, and the former may not even be aware of his/her HIV status. This leads to significant differences in the assumptions underlying our different approaches. For instance, we do not make any assumptions on the connectivity distribution of the population, but rely only on the knowledge of the number of components and their size. Another distinctive feature of our model is in studying information transmission in bipartite environments, with a focus on understanding how various network structures affect gender difference in information concentration. In this regard, we view the identification of female-information-biased economies as a contribution. Lastly, our study distinguishes itself by its applications and empirical implications. The application of our model to HIV/AIDS for instance shows how the basic structure of a mating economy yields different outcomes for men and women. In particular, we have theoretically documented for the first time in a unified framework the role of female discrimination, market segmentation and economic inequality in determining gender difference in HIV/AIDS prevalence.

1.7 Plan of the Paper

The remaining of this paper unfolds as follows. Section 2 introduces the model that forms the basis for our analysis. We characterize pairwise stable networks in Section 3. In Section 4, we study the efficiency of fidelity networks. Section 5 introduces a new approach to analyzing the diffusion of information in a network, which we also use to characterize mating markets in which one side always concentrates more information than the other side in any network that arises. Section 6 introduces two extensions of the fidelity model. We discuss and conclude our study in Section 7, and collect all the proofs in Section 8.

2 The Fidelity Model

The economic environment consists of a non-empty finite set of individuals $N = \{i_1, \ldots, i_n\}$ of size n, partitioned into a set of men M and a set of women W, each of equal size. Each individual derives utility from direct links with opposite sex agents, but engaging in multiple links is an act of infidelity, and is punished if detected by the cheated partner. Detection occurs with positive probability. It is assumed that a woman whose infidelity is detected is more severely punished than a man in a similar situation. Networks that arise from this environment are called *fidelity networks*.

2.1 Utility Functions

Let $M * W$ denote the cartesian product of M and W. A network is a subset of $M * W$. Denote by $\mathcal{G}(N)$ the set of all possible networks, and let $g \in \mathcal{G}(N)$ be a network. Since we are dealing with undirected graphs, if $(i,j) \in g$, we will abuse notation and consider that $(j,i) \in g$ (in fact, (i,j) and (j,i) represent the same relationship). Let $i \in N$ be an individual, and $s_i(g)$ the number of opposite sex partners that i has in the network g.[27] The utility that i derives from g is expressed by the following function:

$$u_i(s_i(g)) = v(s_i(g)) - c(s_i(g))$$

where $v(s_i(g))$ is the utility derived from direct links with opposite sex partners in g, and is concave and strictly increasing in $s_i(g)$; and $c(s_i(g))$ the cost of infidelity.

We shall define the cost function more precisely. Let $j, k \in N$ be such that $(i,j) \in g$ and $(i,k) \in g$. Let π be the probability that j detects the liaison (i,k), and c the cost incurred by i if j detects that liaison. Because i has $s_i(g)$ partners, he/she will be detected $s_i(g)(s_i(g)-1)$ times with probability π, incurring an average cost of $s_i(g)(s_i(g)-1)\pi c$. So we define the cost function as:

$$c(s_i(g)) = s_i(g)(s_i(g)-1)\pi c$$

Assuming that i is an expected utility maximizer, he/she will thus maximize the following utility function:

$$u_i(s_i(g)) = v(s_i(g)) - s_i(g)(s_i(g)-1)\pi c$$

We denote the extension of u_i to the non-negative reals as $\overline{u_i}(s_i)$. Without loss of generality, let $\overline{u}_i$ be twice continuously differentiable. The following remark is straightforward:

Remark 1 *(1)* $\exists s^* \in [1,+\infty[$ *such that* $\overline{u}'(s^*) = 0$, $\forall s \in [0,s^*[$, $\overline{u}'(s) > 0$, *and* $\forall s \in]s^*,+\infty[$, $\overline{u}'(s) < 0$.

(2) $\frac{\partial s^*}{\partial c} \leq 0$

Remark 1 implies that u_i is single-peaked. Given that the cost incurred per detection is equal for all individuals of the same type, they have the same optimal number of partners. Further, the optimal number of partners for women is smaller than the optimal number of partners for men because the former are more severely punished than the latter if their infidelity is detected. Note that if s^* is not an integer, then the optimal number of partners will be either the largest integer smaller than s^* $\lfloor s^* \rfloor$ or the smallest integer greater than

[27] That is, $s_i(g) = |\{j : (i,j) \in g\}|$. s_i can also be regarded as a function that maps any network g into the number $|\{j : (i,j) \in g\}|$.

s^* $\lceil s^* \rceil$. We also postulate that for no $s \geq 0$, $u_i(s) = u_i(s+1)$. These considerations motivate the following assumption:

Assumption A1. Denoting by s_m^* and s_w^* the unique optimal integer number of partners for men and women, respectively, we assume that $s_m^* > s_w^*$.

2.2 Mating economies

This section introduces the notion of mating economies. Let i be an individual and $P(i)$ the set of *feasible* partners of i. We assume that $j \in P(i) \Longrightarrow i \in P(j)$.[28]

A *trivial mating economy (* also called *a mating economy)*, denoted $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$, is a population $N = M \cup W$ endowed with a utility profile $(u_i)_{i \in N}$ such that for any $i \in M$ (resp. $i \in W$), $P(i) = W$ (resp. $P(i) = M$).

In any mating economy, all agents have identical utility functions, but the cost of infidelity after detection is greater for each woman than for each man. Under this assumption, a mating economy $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ corresponds to a triplet $(N = M \cup W, s_m^*, s_w^*)$, and conversely, to any triplet $(N = M \cup W, s_m^*, s_w^*)$, one can associate a mating economy $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$.

A *segmented mating economy* $\mathcal{E}$ is a finite collection of pairwise disjoint mating economies. More formally, $\mathcal{E}$ is segmented if $\mathcal{E} = (\mathcal{E}^t)_{t \in I_T}$ where $I_T = \{1, ..., T\}$; $\mathcal{E}^t = (N^t = M^t \cup W^t, (u_i^t)_{i \in N^t})$ is a mating economy; for any $t \neq t' \in I_T$, $N^t \cap N^{t'} = \phi$; and $N = \bigcup_{t \in I_T} N^t$.

In a segmented mating economy $\mathcal{E}$, each economy $\mathcal{E}^t = (N^t = M^t \cup W^t, (u_i^t)_{i \in N^t})$ is called a segment, and can be associated with a triplet $(N^t = M^t \cup W^t, s_m^{t*}, s_w^{t*})$ where s_m^{t*} and s_w^{t*} are respectively the unique optimal integer number of partners for men and women in that segment. We shall pose $|N^t| = n^t$.

A trivial mating economy as previously defined is therefore simply a segmented mating economy that has only one segment. It can also be called a *non-segmented mating economy.*

A segmented mating economy can be thought of as a collection of mating markets that operate separately. An example of such an economy is the academic job market where candidates and departments are matched based on academic specialization. We provided other examples of such economies in the introduction.

2.3 Elements of Fidelity Networks

Let g be a fidelity network. The elements of N are called vertices. A path in g connecting two vertices i_1 and i_n is a set of distinct nodes in $\{i_1, i_2, \ldots, i_n\} \subset N$ such that for any k, $1 \leq k \leq n-1$, $(i_k, i_{k+1}) \in g$.

Let i be an individual. We denote by $g(i) = \{j \in N : (i, j) \in g\}$ the set of individuals who have i as a partner in the network g. The cardinality of $g(i)$ is called the degree of i. If a set A is included either in M or W, then the image of A in the network g is $g(A) = \bigcup_{i \in A} g(i)$.

[28] Note that if $i \in M$ (resp. $i \in W$), then $P(i)$ is included in W (resp. M), but is not necessarily equal to W (resp. M).

We denote respectively by $M(g) = \{i \in M : \exists j \in W, (i,j) \in g\}$ and by $W(g) = \{i \in W : \exists j \in M, (i,j) \in g\}$ the set of men and women who are matched in the network g. We pose $N(g) = M(g) \cup W(g)$.

A subgraph $g' \subset g$ is a component of g if for any $i \in N(g')$ and $j \in N(g')$ such that $i \neq j$, there is a path in g' connecting i and j, and for any $i \in N(g')$ and $j \in N(g)$ such that $(i,j) \in g$, $(i,j) \in g'$. A network g can always be partitioned into its components. This means that if $C(g)$ is the set of all components of g, then $g = \bigcup_{g' \in C(g)} g'$, and for any $g' \in C(g)$ and $g'' \in C(g)$, $g' \cap g'' = \emptyset$.

2.4 Equilibrium Networks

In a society such as the one we are describing, individuals form new links or sever existing links based on the improvement that the resulting network offers them relative to the current network. We say that a network g is pairwise stable or in equilibrium if (i) no individual has an incentive to sever an existing link she is involved in, and (ii) no pair of a man and a woman have an incentive to form a new link while at the same time severing some of the existing links they are involved in.

More formally, given a profile of utility functions $u = (u_i)_{i \in N}$, a network g is pairwise stable with respect to u if:

(i) $\forall i \in N$, $\forall (i,j) \in g$, $u_i(s_i(g)) > u_i(s_i(g \setminus \{(i,j)\}))$

(ii) $\forall (i,j) \in (M * W) \setminus g$, if network g' is obtained from g by adding the link (i,j) and perhaps severing other links involving i or j, $u_i(s_i(g')) > u_i(s_i(g)) \implies u_j(s_j(g')) \leq u_j(s_j(g))$ and $u_j(s_j(g')) > u_j(s_j(g)) \implies u_i(s_i(g')) \leq u_i(s_i(g))$.

We denote the set of pairwise stable networks of a mating economy $\mathcal{E}$ by $\mathcal{PS}(\mathcal{E})$.

3 Existence and Characterization of Equilibrium Networks

In this section, we prove the existence and provide a complete characterization of equilibrium networks. The proof of the existence is constructive, and easily derives from the characterization result. More generally, all the possible configurations of equilibrium networks can be deduced from this result. Also, for simplicity, we state our results only for non-segmented mating economies. In fact, given that a segmented economy is just a collection of pairwise disjoint non-segmented economies, generalizations are straightforward.[29]

We will see that the main characterization result (Theorem 1) is sensistive to the size of the economy. To facilitate its exposition, we state a few preliminary results as lemmas (Lemmas 1-4), each of which characterizes equilibrium networks in an economy of a given size. Lemma 1 below however applies to economies of any size. It says that in a pairwise stable network, no individual has more than his/her optimal number of partners.

Lemma 1 *Let g be a pairwise stable network. Then,* $\forall (m,w) \in M * W$, $0 \leq s_m \leq s_m^*$ *and* $0 \leq s_w \leq s_w^*$.

[29] Note that only one main result in the paper (see, Section 5, Theorem 3) will explicitly appeal to non-trivial segmented mating economies.

The intuition behind the proof of Lemma 1 is that if an agent is linked to strictly more than his/her optimal number of partners in a network, he/she will be better off by dropping some of them to be at his/her optimum, implying that that network is not pairwise stable.

The following lemma characterizes pairwise stable networks in very small economies and small economies, corresponding formally to $|M| \leq s_w^*$ and $s_w^* < |M| \leq s_m^*$, respectively. It says that in very small economies, there exists a unique pairwise stable network in which all men are matched with all women. In a small economy, a network is pairwise stable if and only if each woman is matched exactly with her optimal number of partners, and each man is matched with anywhere from no woman at all to all women in the economy.

Lemma 2 *Let g be a network. (1) is equivalent to (2) and (3).*

(1) g is pairwise stable

(2) If $|M| \leq s_w^*$, then $\forall (m, w) \in M * W, \;\; s_m = s_w = |M|$

(3) If $s_w^* < |M| \leq s_m^*$, then $\forall (m, w) \in M * W, \;\; 0 \leq s_m \leq |M|$ and $s_w = s_w^*$.

We now turn attention to large economies, which formally correspond to $|M| > s_m^*$. The characterization of pairwise stable networks will depend on the optimal number of partners for each woman (s_w^*). In particular, let us consider two cases: $s_w^* \in \{1, 2\}$ and $s_w^* > 2$. The following lemma says that if $s_w^* \in \{1, 2\}$, a network is pairwise stable if and only if each woman is matched exactly with her optimal number of partners, and each man is matched with no more than his optimal number of partners.

Lemma 3 *Assume that $|M| > s_m^*$ and $s_w^* \in \{1, 2\}$, and let g be a network. Then (1) and (2) are equivalent.*

1) g is pairwise stable

2) $\forall (m, w) \in M * W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

According to Lemma 3, each woman is at her peak in any pairwise stable network whenever the optimal number of partners for women does not exceed 2. The following two examples demonstrate that this is no more true when this number exceeds 2. In fact, they show that a woman may have less than her optimal number of partners in this case.

Example 1 *Consider a mating economy in which there are 5 men and 5 women. An agent i's utility function is $u_i(s) = s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = 10$ if $i \in M$ and $c_i = 14$ if $i \in W$. It can be checked that $s_w^* = 3$ and $s_m^* = 4$. Consider the two networks represented by Figure 1-1 and Figure 1-2, respectively. In the first network, each woman has exactly her optimal number of partners and no man has more than his optimal number. No woman therefore has an incentive sever an existing link she is involved in, or form a new link with a man. This network is therefore pairwise stable.*

In the second, one woman (w_1) has only two partners (less than her optimal number) and four women have their optimal number, while no man exceeds his optimal number. Note that men who are not matched to w_1

are already at their peak, and thus w_1 cannot form a new link even if she has an incentive to. Also, no agent has an incentive to sever an existing link he/she is involved in. So this network is pairwise stable as well. This shows that the characterization of Lemma 3 does not generally hold when the optimal number of partners for each woman is 3.

Consider the following example too, where the optimal number of partners for each woman is 4.

Example 2 *Consider a mating economy in which there are 8 men and 8 women. An agent i's utility function is $u_i(s) = s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = \frac{2}{9}$ if $i \in M$ and $c_i = \frac{2}{7}$ if $i \in W$. It can be checked that $s_w^* = 4$ and $s_m^* = 5$. Consider the two networks represented by Figure 2-1 and Figure 2-2, respectively. In the first network, each woman is at her peak and no man has more than his optimal number. And for the same reason advanced for the network Figure 1-1, it is stable. In the second, one woman (w_1) has only three partners (less than her optimal number) and seven women have their optimal number, while no man exceeds his optimal number of partner. For the same reason advanced for the network represented by Figure 1-2, this network is stable as well, showing that the characterization of Lemma 3 does not hold in this case.*

The following result now generalize some features of the two illustrative examples just given. It shows that in large economies, when the optimal number of partners for women is greater than 2: (1) it is possible that a woman obtain fewer than her optimal number of partners in equilibrium. When such women exist: (2) none of them can be unmatched; (3) they all belong to the same component; and (4) their number cannot exceed $s_w^* - 2$ (so, there can only exist a few such women).

Lemma 4 *Assume that $|M| > s_m^*$, $s_w^* > 2$, and let g be a pairwise network. Let $A = \{w \in W : s_w < s_w^*\}$ be the set of women who are matched with fewer than their optimal number of partners in g.*

1) A may not be empty.

2) If $A \neq \phi$, each woman in A has at least one partner.

3) If $A \neq \phi$, there exists a unique component h of g such that $A \subset W(h)$.

4) $0 \leq |A| \leq s_w^* - 2$.

We are now ready to state our main result, which partially derives from Lemmas 1-4, and provides a complete characterization of pairwise stable networks in a mating economy of any size.

Theorem 1 *Let g be a network. (1) is equivalent to (2)-(5).*

1) g is pairwise stable

2) $|M| \leq s_w^* \Longrightarrow \forall (m,w) \in M * W, \ \ s_m = s_w = |M|$

3) $s_w^* < |M| \leq s_m^* \Longrightarrow \forall (m,w) \in M * W, \ \ 0 \leq s_m \leq |M|$ and $s_w = s_w^*$.

4) $|M| > s_m^*$ and $s_w^* = 1, 2 \Longrightarrow \forall (m,w) \in M * W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

5) $|M| > s_m^*$ and $s_w^* > 2 \Longrightarrow \exists A = \{w \in W : s_w < s_w^*\}$ such that:

- $A = \phi \Longrightarrow \forall (m, w) \in M * W,\ 0 \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$

- $A \neq \phi \Longrightarrow \forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) * (M \setminus \bigcap_{w \in A} g(w)) * A * (W \setminus A)$,

  $|A| \leq s_{m_1} \leq s_m^*,\ s_{m_2} = s_m^*,\ 1 \leq s_{w_1} \leq s_w^* - 1,$ *and* $s_{w_2} = s_w^*$.

In addition, if $|M| \geq s_w^*$ *and* g *is pairwise stable, then the number of women who have fewer than their optimal number of partners if at most* $s_w^* - 2$.

Theorem 1 shows that the characterization of pairwise stable networks in an economy depends on its size. The equivalence between assertation (1) and assertations (3)-(4) is just a re-statement of Lemmas 2-3. Assertation (5) is a bit involved and deserves a few explanatory comments. Its main appeal is that it is constructive. It says that in large economies where the optimal number of partners for each woman exceeds 2, it is possible that a woman be matched with fewer than her optimal number of partners (Lemma 4). Denote by A the set of such women. We illustrate the rest of the assertation by resorting to our previous example 1. A can be empty (as shown in Figure 1-1), or non-empty (as in Figure 1-2 where $A = \{w_1\}$). If $A = \phi$ in a network, then that network is pairwise stable if and only if each woman has exactly her optimal number of partners and each man has no more than his optimal number of partners (this corresponds to the first part of assertion (5), and is illustrated by Figure 1-1).

If $A \neq \phi$ in a pairwise network, then: any man has at least $|A|$ women; in particular, there are two types of men: those who are linked to all women in A and those who are not. (a) Those who are linked to all women in A obviously have at least $|A|$ women and at most their optimal number of partners s_m^* (Lemma 1) (in Figure 1-2, these men are m_1 and m_2); (b) those who are not linked to all women in A have exactly s_m^* women (in Figure 1-2, these men are $m_3 - m_5$). There are also two types of women: Those in A and those not in A. (c) Those in A have at least 1 partner (Lemma 4) and at most $s_w^* - 1$ partners (in Figure 1-2, $A = \{w_1\}$); (d) those not in A have their optimal number of partners s_w^* (in Figure 1-2, $W \setminus A = \{w_2, w_3, w_4, w_5\}$). Conversely, any network in which $A \neq \phi$ and that satisfies (a)-(d) is pairwise stable.

We conclude our comments on Theorem 1 by noting that the fact that the number of women who have fewer than their optimal number of partners does not exceed $s_w^* - 2$ in equilibrium simply derives from the characterization of pairwise stable networks in the case where $s_w^* \leq |M| \leq s_m^*$ and the case where $|M| > s_m^*$ and $s_w^* = 1, 2$. In those cases, each woman has exactly her optimal number of partners (Lemmas 2-3). In the case where $|M| > s_m^*$ and $s_w^* > 2$, this result comes from Lemma 4-4.

We end this section by stating the following straightforward result, which says that the set of pairwise stable networks of any mating economy is never empty.

Remark 2 *Let* $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ *be a mating economy. Then* $\mathcal{PS}(\mathcal{E}) \neq \phi$.

The proof of this existence result is constructive. In fact, it is easy for instance to construct a network in which each agent has the maximal number of partners that each woman can optimally have. Such a network is always pairwise stable.

4 Efficiency and Stability

In this section, we study the welfare properties of pairwise stable networks. We appeal to two concepts of efficiency, namely *Pareto-efficiency* and *strong efficiency*, and we also consider optimality for each side of the market. A network is said to be Pareto-efficient if one cannot increase the utility of one agent without decreasing the utility of another agent. A network is said to be strongly efficient if its total value, given by the sum of individual utilities in the network, is maximal. We say that a network is male-optimal if its total value for men (the sum of utilities accruing to men) is maximal. Female-optimality is similarly defined. Those notions are formalized in the following definition.

Definition 1 *Let $\mathcal{E} = (N = M \cup W, (u_i)_{i\in N})$ be a mating economy and g a network.*

1) g is said to be Pareto-dominated by another network g' if for all agent i, $u_i(s_i(g)) \leq u_i(s_i(g'))$, and $u_j(s_{j(}(g)) < u_j(s_j(g'))$ for some agent j. A network that is not Pareto-dominated is said to be Pareto-optimal or Pareto-efficient.

2) g is said to be strongly efficient if $\sum_{i\in N} u_i(s_i(g)) \geq \sum_{i\in N} u_i(s_i(g'))$ for all network g'.

3) g is said to be male-optimal (resp. female-optimal) if $\sum_{i\in M} u_i(s_i(g)) \geq \sum_{i\in M} u_i(s_i(g'))$ (resp. $\sum_{i\in W} u_i(s_i(g)) \geq \sum_{i\in W} u_i(s_i(g'))$) for all network g'.

The following straightforward proposition says that in a mating economy, egalitarian pairwise stable networks have the maximal total value in the set of pairwise stable networks.

Proposition 1 *: Let $\mathcal{E} = (N = M \cup W, (u_i)_{i\in N})$ be a mating economy and g an egalitarian pairwise stable network. Then, $\sum_{i\in N} u_i(s_i(g)) \geq \sum_{i\in N} u_i(s_i(g'))$ for all network $g' \in \mathcal{PS}(\mathcal{E})$.*

Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997) remark that there exists a tension between stability and strong efficiency. The following example illustrates a similar tension in fidelity networks.

Example 3 *The economy has 4 men and 4 women. An agent i's utility function is $u_i(s) = 10s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = 2$ if $i \in M$ and $c_i = 14$ if $i \in W$. It can be checked that $s^*_w = 1$ and $s^*_m = 3$. We will show that no pairwise stable network is strongly efficient. Given Proposition 1, it suffices to provide an unstable network whose aggregate value is strictly greater than the aggregate value of any egalitarian pairwise stable network. Any egalitarian pairwise stable network g in this economy has the aggregate value $\sum_{i\in N} u_i(s_i(g)) = 80$. Figure 3-1 represents such a network. Now consider the unstable network g' where man m_1 is matched with women w_1 and w_2 , man m_2 with woman w_2, man m_3 with woman w_3, and man m_4 with woman w_4 (Figure 3-2). This network is unstable because w_2 has more than her optimal number of partners which is 1. We have $\sum_{i\in N} u_i(s_i(g')) = 82 > 80 = \sum_{i\in N} u_i(s_i(g))$, which completes our illustration.*

Note however that one can find a utility profile implying no tension between strong efficiency and stability. For instance, consider an economy with 4 men and 4 women. An agent i's utility function is $u_i(s) = s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = \frac{2}{9}$ if $i \in M$ and $c_i = 2$ if $i \in W$. We have $s^*_w = 1$ and $s^*_m = 5$. It can be easily shown that all egalitarian pairwise stable networks are strongly efficient.

We are now ready to state our main result on the relationship between efficiency and stability.

Theorem 2 *Let* $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ *be a mating economy.*

1) If $|M| \leq s_w^*$*, then the unique pairwise stable network that exists is Pareto-efficient, strongly efficient, male-optimal and female-optimal.*

2) If $|M| > s_w^*$*, a Pareto-efficient pairwise stable network always exists, but a strongly efficient pairwise stable network may not exist. In addition:*

(i) A pairwise stable network g *is Pareto-efficient if and only if* $\forall (m, w) \in g,\ \ 0 \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$*.*

(ii) No pairwise stable network is male-optimal.

(iii) Any pairwise stable network that is Pareto-efficient if and only if it is female-optimal.

A few comments on this result are in order. When $|M| \leq s_w^*$, we find that the unique pairwise stable network that exists in the economy is male-optimal and female-optimal. This is uniquely attributed to the fact each agent obtains the maximum possible number of partners in this network. When $|M| > s_w^*$, all pairwise stable networks are female-optimal, except those in which at least one woman is matched to fewer than her optimal number of partners. No pairwise stable network is however male-optimal. In fact, male-optimality to be achieved requires that each man obtain his optimal number of partners, which would imply that the number of links coming from the women side strictly exceed the number of links they can optimally supply, something that is impossible in equilibrium. One therefore notes a tension between male-optimality and network stability, the latter notion implying female-optimality in general. There exists an equivalent tension between female-optimality and male-optimality, the unique underlying factor being female discrimination.

5 Communication Potential and Female-Information-Biased Economies

In this section, we introduce a simple model of communication transmission in a network. This model answers the following questions:

- If a random agent in a network receives from an exogenous source a piece of information that he/she communicates to his/her neighbors who in turn communicate it to their other neighbors and so on, what proportion of the population will end up receiving the information?

- What is the male-female difference in the proportion of such people?

The answers to these two questions define respectively what we call *communication or contagion potential* of a network, and *gender difference in this communication potential*. These definitions are stated more explicitly in Section 5.1 below. With respect to the latter notion in particular, if the gender difference in the communication potential of a network is at most equal to 0, which means that the proportion of women who end up receiving the information is at least as large as that of men, we say that the former concentrate more information than the latter in that network.

The notion of communication potential is subsequently used in Section 5.2 to characterize economies in which women concentrate more information than men in any pairwise stable network. Such economies are said to be *female-information-biased*.

This model of communication transmission may apply to several types of networks. In a social network for instance, news (e.g., fashion, information on a new product or movie, etc.) generally spreads from friends to friends by word-of-mouth. Also, people often communicate their emotion to others. The model might also be useful in evaluating the impact of a random communicable shock. For instance, an idyosyncratic event in financial intermediaries can cause the failure of a single entity (e.g., a bank run), which in turn has a cascading effect on other connected entities. This is known in finance as "systemic risk". A similar phenomenon, called is cascading failure, often occurs in power grids and computer networks, generally due to the failure of one node, which subsequently shifts its load to nearby nodes, which become overloaded and fail in their turn, also shifting their load to neighboring nodes, and so on. Finally, in a sexual network, a random infection shock (such as becoming infected with the AIDS virus due a random event) spreads in the network through sexual interactions. In this respect, our result in Section 5.2 on the characterization of mating economies in which women concentrate more information than men in any pairwise stable network corresponds to a complete identification of sexual markets in which female discrimination necessarily leads to higher HIV/AIDS prevalence among women.

5.1 Communication Potential of a network

Let g be a network. Assume that an agent $i \in N$ is drawn at random to receive a piece of information γ that he/she communicates to his/her partners in $g(i)$, who in turn communicate it to their other partners, and so on. Information is thus supposed to travel the network via word-of-mouth or neighbors' contagion. If i is not matched with any agent, the information does not spread. Suppose that with equal probability, $\frac{1}{|N|}$, each agent receives the information.[30] We define the *communication or contagion potential* of g as the expected proportion of agents who will receive the information following its diffusion in the network. We also define *gender difference in communication potential* or *in information concentration* as the difference in the expected proportion of men and women who will receive the information. To formalize these notions, we need a few additional definitions.

Let $i \in N$ be an agent such that $g(i) = \emptyset$. We say that i is isolated in the network g. We abuse language and call $\{i\}$ an isolated component of g, thus consisting only of one agent. We denote by $\mathcal{I}(g)$ and $\mathcal{J}(g)$ respectively the set of isolated and non-isolated components of g. Clearly, the set of components of g $C(g) = \mathcal{I}(g) \cup \mathcal{J}(g)$.

Assume that g is a k-component network, and let $C(g) = \{g_1, \ldots, g_k\}$ be the set of its components. Pose $I_k = \{1, \ldots, k\}$. To simplify notation, we write $N(g_i) = N_i$, $M(g_i) = M_i$, $W(g_i) = W_i$, and $|N_i| = n_i$, $|M_i| = m_i$, and $|W_i| = w_i$ for $i \in I_k$. We associate each component g_i with the number n_i and its bipartite component vector (m_i, w_i), and g with the vector $[(n_i)]_{i \in I_k}$ and its bipartite vector $[(m_i, w_i)]_{i \in I_k}$. Also, if g_i is an isolated component, its associated vector is either $(1, 0)$ or $(0, 1)$.

[30] This could be an instance of becoming infected with the AIDS virus due to a random event.

Denote by $\rho(z,\gamma)$ the status of an agent z with respect to the information γ. We pose $\rho(z,\gamma)=1$ if z has received the information and 0 if he/she has not. For any set $B=N,M,W$, let $\Pr(\gamma|B)=\frac{|\{z\in B:\rho(z,\gamma)=1\}|}{|B|}$ be the proportion of agents who have received the information in the population B. We provide below a formula for the expected value of $\Pr(\gamma|N)$ and $\Pr(\gamma|M)-\Pr(\gamma|W)$, denoted respectively $E[\Pr(\gamma|N)]$ and $E[\Pr(\gamma|M)-\Pr(\gamma|W)]$. We have the following result.

Claim 1: *1)* $E[\Pr(\gamma|N)]=\frac{1}{n^2}\sum_{i\in I_k}n_i^2$.

2) $E[\Pr(\gamma|M)-\Pr(\gamma|W)]=\frac{2}{n^2}\sum_{i\in I_k}(m_i^2-w_i^2)$.

Note that the proof of part 1) of Claim 1 does not use the fact that the network is a bipartite graph. This proof is therefore valid for any network.

Claim 1 provides the foundation for the following definition.

Definition 2 *Let g be a k-component network with the corresponding component vector $[(n_i)]_{i\in I_k}$.*

(1) The communication or contagion potential of g is defined as

$$\mathcal{P}(g)=\frac{1}{n^2}\sum_{i\in I_k}n_i^2.$$

(2) If g is a bipartite graph with the corresponding component vector $[(m_i,w_i)]_{i\in I_k}$, the gender difference in the communication potential in g is defined as

$$\mathcal{F}(g)=\frac{2}{n^2}\sum_{i\in I_k}(m_i^2-w_i^2).$$

We will say that a mating economy is female-information-biased if for any equilibrium network g of that economy, $\mathcal{F}(g)\leq 0$ (that is, in a female-information-biased mating economy, women concentrate more information than men in any equilibrium network). Similarly, a mating economy will be said to be male-information-biased if for any equilibrium network g of that economy, $\mathcal{F}(g)\geq 0$. An economy that is female-information-biased and male-information-biased will be said to be gender-information-neutral.

Now, let us illustrate Definition 2 through the following example.

Example 4 *Consider a population that consists of 10 men and 10 women. Consider the networks g and h represented by Figure 4-1 and Figure 4-2, respectively. Their respective component vectors are $[(19);(1)]$ and $[(13);(6);(1)]$, and their respective bipartite component vectors are $[(9,10);(1,0)]$ and $[(7,6);(2,4);(1,0)]$. One can easily check that the communication potential of these networks is $\mathcal{P}(g)=\frac{362}{400}$ and $\mathcal{P}(h)=\frac{206}{400}$. Similarly, the gender difference in their communication potential is $\mathcal{F}(g)=-\frac{36}{400}$ and $\mathcal{F}(h)=\frac{4}{400}$. Thus g has a greater communication potential than h. This means that if information is the AIDS virus for instance, the expected prevalence of HIV/AIDS will be higher in the former as compared to the latter network. Note however that g and h have the same degree distribution, as the number of partners that each agent has does not change across the two networks. But these networks differ in their structure (here captured by their component vectors), which* *explains all the difference in their communication potential as well as the gender difference in this potential. This highlights the role of network structure in the diffusion of information. To further illustrate this latter point, consider the network h' represented by Figure 4-3. Its component vector and bipartite component vector are respectively* $[(13);(6);(1)]$ *and* $[(7,6);(2,4);(1,0)]$. *It therefore has the same structure as h, and thus $\mathcal{P}(h') = \frac{206}{400}$ and $\mathcal{F}(h') = \frac{4}{400}$. But despite having the same structure and communication potential, h and h' differ in their degree distribution, which again shows that network structure plays a significant and independent role in the diffusion of information.*

5.2 Female-information-biased Economies

In this section, our goal is to characterize female-information-biased economies. To motivate this question, consider an economy of 10 men and 10 women, and assume that agents' utility functions are such that $s_w^* = 2$ and $s_m^* = 4$. Then networks g and h given in Example 4 are two pairwise stable networks of that economy, while network h' is not. We have shown that women concentrate more information than men in g, while they concentrate less information than men in h. This means that this economy is not female-information-biased. Given an economy $\mathcal{E}$, we would like to find necessary and sufficient conditions on $\mathcal{E}$ for $\mathcal{E}$ to be female-information-biased. We will need two preliminary results (Lemmas 5-6).

Let g be a pairwise stable network, g' a non-isolated component of g, and $A = \{w \in W : s_w < s_w^*\}$ the set of women who are matched to fewer than their optimal number of partners. The following lemma gives a formula for the number of women involved in g', and provides a lower bound and an upper bound for the number of men in this component under the assumption that A is empty.

Lemma 5 *Let g be a pairwise stable network such that $A = \phi$, and $g' \in \mathcal{J}(g)$ a non-isolated component of g. Then:*

1) $|W(g')| = \frac{|g'|}{s_w^*}$.

2) $\max(\left\lceil \frac{|g'|}{s_m^*} \right\rceil, s_w^*) \leq |M(g')| \leq |g'| - \frac{|g'|}{s_w^*} + 1$.

Combining items (1) and (2) in Lemma 5 yields a complete identification of pairs $(X,Y) \subset M * W$ such that there exists a matching between X and Y that is pairwise stable and that directly or indirectly connects any two elements of $X \cup Y$.

The following lemma will be needed too. It says that if each non-isolated component of a bipartite (not necessarily pairwise) network g is such that the number of women weakly exceeds the number of men, then women concentrate more information than men in that network.

Lemma 6 *Let g be a network.*

If $\forall g' \in \mathcal{J}(g)$, $|M(g')| \leq |W(g')|$, then $\mathcal{F}(g) \leq 0$.

It is important to remark that the assumption is made only on the non-isolated components of a bipartite graph, but the implication is derived for the entire graph. A good example of a network in which the number of women weakly exceeds the number of men in any non-isolated component is a polygynous network. In such a network, each matched woman has only one partner and each matched man may have more than one partner. Following Lemma 6, women concentrate more information than men in such a network.

The following result provides a complete characterization of female-information-biased mating economies. It says that a mating economy is female-information-biased if and only if the optimal number of partners for each woman is 1 or the size of the economy does not exceed $4s_w^* + 2$.

Lemma 7 *Let $\mathcal{E} = (N = M \cup W, s_m^*, s_w^*)$ be a mating economy. Assertions (1) and (2) are equivalent.*

1) $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$.

2) $s_w^* = 1$ or $n \leq 4s_w^* + 2$.

It is easy to see that if $s_w^* = 1$, then the economy is female-information-biased. This is because if $s_w^* = 1$, then any pairwise stable network in the economy is a polygynous network (Lemma 6). If $n \leq 4s_w^* + 2$, we prove that any possible pairwise stable network g is such that $\mathcal{F}(g) \leq 0$. The intuition behind why these two conditions are also necessary for the economy to be female-information-biased might not be easy to pin down. However we show by contradiction that if those two conditions are violated (that is, $s_w^* > 1$ and $n > 4s_w^* + 2$), then it is always possible to construct a pairwise stable network g such that $\mathcal{F}(g) > 0$ (Lemma 5 is particularly useful to this part of the proof). Note for instance that the economy of 10 men and 10 women such that $s_w^* = 2$ and $s_m^* = 4$ considered in the introduction of Section 5.2 violates these two conditions. This is the reason why it is not female-information-biased.

Note however that if we were to partition the economy just described into two segments, each containing 5 men and 5 women, keeping $s_w^* = 2$ in each segment, then condition $n \leq 4s_w^* + 2$ would be satisfied, and the economy would become female-information-biased. This observation motivates our main result for this section. This result says that a non necessarily trivial mating economy is female-information-biased if and only if that economy is segmented such that in each segment t, there are no more than $4s_w^{t*} + 2$ men and women whenever the optimal number of partners for each woman is greater than 1.

Theorem 3 *Let $\mathcal{E}$ be a (non necessarily trivial) mating economy. Assertions (1) and (2) are equivalent.*

1) $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$.

2) $\mathcal{E}$ is a segmented mating economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t \in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$.

A segment in which each woman can optimally have only one partner uniquely corresponds to a mating economy in which the punishment to female infidelity is sufficiently severe. Note also that all segments need not have the same characteristics. For instance, if an economy has two segments, the optimal number of partners for each woman could be 1 in one segment and 3 in the other. In this case, for women to always concentrate more information than men in any network likely to form in the economy would require that there be at most 14 individuals in the second segment, and any number of individuals in the first one. One therefore sees that segments may have completely different characteristics.

The following example illustrates a sexual market in which segmentation (due to an exogenous factor) increases the vulnerability of women to HIV/AIDS.

Example 5 *Consider a sexual market of 10 men and 10 women, and assume that agents' utility functions are such that* $s_w^* = 2$ *and* $s_m^* = 4$. *The network g , associated with the bipartite component vector* $[(7,6);(2,4);(1,0)]$, *and represented by Figure 5-1 is a pairwise stable network in this economy.*

Now, assume that the economy is segmented due to an exogenous factor, and two segments, each consisting of 5 men and 5 women, result. The first segment contains men $m_1 - m_5$ *and women* $w_1 - w_5$, *and the second segment contains men* $m_6 - m_{10}$ *and women* $w_6 - w_{10}$. *Cross-segment relationships are not possible. The network h, associated with the bipartite component vector* $[(5,5);(2,1);(2,4),(1,0)]$, *and represented by Figure 5-2 is a pairwise stable network of this new economy. Note that the only difference between g and h is that woman* w_5 *has severed her link with* m_6 *in g to form a new link with* m_1 *in h (one can think of this link severance as resulting from market segmentation).*

The gender difference in the contagion potential of g and h is respectively $\mathcal{F}(g) = \frac{4}{400}$ *and* $\mathcal{F}(h) = -\frac{16}{400}$. *Men concentrate more information in g while women do in h. Note that in general, women hold at least as much information as men in any network that is likely to form in the segmented economy, because in each segment t of this economy, we have* $n^t \leq 4s_w^{t*} + 2$ *(in fact,* $n^t = 4s_w^{t*} + 2 = 10$*) (see Theorem 3). Therefore, if information is the AIDS virus, then, market segmentation causes women to bear a greater share of the HIV/AIDS burden than men.*

The second economy of Example 5 shows how in a sexual market, discrimination and segmentation combine to produce networks that do not favor women when it comes to HIV/AIDS or other sexually transmitted diseases. However in a sufficiently large and homogeneous population where segmentation is unlikely as in the first economy of the same example, women do not necessarily bear a greater share of the HIV/AIDS burden than men, despite the presence of female discrimination.

The following example shows the relationship between the level of market segmentation and the maximal market size that guarantees that women concentrate more information than men in any network that arises in the economy.

Example 6 *Assume that in a mating economy, individuals choose their partners based on a set of e criteria* $\{x_1, ..., x_e\}$.[31] *Each criterium* x_i *is a categorical variable with* y_i *categories.*[32] *So, there are* $z = \prod_{1 \leq i \leq e} y_i$ *segments in the economy. Assume that in each segment* $t \in I_z$, *the optimal number of partners for each woman is* $s_w^{t*} > 1$. *Following Theorem 3, the maximal market size that guarantees that women concentrate at least as much information as men in any network that arises in the economy is* $\overline{n} = \sum_{t \in I_z} (4s_w^{t*} + 2)$. *This latter equation shows the relationship between this maximal market size and the level of market segmentation.*

[31] In a sexual market for instance, these criteria would generally include socioeconomic and biological considerations such as income, education, occupation, religion, family background, biological distance, age, height, etc.

[32] Religion for instance would have christianity, judaism, islam, induism, buddhism, etc. as categories.

To further illustrate this relationship, assume in particular that each of the e criteria for partners selection has $y = 5$ categories, and that in each segment of the economy, the optimal number of partners for each woman is $s_w^* = 2$. Then it is clear that the maximal market size that guarantees that the economy is female-information-biased is $\overline{n} = y^e(4s_w^* + 2) = 10 * 5^e$. Figure 6 shows how $\overline{n}$ varies as a function of e. We note in particular that if $e = 1$, then $\overline{n} = 50$, and if $e = 10$, then $\overline{n} = 97,656,250$.

6 Two Extensions of the Fidelity Model

In this section, we extend the fidelity model to two natural classes of economies. The first extension is to mating economies of female-to-male subjugation, and the second is to mating economies of class societies. A mating economy of female-to-male subjugation is a mating economy in which within each couple, the woman is subjugated to the man in the sense that she is always available to him whenever he needs her. A mating economy of class societies is a mating economy in which agents on each side of the market are ranked according to their social class, and higher-ranked agents are more preferred as partners by agents of the opposite type.

6.1 Economies of Female-to-Male Subjugation

Discrimination takes several forms. The type of discrimination uncovered so far postulates only gender asymmetry in the punishment of infidelity. In this section, we additionally assume female-to-male subjugation in the sense that within each couple, the woman is always available to the man whenever he needs her. This assumption, that we denote $(\mathcal{S})$, translates into gender differential time investment in a relationship as follows: Assuming that each agent is endowed with one unit of time that he/she splits equally among his/her partners, $(\mathcal{S})$ is equivalent to saying that within each couple, the woman invests at least as much time as the man. More formally, let (m, w) be a pair of a man and a woman on a relationship, and s_m and s_w their respective number of partners. The assumption that w is subjugated to m is expressed as :

$$(\mathcal{S}): \quad \frac{1}{s_w} \geq \frac{1}{s_m}$$

where $\frac{1}{s_i}$ is the amount of time that each agent $i \in \{m, w\}$ invests in each of his/her relationships.

We define a mating economy of female-to-male subjugation as a mating economy subject to the constraint of female-to-male subjugation $\mathcal{S}$. It is formalized by the 4-tuple $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^*, (\mathcal{S}))$.

The following result provides a complete characterization of pairwise stable networks in a mating economy of male-to-female subjugation. In addition, it shows that such economies are always female-information-biased.

Theorem 4 *Let $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^*, (\mathcal{S}))$ be an economy of female-to-male subjugation and g a network. (1) is equivalent to (2)-(5).*

1) g is pairwise stable

2) $|M| < s_w^* \Longrightarrow \forall (m,w) \in M * W, \;\; s_m = s_w = |M|$

3) $s_w^* \leq |M| \leq s_m^* \Longrightarrow \forall (m,w) \in g, \;\; s_w^* \leq s_m \leq |M|$ *and* $s_w = s_w^*$.

4) $|M| > s_m^*$ *and* $s_w^* = 1, 2 \Longrightarrow \forall (m,w) \in g,\ s_w^* \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$.

5) $|M| > s_m^*$ *and* $s_w^* > 2 \Longrightarrow \exists A = \{w \in W : s_w < s_w^*\}$ *such that:*

- $A = \phi \Longrightarrow \forall (m,w) \in g,\ s_w^* \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$
- $A \neq \phi \Longrightarrow \forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) * (M \setminus \bigcap_{w \in A} g(w)) * A * (W \setminus A)$, $|A| \leq s_{m_1} \leq s_m^*,\ s_{m_2} = s_m^*,\ 1 \leq s_{w_1} \leq |A|$, *and* $s_{w_2} = s_w^*$.

6) In addition, for any pairwise stable network g, $\mathcal{F}(g) \leq 0$.

A few comments on this result are necessary. First, notice that if the optimal number of partners for each woman is 1, the constraint $(\mathcal{S})$ is always satisfied. This implies that any mating economy of female-to-male subjugation $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^* = 1, (\mathcal{S}))$ admits the exact same set of equilibrium networks as the corresponding economy without constraint $\mathcal{E} = (N = M \cup W, s_m^*, s_w^* = 1)$. This set consists solely of monogamous and polygynous networks in which each man is matched to no more than his optimal number of partners. This shows that the constraint $(\mathcal{S})$ is far from being restrictive, and can in fact be viewed as a generalization of the normative principle that governs the formation of monogamous and polygynous networks.

Second, we note that in a mating economy of female-to-male subjugation, women always concentrate more information than men in any network that arises. In a sufficiently large economy without constraint, this is the case if and only if $s_w^* = 1$ (Lemma 7). This again shows that the constraint $(\mathcal{S})$ can be viewed as an extension of the principle underlying the formation of female-biased networks.

Third, one might be tempted to view the set of pairwise stable networks of an economy of female-to-male subjugation as a refinement of the set of pairwise stable networks of the corresponding economy without constraint. But the following example shows that this is generally not the case.

Example 7 *Consider a mating economy of 4 men and 4 women, and assume that agents' utility functions are such that* $s_w^* = 2$ *and* $s_m^* = 3$. *Consider the networks* g *and* h, *associated respectively with the bipartite component vectors* $[(2,1);(2,3)]$ *and* $[(1,1);(1,0);(2,3)]$, *and represented by Figure 7-1 and Figure 7-2. We note that* g *is not pairwise stable if the economy is subject to the constraint* $(\mathcal{S})$ *(because woman* w_1 *has two partners* m_1 *and* m_2, *each of whom has only* w_1 *as partner, thus the constraint* $(\mathcal{S})$ *is violated); but* g *is pairwise stable if the economy is not constrained. As for* h, *it is pairwise stable in the former economy, but is not in the latter (because man* m_2 *and woman* w_1 *have an incentive to form a link). This shows that the set of pairwise stable networks of an economy of female-to-male subjugation is not generally a refinement of the set of pairwise stable networks of the corresponding economy without constraint.*

6.2 Mating Economies of Class Societies

A mating economy of class societies or a class economy is a mating economy in which each agent has a distinct social rank, and higher-ranked agents are more preferred as partners. More formally, a class economy is a 5-tuplet $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ where $\succ_m$ and $\succ_w$ are complete linear orders on M and W, respectively; $\succ_m$ also represents women's preferences for men, and $\succ_w$ represents men's preferences for women.[33] Each agent prefers to be rather matched than unmatched, and the optimal of number of partners is fixed for all agents (s_m^* for men and s_w^* for women of all ranks).

Intuitively, a mating economy of class societies is a two-sided market in which each agent falls into a certain social class, which in turn determines his/her desirability as a partner: the higher in the social hierarchy, the more desired by agents of the opposite type. A real-life example is a sexual market where partners selection is based on income or social class. An example in a non-fidelity context is the academic job market where candidates in a certain field are ranked by potential employers according to the quality of their job market papers, and employers are also ranked by candidates on the basis of the work environment they offer.

6.2.1 Characterization

The following lemma will be needed in the proof of our main result.

Lemma 8 *(Pongou 2008) Let g be a network such that $\forall g' \in \mathcal{J}(g)$, $|M(g')| > |W(g')| \Longrightarrow |M(g')| = s_w^*$, and $|M(g')| \leq |W(g')| \Longrightarrow |M(g')| \geq s_w^*$. Then, $\mathcal{F}(g) < 0$.*

Our main result for this section says that a mating economy of class societies admits a unique equilibrium network, and is female-information-biased.

Theorem 5 *Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a mating economy of class societies.*

1) There exists a unique pairwise stable network $g \in \mathcal{PS}(\mathcal{E}^{\succ})$.

2) - If $|M| \leq s_w^*$, then $\mathcal{F}(g) = 0$.

- If $|M| > s_w^*$, then $\mathcal{F}(g) < 0$.

The proof of the first part is constructive. The unique equilibrium network that exists presents a pattern of matching in which women from lower ranks match with men from higher ranks. When it comes to HIV/AIDS, our findings shed new light on the relationship between economic inequality and gender difference in the prevalence of HIV/AIDS and other sexually transmitted diseases. According to Theorem 7, in the prsence of female discrimination in infidelity punishment, inequality leads higher HIV/AIDS prevalence among women in sufficiently large markets. And here, it seems important to stress that it is inequality, not poverty, that matters. In

[33] On the second interpretation of $\succ_m$and $\succ_w$, note that $\succ_m$ for instance is not a ranking of the subsets of the set of men by women as it is often the case in traditional matching problems; $\succ_m$ is a ranking of individual (or singleton) men by women. For our purpose, we do not need a ranking of the subsets of the set of agents on each side of the market.

fact, if we assume that preferences for partners are determined by income only, in a generalized poverty context, we have a large and homogeneous population; appealing to Theorem 3, we know that networks that arise in such a market are not necessarily unfavorable to women, despite the presence of discrimination.

Let us further illustrate this result through the following example.

Example 8 *Assume a mating economy of class societies* $\mathcal{E}^{\succ} = (N = M \cup W, s^*_m, s^*_w, \succ_m, \succ_w)$ *such that* $M = \{i_1, ..., i_{|M|}\}$, $W = \{j_1, ..., j_{|W|}\}$, $i_1 \succ_m i_2 \succ_m ... \succ_m i_{|M|}$ *and* $j_1 \succ_w j_2 \succ_w ... \succ_w j_{|W|}$. *If* $|M| = |W| = 11$, $s^*_m = 7$ *and* $s^*_w = 5$, *the unique pairwise stable network is the network* g, *associated with the component vector* $[(5,7);(5,4);(1,0)]$, *and represented in Figure 8-1. We have* $\mathcal{F}(g) = -\frac{28}{484} < 0$.

If $|M| = |W| = 11$, $s^*_m = 8$ *and* $s^*_w = 5$, *the unique pairwise stable network is the network* h, *associated with the component vector* $[(5,8);(5,3);(1,0)]$, *and represented in Figure 8-2. We have* $\mathcal{F}(h) = -\frac{44}{484} < 0$.

If $|M| = |W| = 8$, $s^*_m = 7$ *and* $s^*_w = 5$, *the unique pairwise stable network is the network* l, *associated with the component vector* $[(5,7);(3,1)]$, *and represented in Figure 8-3. We have* $\mathcal{F}(l) = -\frac{32}{256} < 0$.

If $|M| = |W| = 9$, $s^*_m = 7$ *and* $s^*_w = 5$, *the unique pairwise stable network is the network* q, *associated with the component vector* $[(5,7);(4,2)]$, *and represented in Figure 8-4. We have* $\mathcal{F}(q) = -\frac{24}{256} < 0$.

Assuming that social class is determined by income, it is interesting to see that in all networks, despite the fact that poor women match with richer men, the poorest women do not match with the richest men in general, unless the level of gender asymmetry in the optimal number of partners is sufficiently high. In this respect, we note for instance that when $s^*_m = 7$ *and* $s^*_w = 5$, *woman* j_8 *matches with men* $i_6 - i_{10}$ *(see Figure 8-2), but when* $s^*_m = 8$ *and* $s^*_w = 5$, *woman* j_8 *now matches with men* $i_1 - i_5$, *who are of course richer than men* $i_6 - i_{10}$ *(see Figure 8-1). This shows that an increase in gender asymmetry increases the quality of the matches of some of the poorest women (see Proposition 2 in Section 6.2.2 below).*

We also note that on both sides of the market, the number of partners that each agent has weakly increases with his/her social rank, more so for men than for women. While Figures 8-1 and 8-2 show that all women have the same number of partners, Figures 8-3 and 8-4 illustrate that the women on the bottom have fewer partners than those on the top. In all four figures, lower ranked men have fewer partners.

Figures 8-1, 8-2, 8-3 and 8-4 also allow us to appreciate some of the difficulties inherent in the general proof of the sign of the function $\mathcal{F}(.)$ *in Theorem 5. We note that in the four figures, there are more women than men in the first component, but more men than women in the second component. It is not therefore possible to appeal to Lemma 6 to figure out the sign of* $\mathcal{F}(g)$, $\mathcal{F}(h)$, $\mathcal{F}(l)$ *or* $\mathcal{F}(q)$.[34] *We however note that the assumptions of Lemma 8 are satisfied in all these cases, which allows to answer the question.*

6.2.2 Comparative Statics

In this section, we are going to conduct two comparative statics exercises. The first one studies the effect of an increase in female discrimination on the outcomes of women. The second studies the effect of economic inequality

[34] It is perhaps important to remind that Lemma 6 helps in the determination of the sign of $\mathcal{F}(.)$ only when the number of women weakly execeeds the number of men in each non-isolated component of a network.

on the level of information concentration. In particular, we find that increasing female discrimination increases the quality of women's matches as well as the concentration of information among them, and that economic inequality leads to higher concentration of information if and only if female discrimination is sufficiently severe. We need a few preliminary definitions.

Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a mating economy of class societies. Let $2^M(s_w^*)$ be the set of subsets of M with cardinality not exceeding s_w^*. Define over $2^M(s_w^*)$ the following binary relation denoted R_M: Let $X, Y \in 2^M(s_w^*)$ be two elements of $2^M(s_w^*)$. We say that X is better than Y, denoted XR_MY, if:

- the cardinality of X is at least equal to that of Y
- and any element of X that is not in Y is ranked higher than all elements of $Y \setminus X$ by $\succ_m$.[35]

Denote by P_M and I_M the strict component and the symmetric component of R_M, respectively. If XR_MY, one can say that a woman who is matched to all men in X has better matches than a woman who is match to all men in Y.

We are now ready to state our result.

Proposition 2 *: Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ and $\mathcal{E}'^{\succ} = (N = M \cup W, s_m^{*'}, s_w^*, \succ_m, \succ_w)$ be two mating economies of class societies such that $s_m^{*'} > s_m^*$. Let g and g' be their respective unique equilibria. Then:*

1) If $|M| \leq s_w^*$, then $\forall w \in W$, $s_w(g')I_M \ s_w(g)$; in addition, $\mathcal{F}(g) = \mathcal{F}(g') = 0$

2) If $|M| > s_w^*$, then $\forall w \in W$, $s_w(g')R_M \ s_w(g)$, and $\exists w_0 \in W$ such that $s_{w_0}(g')P_M \ s_{w_0}(g)$; in addition, $\mathcal{F}(g') < \mathcal{F}(g) < 0$.

The following result states that inequality leads to higher concentration of information in small and large economies if and only if the optimal number of partners for each woman is 1, that is, if female discrimination is sufficiently severe.

Proposition 3 *: Let $\mathcal{E} = (N = M \cup W, s_m^{*'}, s_w^*)$ a mating economy and $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ a corresponding mating economy of class societies.*

1) If $|M| \leq s_w^*$, $\forall g \in \mathcal{PS}(\mathcal{E})$, $\forall g' \in \mathcal{PS}(\mathcal{E}^{\succ})$, $\mathcal{P}(g) = \mathcal{P}(g')$.

2) If $|M| > s_w^*$, $\forall g \in \mathcal{PS}(\mathcal{E})$, $\forall g' \in \mathcal{PS}(\mathcal{E}^{\succ})$, $\mathcal{P}(g) \leq \mathcal{P}(g')$ if and only if $s_w^* = 1$.

When information is the AIDS virus, the first result suggests that an increase in female discrimination increases the quality of women's matches, but it increases their vulnerability to HIV/AIDS as well. The second result says that unless discrimination against women is sufficiently severe, economic inequality does not necessarily lead to higher overall HIV prevalence.

[35] One can show that if X is better than Y, then any element of Y that is not in X is ranked lower than all elements of $X \setminus Y$ by $\succ_m$.

6.2.3 Rank difference and rank-adjusted gender difference in information concentration

In this section, we study the relationship between an individual's rank and his/her likelihood of concentrating information. We also study the difference in the likelihood of concentrating information between a man and a woman of the same rank.

Let us state the problem more clearly. Consider two individuals i and j on the same side of the market. They have different ranks. If an agent z is drawn at random to receive a piece of information that spreads to his/her direct and indirect neighbors in the unique pairwise stable network that exists in the economy, is the higher ranked agent more likely to receive the information than the lower ranked agent?

Now assume that i is a man and j is a woman, and both have the same rank. Is i more likely to receive the information than j?

On the first question, we find that on each side of the market, higher ranked individuals are more likely to receive the information. On the second question, a woman is more likely to receive the information than a man of the same rank.

To state this result more formally, consider an agent i_v of rank v, and denote by $p(i_v)$ the probability that i_v receives the information transmitted exogenously to a randomly selected agent in the unique pairwise stable network that exists in a mating economy of class societies. We have the following proposition.

Proposition 4 *: Let* $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ *be a mating economy of class societies, and* i_v *and* j_v *a man and a woman of rank* v*.*

1) $p(i_v)$ *and* $p(j_v)$ *weakly increase in* v*.*

2) $p(i_v) - p(j_v) \leq 0$*.*

3) $p(i_v) - p(j_v)$ *is non-monotonic in* v *in general.*

Apart from the theoretical interests of Proposition 4, the results also speak to the effects of economic status on HIV/AIDS infection and other sexually transmitted diseases. In this respect, Proposition 4 tells us that richer and more educated men and women are more likely to be infected with the AIDS virus, as a result of their position in the existing sexual network. Also, a woman is more likely to be infected than a man of the same economic status. However, gender difference in prevalence is a non-monotonic function of social rank.

7 Conclusion

We have studied *fidelity networks*, which are networks that form in a mating economy of agents of two types (e.g., men and women), where each agent enjoys having direct relationships with the opposite type, while having multiple partners is viewed as infidelity and is punished if detected by the cheated partner. There is female discrimination, that is, women are more severely punished than men, which results in each woman desiring fewer partners than each man. The characterization of pairwise stable or equilibrium networks is sensitive to the size of the economy. In very small economies ($n \leq 2s_w^*$), there is a unique equilibrium network in which all men are matched to all women. In small economies ($2s_w^* < n \leq 2s_m^*$), women obtain their optimal number of partners in equilibrium, while each man is matched to anywhere from no woman to all women in the economy. Finally, in large economies ($n > 2s_m^*$), each man obtains his optimal number of partners at most, and each woman is matched exactly to her optimal number of partners, except possibly at most $s_w^* - 2$ women.[36] When a woman has fewer than her desired number of partners in equilibrium, this uniquely reflects an absence of coordination in network formation. In a small or large economy, one notes that while a man can be isolated in equilibrium, a woman always find a match. In fact, due to female discrimination in infidelity punishment, women supply fewer links than the ones demanded by men, which leads to men competing for them.

The study of the relationship between stability and efficiency has revealed a tension between stability and strong efficiency when the economy is small or large, but stability agrees with Pareto-efficiency in general. In fact, in small and large economies, all equilibrium networks are Pareto-efficient, except those in which a woman has fewer than her optimal number of partners. In such economies, all Pareto-efficient equilibrium networks are female-optimal, but none is male-optimal. This shows that the tension between stability and strong efficiency exists only for the men's side of the market. It is also important to note that the tension between female-optimality and male-optimality is uniquely rooted in female discrimination, which paradoxically appears to be positive for women, but negative for men.

We have also introduced a new approach to analyzing the diffusion of information in a network, and have subsequently used it to identify female-information-biased economies, which are economies in which women concentrate more information than men in any equilibrium network. We have found that female-information-biased economies are segmented such that in each segment, the population size does not exceed a certain threshold whenever female discrimination is not sufficiently severe. Segmented mating economies abound in real life, and generally correspond to economies in which the supply and demand for partners obey to rules that partition the population into pairwise disjoint groups of agents.

We have extended the fidelity model to two natural classes of economies, namely the *mating economies of female-to-male subjugation*, and the *mating economies of class societies*. The first class corresponds to economies in which within each agreed relationship, the woman is subjugated to the man in the sense that she is always available to him whenever he needs her. The constraint of female subjugation appears to be a generalization of the normative principle that governs the formation of monogamous and polygynous networks. We have found that these economies are female-information-biased.

Economies of class societies are those in which agents have distinct social ranks, and higher-ranked agents are more preferred as partners. Each such economy admits a unique equilibrium network, and is female-information-biased. In these economies, an increase in the level of female discrimination paradoxically increases the quality of women's matches, and their share of information. We have also found that inequality leads to higher concentration of information if and only if female discrimination is sufficiently severe. Also, the likelihood

[36] This implies that if $s_w^* = 1, 2$, no woman will have fewer than her optimal number of partners in equilibrium.

of concentrating information weakly increases with social rank, and a woman is more likely to concentrate information than a man of the same rank.

Apart from the theoretical interests of the analysis conducted in this study, the findings may be used to understand the structure of relationships in some real-life two-sided economies. For instance, the identification of sexual markets that are prone to greater female vulnerability to HIV/AIDS is likely to inform policies that seek to address this very crucial issue of gender equity. In this respect, we have learned about the role of female discrimination, market segmentation and economic inequality in the greater vulnerability of women. Also, the unique equilibrium generated by an economy of class societies captures the reality of age mixing often observed in empirical studies. If age reflects wealth, this equilibrium shows a pattern in which older men match with younger women. Further, the finding that the number of partners and the likelihood of concentrating information increase with social rank provides the first theoretical backing to empirical studies showing that richer and more educated men and women have more sexual partners and concentrate more HIV/AIDS than their poorer and less educated counterparts. In so doing, we call into question the prevailing perception about the role of poverty in the spread of the AIDS epidemic.

Finally, one may think of possible generalizations of the fidelity model. In the current model, agents on both sides of the market can be punished for infidelity. In some markets however, punishment may be only one-sided. For instance, while one may think of a citizen of, say Cameroon, being disloyal to his/her country when acquiring a foreign nationality, one may hardly think of Cameroon being unfaithful to its current citizens when admitting new citizens. In this case, the requirement of fidelity lies only with citizens. In such a market, agents on one side have a finite optimal number of partners, while agents on the other side can have as many partners as possible. One can show that if such an economy is relatively large, agents on the weak side of the market will have their optimal number of partners, while agents on the strong side will have anywhere from no partner to all agents on the weak side in equilibrium. One notes here that despite the absence of coordination in network formation, no agent on the weak side has fewer than his/her optimal number of partners, unlike in economies where punishment is two-sided.

A second generalization may distinguish between partners in terms of who has the right to punish the cheater. In the current model, we assume that a cheater can be punished by any of his/her partners. One can imagine a situation in which each matched agent has a principal partner who has the right to punish his/her infidelity, while other partners are secondary and deprived of that right. In a sexual market for example, a married man who visits prostitutes is likely to be punished by his wife only, that is, the prostitutes that he visits may not punish him for his relationship with his wife. Allowing for such an asymmetry will make it possible for some agents (such as the prostitutes) to be aware of the fact that some of their potential partners have other partners. This in turn might affect the incentive of the informed parties (that is, the prostitutes) to engage in certain relationships, especially if they take into account the externalities generated by their potential partners' other links. One might endeavor to study the structure of networks that will form in such an asymmetric environment, as well as their implications for the diffusion of information.

8 Proofs

Proof of Remark 1

Proof. The proof is easy and left to the reader. ■

Proof of Lemma 1

Proof. Let g be a pairwise stable network. It is obvious that for any pair $(m, w) \in M * W$, the inequalities $0 \leq s_m$ and $0 \leq s_w$ hold. Now assume by contradiction that there exists an agent i who is matched with more than his/her optimal number of partners. That agent will improve by unilaterally severing one of the links he/she is involved in, which implies that g is not pairwise stable, a contradiction. It follows that any pair $(m, w) \in M * W$, $s_m \leq s_m^*$ and $s_w \leq s_w^*$. This completes the proof. ■

Proof of Lemma 2

Proof. The proof of the equivalence of (1) and (2) is straightforward and simply says tha if $|M| < s_w^*$, the unique pairwise stable network is the one in which each man is matched with all women.

Let us now show the equivalence of (1) and (3). We will proceed in two steps.

(1) $\Longrightarrow$ (3) : Assume that g is pairwise stable. Assume also that $s_w^* < |M| \leq s_m^*$. Given that the number of women is no more than the optimal number of partners for each man (because $|W| = |M| \leq s_m^*$), it is clear that each man is matched to at most $|M|$ women: $\forall m \in M,\ 0 \leq s_m \leq |M|$.

It just remains to prove that $\forall w \in W$, $s_w = s_w^*$. Assume by contradiction that there exists $w_0 \in W$ such that $s_{w_0} < s_w^*$. Then for any man m not matched with w_0, it should be the case that $s_m = |W| = |M|$ (in fact, if there exists a man m_0 not matched with w_0 such that $s_{m_0} < |W|$, it is clear that m_0 and w_0 will both improve by forming a new link, which contradicts the fact that g is pairwise stable). But this is impossible because given that $|W| = |M| \leq s_m^*$, there cannot exist any man m not matched with w_0 who has $|W|$ partners. It follows that $\forall w \in W$, $s_w = s_w^*$.

(3) $\Longrightarrow$ (1) : Assume that $s_w^* < |M| \leq s_m^*$ and that $\forall (m, w) \in M * W,\ 0 \leq s_m \leq |M|$ and $s_w = s_w^*$. Let us show that g is pairwise stable. A man alone cannot improve by severing a link since he is at the upward sloping part of his utility function. He cannot form a new link with another woman since each woman has her optimal number of partners. And a woman cannot be part of any blocking move (either by herself or with a man) since she is at her peak. Therefore, g is a pairwise stable network. This completes the proof. ■

Proof of Lemma 3

Proof. Assume that $s_w^* \in \{1, 2\}$ and $|M| > s_m^*$ and let g be a network.

(1) $\Longrightarrow$ (2) : Assume that g is pairwise stable. Given Lemma 1, we just have to prove that $\forall w \in W$, $s_w = s_w^*$. Assume by contradiction that there exists $w_0 \in W$ such that $s_{w_0} < s_w^*$. Then for any man m not matched with w_0, it should be the case that $s_m = s_m^*$. In fact, if there exists a man m_0 not matched with w_0 such that $s_{m_0} < s_m^*$, it is clear that m_0 and w_0 will both improve by forming a new link, which contradicts the fact that

g is pairwise stable.

It then follows that the number of links coming from the men side is at least $s_m^*(|M| - s_{w_0})$. Notice that $s_m^*(|M| - s_{w_0}) \geq s_m^*(|M| - 1)$ because $s_{w_0} < s_w^* \in \{1, 2\}$ by contradiction. But $s_m^*(|M| - 1) \geq (s_w^* + 1)(|M| - 1)$ (because $s_w^* < s_m^*$), and $(s_w^* + 1)(|M| - 1) > s_w^*|M| = s_w^*|W|$ (because $|M| > s_m^* \geq s_w^* + 1$). It therefore follows that $s_m^*(|M| - s_{w_0}) > s_w^*|W|$, which means that the number of links coming from the men side exceeds the maximal number of links that women can supply, which is impossible. We therefore conclude that $\forall w \in W$, $s_w = s_w^*$.

(2) $\Longrightarrow$ (1) : The argument here is similar to that of (3) $\Longrightarrow$ (1) of the proof of Lemma 2. ■

Proof of Lemma 4

Proof. Assume that $s_w^* > 2$ and $|M| > s_m^*$, and let g be a pairwise stable network.

1) Based on Example 1 and the network represented by Figure 1-2 in that example, we know that A is not always empty.

2) Assume that $A \neq \phi$, and assume by contradiction that there exists a woman w_0 in A who is not matched to any man. Then, it should be the case that each man has s_m^* partners, because if there exists a man m_0 who has fewer than s_m^*, then m_0 and w_0 will both improve by forming a new link, which implies that g is not a pairwise stable network, a contradiction. But given that each man has s_m^* partners, the number of links coming from the men side is $s_m^*|M|$, which exceeds the number of links that women can supply, which is impossible.

3) Assume that $A \neq \phi$. To show that there exists a unique component h of g such that $A \subset W(h)$, it suffices to prove that $\bigcap_{w \in A} g(w) \neq \phi$. In fact, if $\bigcap_{w \in A} g(w) = \phi$, then $\exists w_0 \in A$ such that $g(w_0) \cap (\bigcap_{w \in A \setminus \{w_0\}} g(w)) = \phi$. It is also the case that $\forall m \in M \setminus g(w_0)$, $s_m = s_m^*$. Also, we have: $\forall m \in g(w_0)$, $s_m = s_m^*$ (in fact, suppose that there exists $m_0 \in g(w_0)$ such that $s_{m_0} < s_m^*$; then, since $g(w_0) \cap (\bigcap_{w \in A \setminus \{w_0\}} g(w)) = \phi$, there necessarily exists $w_1 \in A \setminus \{w_0\}$ such that $m_0 \notin g(w_1)$; but given that $s_{m_0} < s_m^*$ and $s_{w_1} < s_w^*$, it will be beneficial to both m_0 and w_1 to form a new link, contradicting the fact that g is pairwise stable). This implies that $\forall m \in g(w_0) \cup (M \setminus g(w_0)) = M$, $s_m = s_m^*$, also implying that $\sum_{m \in M} s_m = s_m^*|M| > s_w^*|W| > \sum_{w \in W} s_w$, which is impossible. Therefore, $\bigcap_{w \in A} g(w) \neq \phi$.

4) We now want to show that $0 \leq |A| \leq s_w^* - 2$. Based on Example 1 and the network represented by Figure 1-1 in that example, we know that A may be empty (that is, $|A| = 0$). In addition, A being a finite set, it is the case that $|A|$ is a natural number. It follows that $|A| \geq 0$. It remains to show that $|A| \leq s_w^* - 2$. Let $h \in C(g)$ be the unique component in which the elements of A are vertices. We shall distinguish two cases: $W(h) = A$ and $W(h) \neq A$.

4-a) Suppose that $W(h) = A$. We shall first show that $M(h) = \bigcap_{w \in A} g(w) = g(A)$. Since $W(h) = A$, it is obvious that $g(\bigcap_{w \in A} g(w)) \subset A$, which means that no man in $\bigcap_{w \in A} g(w)$ is matched with a woman outside of A, because otherwise, $W(h) \neq A$, which is a contradiction. $W(h) = A$ also obviously implies that $\bigcap_{w \in A} g(w) \subset M(h)$. Now, let us assume by contradiction that $M(h)$ is not included in $\bigcap_{w \in A} g(w)$. This implies that $\exists m_0 \in$

$M(h) \setminus \bigcap_{w\in A} g(w)$ such that $m_0 \in g(A)$. But since $W(h) = A$, m_0 cannot have a partner outside of A, because otherwise, $W(h) \neq A$, which is a contradiction. Also, it is necessarily the case that $s_{m_0} = s^*_m$ (because if $s_{m_0} < s^*_m$, since $\forall w \in A$, $s_w < s^*_w$, each woman in A not matched with m_0 will form a link with m_0). But given that $s_{m_0} = s^*_m$ and the fact that m_0 has all his partners in A, it follows that $|A| \geq s^*_m$. However, since by definition, $\forall (m,w) \in \bigcap_{w\in A} g(w) * A$, $(m,w) \in g$, and because of $|A| \geq s^*_m$, it is necessarily the case that $\forall m \in \bigcap_{w\in A} g(w)$, $s_m = s^*_m$. This implies that $\forall m \in (\bigcap_{w\in A} g(w)) \cup (M \setminus \bigcap_{w\in A} g(w))$, $s_m = s^*_m$, also implying that $\sum_{m\in M} s_m = s^*_m|M| > s^*_w|W| > \sum_{w\in W} s_w$, which is impossible. Therefore, $M(h) = \bigcap_{w\in A} g(w) = g(A)$.

Now, because of $\forall (m,w) \in \bigcap_{w\in A} g(w) * A$, $(m,w) \in g$ and $s_w < s^*_w$, it is necessarily the case that $|\bigcap_{w\in A} g(w)| < s^*_w$. It is also the case that $\forall (m,w) \in (M \setminus \bigcap_{w\in A} g(w)) * (W \setminus A)$, $s_m = s^*_m$ and $s_w = s^*_w$, therefore implying that $|M \setminus \bigcap_{w\in A} g(w)| s^*_m = |W \setminus A| s^*_w$. Given that $s^*_m > s^*_w$, this equation implies $|M \setminus \bigcap_{w\in A} g(w)| < |W \setminus A|$, and thus $|\bigcap_{w\in A} g(w)| > |A|$. It therefore follows from $|\bigcap_{w\in A} g(w)| < s^*_w$ that $|A| \leq s^*_w - 2$.

4-b) Now suppose that $W(h) \neq A$. Given that h is unique, it is straightforward that $A \subset W(h)$ and this inclusion is strict. This is equivalent to saying that a man in $\bigcap_{w\in A} g(w)$ is matched with a woman outside of A or ("or" here is inclusive) that a woman in A is matched wth a man in $M \setminus \bigcap_{w\in A} g(w)$ (one can show that such a man is necessarily linked to a woman outside of A). In general, let: $B = \{(m,w) \in g : (m,w) \in \bigcap_{w\in A} g(w) * (W \setminus A)\}$, $C = \{(m,w) \in g : (m,w) \in (M \setminus \bigcap_{w\in A} g(w)) * A)\}$, $D = \{(m,w) \in g : (m,w) \in \bigcap_{w\in A} g(w) * A\}$ and $E = \{(m,w) \in g : (m,w) \in M \setminus \bigcap_{w\in A} g(w)) * (W \setminus A)\}$. We have $g = B \cup C \cup D \cup E$. Each man in $M \setminus \bigcap_{w\in A} g(w)$ has s^*_m partners and each woman in $W \setminus A$ has s^*_w partners. Thus, the following equality holds: $|M \setminus \bigcap_{w\in A} g(w)| s^*_m - |C| = |W \setminus A| s^*_w - |B|$. This implies $|B| = |W \setminus A| s^*_w - |M \setminus \bigcap_{w\in A} g(w)| s^*_m + |C|$. But $|C| = \sum_{w\in A} s_w - |D|$, and $|D| = |\bigcap_{w\in A} g(w)||A|$, thus $|B| = |W \setminus A| s^*_w - |M \setminus \bigcap_{w\in A} g(w)| s^*_m + \sum_{w\in A} s_w - |\bigcap_{w\in A} g(w)||A|$. Beacuse each woman in A has at most $s^*_w - 1$ partners, the greatest possible value of $\sum_{w\in A} s_w$ is $|A|(s^*_w - 1)$. This implies that the greatest possible value of $|B|$ is $\max |B| = |W \setminus A| s^*_w - |M \setminus \bigcap_{w\in A} g(w)| s^*_m + |A|(s^*_w - 1) - |\bigcap_{w\in A} g(w)||A| = (s^*_m - |A|)|\bigcap_{w\in A} g(w)| - |A| + |W| s^*_w - |M| s^*_m$. But $\max |B| > 0$, which implies that $|A| < \frac{s^*_m|\bigcap_{w\in A} g(w)| - |M|(s^*_m - s^*_w)}{|\bigcap_{w\in A} g(w)| + 1} < s^*_w - 1$. This obviously implies that $|A| \leq s^*_w - 2$, which completes our proof.

■

Proof of Theorem 1

Proof. The proof of the equivalence of (1) on one hand, and (2) – (4) on the other hand derives directly from Lemmas 2 and 3. It remains to prove the equivalence of (1) and (5).

(1) $\Longrightarrow$ (5) : Assume that g is pairwise stable, and that $s^*_w > 2$ and $|M| > s^*_m$. Let $\exists A = \{w \in W : s_w < s^*_w\}$ be the set of women who are matched to fewer than their optimal number of partners. We know that A may or may not be empty.

Assume that $A = \phi$. Then, it directly follows from that assumption and from Lemma 1 that $\forall (m, w) \in M{*}W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

Assume that $A \neq \phi$. Let $(m_1, m_2, w_1, w_2) \in \bigcap\limits_{w \in A} g(w) * (M \setminus \bigcap\limits_{w \in A} g(w)) * A * (W \setminus A)$. m_1 is matched to all women in A; in addition, we have $0 \leq s_{m_1} \leq s_m^*$ (from Lemma 1); it thus follows that $|A| \leq s_{m_1} \leq s_m^*$.

It is also clear that $s_{m_2} = s_m^*$. In fact, if $s_{m_2} < s_m^*$, given that $m_2 \in M \setminus \bigcap\limits_{w \in A} g(w)$, there necessarily exists a woman w_0 in A who is not matched to m_2. Both m_2 and w_0 would therefore improve by forming a new link, which contradicts the fact that g is pairwise stable.

Regarding w_1, the first inequality $1 \leq s_{w_1}$ comes from Lemma 4, and the second inequality $s_{w_1} \leq s_w^* - 1$ comes from the fact that $w_1 \in A$.

Finally, we obviously have $s_{w_2} = s_w^*$ from the fact that $0 \leq s_{w_2} \leq s_w^*$ (Lemma 1) and from $w_2 \notin A$. This completes the proof of $(1) \Longrightarrow (2 - iv)$.

$(5) \Longrightarrow (1)$. Let g be a network. Suppose that $s_w^* > 2$, and let $A = \{w \in W : s_w < s_w^*\}$.

Assume that $A = \phi$ and that $\forall (m, w) \in M * W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$; and prove that g is a pairwise stable network. The proof is similar to that of $(3) \Longrightarrow (1)$ of Lemma 2.

Assume that $A \neq \phi$ and that $\forall (m_1, m_2, w_1, w_2) \in \bigcap\limits_{w \in A} g(w) * (M \setminus \bigcap\limits_{w \in A} g(w)) * A * (W \setminus A)$, $|A| \leq s_{m_1} \leq s_m^*$, $s_{m_2} = s_m^*$, $1 \leq s_{w_1} \leq s_w^* - 1$, and $s_{w_2} = s_w^*$; and prove that g is a pairwise stable network. No agent will improve by unilaterally severing an existing link he/she is involved in g since he/she is either at the upward sloping part of his/her utility function or at his/her peak. No man in $\bigcap\limits_{w \in A} g(w)$ cannot be part of a blocking move with a woman in A (since both are already matched to each other) or with a woman in $W \setminus A$ since she is at her peak. Similarly, no woman in $W \setminus A$ cannot be part of a blocking move with any man since she is at her peak. Finally, no pair of a man and a woman in $(M \setminus \bigcap\limits_{w \in A} g(w)) * (W \setminus A)$ (not matched in g, if any) will not improve by forming a new link since they are at their optimum. This shows that g is a pairwise stable network.

The proof of the last assertion immediately follows from items 2-4 in Theorem 1 for the case where $|M| = s_w^*$, $s_w^* < |M| \leq s_m^*$, or $|M| > s_m^*$ and $s_w^* = 1, 2$; in each of these cases, each woman obtains her optimal number of partners. When $|M| > s_m^*$ and $s_w^* > 2$, it follows from item 4 in Lemma 4; in that case, at most $s_w^* - 2$ women have fewer than their optimal number number of partners. ■

Proof of Remark 2

Proof. The proof is simple by noticing that: if $|M| \leq s_w^*$, there is a unique pairwise stable network (Lemma 2); and if $|M| > s_w^*$, then all egalitarian networks where each agent has s_w^* partners are pairwise stable. Such networks always exist and are very easy to construct. ■

Proof of Proposition 1

Proof. The proof is easy and left to the reader. ■

Proof of Theorem 2

Proof. 1) If $|M| \leq s_w^*$, the only pairwise stable network that exists in the economy is the one in which each man is matched with all women. This network is Pareto-efficient because each agent being at the upward sloping part of his/her utility function has the maximal number of partners he/she can have; thus this network cannot be improved upon. It follows from the same argument that the aggregate value of this network is maximal overall and on each side of the economy. This network is therefore strongly efficient, male-optimal and female-optimal.

2) If $|M| > s^*_w$, a Pareto-efficient pairwise stable network always exists because any egalitarian pairwise stable network for instance is Pareto-efficient; but Example 3 shows that a strongly efficient pairwise stable network may not exist.

2-i) Let us now prove 2-i). Let g be a pairwise stable network. Assume that g is Pareto-efficient. Then, it follows from Lemma 1 that $\forall (m,w) \in g$, $0 \leq s_m \leq s^*_m$ and $0 \leq s_w \leq s^*_w$. It remains to show that $\forall w \in W$, $s_w = s^*_w$. Assume by contradiction that there exists a woman w_1 such that $s_{w_1} < s^*_w$. This implies that the set $A = \{w \in W : s_w < s^*_w\}$ is not empty. Thus by Theorem 1, $\forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) * (M \setminus \bigcap_{w \in A} g(w)) * A * (W \setminus A)$, $s^*_w - 2 \leq s_{m_1} \leq s^*_m$, $s_{m_2} = s^*_m$, $1 \leq s_{w_1} \leq s^*_w - 1$, and $s_{w_2} = s^*_w$. It is also the case that there exists a man $m_1 \in \bigcap_{w \in A} g(w)$ such that $s_{m_1} < s^*_m$. There also exists a man $m_2 \in (M \setminus \bigcap_{w \in A} g(w))$ who is not matched with w_1 and who is matched with a woman $w_2 \in W \setminus A$ not matched with m_1. Delete the link (m_2, w_2), and add the links (m_2, w_1), (m_1, w_2), resulting in a new network g'. Note that g' is pairwise stable. In this new network, m_1 and w_1 have strictly improved relative to the network g, and no one's utility has decreased. It therefore follows that g is Pareto-dominated by g', which is a contradiction. Thus, $\forall w \in W$, $s_w = s^*_w$.

Conversely, assume that $\forall (m,w) \in g$, $0 \leq s_m \leq s^*_m$ and $s_w = s^*_w$, and show that g is Pareto-efficient. Assume by contradiction that it is not. Therefore, it is Pareto-dominated by another network g', which implies that for all agent i, $u_i(s_i(g)) \leq u_i(s_i(g'))$, and $u_j(s_j(g)) < u_j(s_j(g'))$ for some agent j. Note that each woman is necessarily at her peak in both g and g', which implies that j is a man. Given that g is pairwise stable, $s_j(g) < s^*_m$. Therefore, $u_j(s_j(g)) < u_j(s_j(g'))$ implies that $s_j(g) < s_j(g')$. Since no man becomes worse off in g' relative to g, the number of links coming from the men side (or from the women side) is strictly greater in g' than in g (that is, $|g'| > |g|$); this is impossible because $|g'| = |g| = s^*_w|W|$. Thus, g is Pareto-efficient.

2-ii) A network is male-optimal if and only if each man is at his peak. In no pairwise stable network is this possible.

2-iii) A pairwise stable network that is Pareto-efficient is female-optimal because each woman is at her peak in such a network, and thus its aggregate value for women is maximal. Conversely, in any pairwise stable network whose aggregate value for women is maximal, each woman is at her peak, and thus it follows from part 2-i) that such a network is always Pareto-efficient. ■

Proof of Claim 1

Proof. Assume that an agent $z \in N = M \cup W$ is drawn at random to receive the piece of information γ.

1) Let $\Pr(\gamma | N, \rho(z, \gamma) = 1)$ be the proportion of agents who will receive the information given that z has received it. If z belongs to component g_i, it is obvious that the information will spread only to agents in that component. Thus, $\Pr(\gamma | N, \rho(z, \gamma) = 1) = \frac{n_i}{n}$. Given that each agent is drawn with equal probability, we have:

$$
\begin{aligned}
E[\Pr(\gamma|N)] &= \tfrac{1}{n}\textstyle\sum_{z\in N}(\Pr(\gamma|N,\rho(z,\gamma)=1) \\
&= \tfrac{1}{n}\textstyle\sum_{z\in N_1\cup\ldots\cup N_k}\tfrac{n_i}{n} \\
&= \tfrac{1}{n}\textstyle\sum_{i\in I_k}\sum_{z\in N_i}\tfrac{n_i}{n} \\
&= \tfrac{1}{n}\textstyle\sum_{i\in I_k}n_i\tfrac{n_i}{n} \\
&= \tfrac{1}{n^2}\textstyle\sum_{i\in I_k}n_i^2.
\end{aligned}
$$

2) Still assuming that z belongs to component g_i, m_i men and w_i women will receive the information. Thus, the gender difference in the concentration of information is $\Pr(\gamma|M,\rho(z,\gamma)=1)-\Pr(\gamma|W,\rho(z,\gamma)=1)=\frac{m_i}{|M|}-\frac{w_i}{|W|}=\frac{2}{n}(m_i-w_i)$ because $|M|=|W|=\frac{n}{2}$. This implies that the expected value of $\Pr(\gamma|M)-\Pr(\gamma|W)$ is:

$$
\begin{aligned}
E[\Pr(\gamma|M)-\Pr(\gamma|W)] &= \tfrac{1}{n}\textstyle\sum_{z\in N}(\Pr(\gamma|M,\rho(z,\gamma)=1)-\Pr(\gamma|W,\rho(z,\gamma)=1)) \\
&= \tfrac{1}{n}\textstyle\sum_{z\in N}\tfrac{2}{n}(m_i-w_i) \\
&= \tfrac{2}{n^2}\textstyle\sum_{z\in N_1\cup\ldots\cup N_k}(m_i-w_i) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}\sum_{z\in N_i}(m_i-w_i) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}n_i(m_i-w_i) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}(m_i+w_i)(m_i-w_i) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}(m_i^2-w_i^2).
\end{aligned}
$$

This completes the proof. ■

Proof of Lemma 5

Proof. Let g be a pairwise stable network such that $A=\phi$, and $g'\in\mathcal{J}(g)$ a non-isolated component of g.

1) We want to show that $|W(g')|=\frac{|g'|}{s_w^*}$. Given that each woman $w\in W(g')$ has s_w^* partners, the cardinality of the subgraph g' is obviously $|g'|=s_w^*|W(g')|$, which implies $|W(g')|=\frac{|g'|}{s_w^*}$.

2) We now want to show that $\max(\left\lceil\frac{|g'|}{s_m^*}\right\rceil,s_w^*)\leq|M(g')|\leq|g'|-\frac{|g'|}{s_w^*}+1$. In the subgraph g', $|M(g')|$ men are involved in $|g'|$ relationships; and given that each man is matched to a maximum of s_m^* partners, these $|g'|$ relationships can only be shared by a minimum of $\left\lceil\frac{|g'|}{s_m^*}\right\rceil$ men, implying $\left\lceil\frac{|g'|}{s_m^*}\right\rceil\leq|M(g')|$. Also, given that each woman $w\in W(g')$ should be linked to exactly s_w^* partners, it follows that $s_w^*\leq|M(g')|$. But $\left\lceil\frac{|g'|}{s_m^*}\right\rceil\leq|M(g')|$ and $s_w^*\leq|M(g')|$ imply $\max(\left\lceil\frac{|g'|}{s_m^*}\right\rceil,s_w^*)\leq|M(g')|$.

The second inequality $|M(g')|\leq|g'|-\frac{|g'|}{s_w^*}+1$ comes from the fact that $|g'|-\frac{|g'|}{s_w^*}+1$ is the largest number of men that is required for all vertices of the subgraph g' to remain directly or indirectly connected. In fact, let $M(g')=\{p_1,...,p_{|M(g')|}\}$ be the set of men, and $W(g')=\{q_1,...,q_{|W(g')|}\}$ be the set of women. We want to construct the component g' so that $|M(g')|$ is the largest possible. Construct g' by linking w_i to $\{p_{(i-1)s_w^*-i+2},...,p_{is_w^*-i+1}\}$ for each $i\in\{1,...,|W(g')|\}$. Since the function is_w^*-i+1 is increasing in i, it reaches its maximum at $i=|W(g')|$, implying that $|W(g')|s_w^*-|W(g')|+1$ is the largest possible value of $|M(g')|$; and given that $|W(g')|=\frac{|g'|}{s_w^*}$, this value is equal to $|g'|-\frac{|g'|}{s_w^*}+1$. It is easy to see that each q_i is matched to exactly s_w^* partners and all elements of $M(g')$ and $W(g')$ are directly or indirectly linked; in fact, p_1 is linked to q_1, $p_{|W(g')|s_w^*-|W(g')|+1}$ is linked to $w_{|W(g')|}$, and each p_j such that there exists $i\in\{2,...,|W(g')|-1\}$ such that $j=is_w^*-i+1$ is linked to q_i and q_{i+1}, and each p_j such that there exists $i\in\{1,...,|W(g')|-1\}$ such that $(i-1)s_w^*-i+2<j<is_w^*-i+1$ is linked to q_i. ■

Proof of Lemma 6

Proof. Let g be a k-component network with the corresponding bipartite component vector $[(m_i, w_i)]_{i\in I_k}$. Assume that $\forall g' \in \mathcal{J}(g)$, $|M(g')| \leq |W(g')|$, and let us show that $\mathcal{F}(g) \leq 0$. Assume that there are ℓ non-isolated components, l isolated components of men, and $k-\ell-l$ isolated components of women. Without loss of generality, assume that the first ℓ components of the vector $[(m_i, w_i)]_{i\in I_k}$ represent the non-isolated components of g, the l next components represent the isolated components of men, and the remaining components represent the isolated components of women. There are therefore l components $(1,0)$ and $k-\ell-l$ components $(0,1)$. Remark that each non-isolated component vector (m_i, w_i) is such that $m_i + w_i = n_i \geq 2$ since it contains at least one man and one woman. Also, we have $\sum_{i\in I_k} m_i = \sum_{i\in I_\ell} m_i + l$ and $\sum_{i\in I_k} w_i = \sum_{i\in I_\ell} w_i + (k-\ell-l)$, which, given the fact that $\sum_{i\in I_k} m_i = \sum_{i\in I_k} w_i$, implies that $\sum_{i\in I_\ell}(m_i - w_i) = k-\ell-2l$. Because $m_i \leq w_i$ for each $i \in I_\ell$, it thus follows that $\sum_{i\in I_\ell}(m_i - w_i) = k-\ell-2l \leq 0$. We have the following results:

$$\begin{aligned}
\mathcal{F}(g) &= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}(m_i^2 - w_i^2) \\
&= \tfrac{2}{n^2}\{\textstyle\sum_{i\in I_\ell}(m_i^2 - w_i^2) + \sum_{\ell+1\leq i\leq \ell+l}(m_i^2 - w_i^2) + \sum_{\ell+l+1\leq i\leq k}(m_i^2 - w_i^2)\} \\
&= \tfrac{2}{n^2}\{\textstyle\sum_{i\in I_\ell}(m_i - w_i)(m_i + w_i) + \sum_{\ell+1\leq i\leq \ell+l}(1^2 - 0^2) + \sum_{\ell+l+1\leq i\leq k}(0^2 - 1^2)\} \\
&= \tfrac{2}{n^2}\{\textstyle\sum_{i\in I_\ell}(m_i - w_i)n_i + l - (k-\ell-l)\} \\
&\leq \tfrac{2}{n^2}\{2\textstyle\sum_{i\in I_\ell}(m_i - w_i) - k + \ell + 2l\} \\
&= \tfrac{2}{n^2}\{2(k-\ell-2l) - k + \ell + 2l\} \\
&= \tfrac{2}{n^2}(k-\ell-2l) \\
&\leq 0
\end{aligned}$$

Note that the last inequality is strict if at least one man is isolated. ■

Proof of Lemma 7

Proof. Let $\mathcal{E} = (N = M \cup W, s_m^*, s_w^*)$ be a mating economy.

(2) $\Longrightarrow$ (1): a) Assume that $s_w^* = 1$ and show that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let $g \in \mathcal{PS}(\mathcal{E})$ be a pairwise stable network and $g' \in \mathcal{J}(g)$ a non-isolated component of g. It is straightforward from Lemma 5 that $|M(g')| = 1$. Given that $|W(g')| \geq 1$, it follows that $|M(g')| \leq |W(g')|$. Thus, each non-isolated component of g is such that the number of women weakly exceeds the number of men. It therefore follows from Lemma 6 that $\mathcal{F}(g) \leq 0$.

b) Assume that $n \leq 4s_w^* + 2$ and show that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let $g \in \mathcal{PS}(\mathcal{E})$ be a pairwise stable network and A be the set of women who have less than their optimal number of partners. We shall distinguish two cases: $A = \phi$ and $A \neq \phi$.

$b-1$) Assume that $A = \phi$.

- First assume that $n < 4s_w^*$. For any $g \in \mathcal{PS}(\mathcal{E})$, let us show that there is only one non-isolated component $g' \in \mathcal{J}(g)$. Assume by contradiction that there are two such components g_1 and g_2. Then by Lemma 5, $s_w^* \leq m_1$ and $s_w^* \leq m_2$ (remember that m_1 and m_2 are respectively the number of men in g_1 and g_2), which implies that $|M| \geq m_1 + m_2 \geq 2s_w^*$, and $n = 2|M| \geq 4s_w^*$, thus contradicting our assumption. So there is only one non-isolated component $g' \in \mathcal{J}(g)$; since $|W(g')| = |W|$ and $\left\lceil \frac{s_w^*|W|}{s_m^*} \right\rceil \leq |M(g')| \leq |M| = |W|$, it follows that $|M(g')| \leq |W(g')|$, which by Lemma 6, implies that for any $g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$.

- Now assume that $n = 4s_w^*$. This implies that $|M| = |W| = 2s_w^*$. There exist at most two non-isolated components. If there is only one such component, then the proof follows as for the case where $|W| < 2s_w^*$. Now, suppose that there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$; by Lemma 5, $m_1 = m_2 = s_w^*$. We thus have the following:

$$\begin{aligned} \mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_2}(m_i^2 - w_i^2) \\ &= \frac{2}{n^2}(2s_w^{*2} - w_1^2 - w_2^2) \\ &= \frac{2}{n^2}(2s_w^{*2} - w_1^2 - (2s_w^* - w_1)^2) \\ &= \frac{2}{n^2}\{-\ 2(s_w^* - w_1)^2\} \\ &\leq 0 \end{aligned}$$

- Assume that $n = 4s_w^* + 2$. This implies that $|M| = |W| = 2s_w^* + 1$. There are at most two non-isolated components. If there is only one such component, then the proof is similar to that of the case where $|W| < 2s_w^*$. Suppose that there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$; then by Lemma 5, the number of men is s_w^* in one component and $s_w^* + 1$ in the other component. Without loss of generality, assume that $m_1 = s_w^*$ and $m_2 = s_w^* + 1$. We thus have the following:

$$\begin{aligned} \mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_2}(m_i^2 - w_i^2) \\ &= \frac{2}{n^2}(s_w^{*2} + (s_w^* + 1)^2 - w_1^2 - (2s_w^* + 1 - w_1)^2) \\ &= \frac{2}{n^2}\{-2(s_w^* - w_1)(s_w^* - w_1 - 1)\} \\ &\leq 0 \end{aligned}$$

Note that the last inequality comes from the fact that the expression $-2(s_w^* - w_1)(s_w^* - w_1 - 1)$ is strictly positive if and only if $w_1 \in (s_w^* - 1, s_w^*)$, which is impossible because s_w^* and w_1 are integers.

$b - 2)$ Assume that $A \neq \phi$. This implies that no man is isolated, and there are at most two non-isolated components. If there is only one non-isolated component, then, it is straightforward that all men and women in the economy belong to that component, which implies that $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i \in I_1}(m_i^2 - w_i^2) = 0$.

If there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$, it should be the case that one of them, say g_1, is such that $W(g_1) = A$. Therefore, all men involved in g_1 are in the set $M(g_1) = g(A) = \bigcap_{w \in A} g(w)$. From Lemma 4, we know that g_1 has at most $s_w^* - 1$ men and $s_w^* - 2$ women, leaving g_2 with at least $|M| - s_w^* + 1$ men and $|W| - s_w^* + 2$ women. Also, because each man involved in g_2 has s_m^* women (if not, all men in g_2 will have an incentive to form a link with women in g_1), it follows that $|W| - s_w^* + 2 \geq s_m^*$, which also implies that $|M| - s_w^* + 1 \geq s_m^* - 1 > s_w^* - 1$. It results from all these assertions that $m_1 < m_2$ and $w_1 < w_2$, which implies that $n_1 = m_1 + w_1 < m_2 + w_2 = n_2$. Also note that because no man is isolated, $m_1 - w_1 = -(m_2 - w_2) > 0$. We therefore have the following:

$$\begin{aligned} \mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_2}(m_i^2 - w_i^2) \\ &= \frac{2}{n^2}\{(m_1 - w_1)n_1 + (m_2 - w_2)n_2 \\ &= \frac{2}{n^2}(m_1 - w_1)(n_1 - n_2) \\ &< 0 \end{aligned}$$

This concludes the proof of (2) $\Longrightarrow$ (1).

(1) $\Longrightarrow$ (2): Assume that for any $g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. We want to show that $s_w^* = 1$ or $n \leq 4s_w^* + 2$.

c) Assume that $s_w^* > 1$ and show that $n \leq 4s_w^* + 2$. Assume by contradiction that $n > 4s_w^* + 2$. Construct a pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ with two non-isolated components g_1 and g_2 such that $m_1 = s_w^*$, $m_2 = |M| - s_w^*$, $w_1 = s_w^* + 1$, and $w_2 = |W| - s_w^* - 1 = |M| - s_w^* - 1$. Remark that this network satisfies the bounds conditions of Lemma 5 because $s_w^* > 1$. We want to show that $\mathcal{F}(g) > 0$. We have:

$$\begin{aligned}
\mathcal{F}(g) &= \frac{2}{n^2}\sum_{i\in I_2}(m_i^2 - w_i^2) \\
&= \frac{2}{n^2}(s_w^{*2} + (s_w^* + 1)^2 - (|M| - s_w^*)^2 - (|M| - s_w^* - 1)^2) \\
&= -4s_w^* + 2|M| - 2 \\
&= -4s_w^* + -2 + n \\
&> 0
\end{aligned}$$

$\mathcal{F}(g) > 0$ is a contradiction of our assumption, so we conclude that $n \leq 4s_w^* + 2$.

d) Assume that $n > 4s_w^* + 2$ and show that $s_w^* = 1$. Assume by contradiction that $s_w^* > 1$. Then any pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ with two non-isolated components g_1 and g_2 such that $m_1 = s_w^*$, $m_2 = |M| - s_w^*$, $w_1 = s_w^* + 1$, and $w_2 = |W| - s_w^* - 1 = |M| - s_w^* - 1$ is such that $\mathcal{F}(g) > 0$. The proof is exactly as in part *c*). This completes the proof. ■

Proof of Theorem 3

Proof. Let $\mathcal{E}$ be a (non necessarily trivial) mating economy.

(2) $\Longrightarrow$ (1): Assume that $\mathcal{E}$ is a mating segmented economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t\in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*}+2$. Let us show that any network $g \in \mathcal{PS}(\mathcal{E})$ is such that $\mathcal{F}(g) \leq 0$. Call g^t the sub-network (or sub-graph) of g that forms in the segment $\mathcal{E}^t$ of the economy. It can be shown that $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t)$ where $n^t = |N^t|$. It also follows from the assumption that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$ that $\forall t \in I_T$, $\mathcal{F}(g^t) \leq 0$ (Lemma 7). Thus $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t) \leq 0$.

(1) $\Longrightarrow$ (2): Assume that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let us show that $\mathcal{E}$ is a segmented mating economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t\in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$. Assume by contradiction that there exists a segment $\mathcal{E}^{t_0}$ such that $s_w^{t_0*} > 1$ and $n^{t_0} > 4s_w^{t_0*} + 2$. Then following Lemma 7, we can construct a pairwise stable network $g^{t_0} \in \mathcal{PS}(\mathcal{E}^{t_0})$ such that $\mathcal{F}(g^{t_0}) > 0$. Construct such a g^{t_0}. For any other segment $\mathcal{E}^t \neq \mathcal{E}^{t_0}$, construct an egalitarian pairwise stable network g^t (this is always possible and is easy); we thus have $\mathcal{F}(g^t) = 0$. Call the resulting network g. It is clear that g is a pairwise stable network of the economy $\mathcal{E}$. In addition, we have $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t) = \frac{2}{n^2}\frac{n^{t_0 2}}{2}\mathcal{F}(g^{t_0}) > 0$, which is a contradiction ■

Proof of Theorem 4

Proof. The proof of the equivalence between (1) on one hand and (2)-(5) one the other follows from that of Theorem 1, the only difference being that the constraint $(\mathcal{S})$ is taken into account in the current proof. This constraint implies for instance that if a woman is matched to her optimal number of partners s_w^* in a pairwise stable network g, each of her s_w^* male partners in that network should be matched to at least s_w^* female partners. The proof is therefore made easier by that of Theorem 1 and is left to the reader.

6) To prove that any pairwise stable network g is such that $\mathcal{F}(g) \leq 0$, it suffices to show that each non-isolated component of g has at least as many women as men. We can show this by noticing that in each non-isolated

component of g, no woman has more partners than her least connected male partner. The rest then follows by invoking Lemma 6. ■

Proof of Theorem 5

Proof. 1) Let $M = \{i_1, ..., i_{|M|}\}$ and $W = \{j_1, ..., j_{|W|}\}$ be the sets of men and women, respectively. Without loss of generality, we assume that the label of each agent indicates his/her position in the social hierarchy (that is, i_1 is the highest ranked man, i_2 the second highest ranked man, and so on). Let us distinguish three cases: $|M| \leq s_w^*$, $s_w^* < |M| \leq s_m^*$, and $|M| > s_m^*$.

a) If $|M| \leq s_w^*$, then the unique pairwise stable network is the one in which each woman is matched with all men.

b) If $s_w^* < |M| \leq s_m^*$, the unique pairwise stable network is the one in which all women are matched with the s_w^* most highly ranked men, and all other men are unmatched.

c) If $|M| > s_m^*$, write $|M| = |W| = ks_m^* + r = k's_w^* + r'$ where k, r, k' and r' are integers such that $0 \leq r < s_m^*$ and $0 \leq r' < s_w^*$. Partition all women into $k+1$ sets $W_1, .., W_{k+1}$ such that for any $\ell \in I_k$, W_ℓ contains s_m^* individuals, the set W_{k+1} contains r individuals, and all women in each set W_ℓ are more highly ranked than all women in the set $W_{\ell+1}$. Similarly, partition all men into $k'+1$ sets $M_1, .., M_{k'+1}$ such that for any $\ell \in I_{k'}$, M_ℓ contains s_w^* individuals, the set $W_{k'+1}$ contains r' individuals, and all men in each set M_ℓ are more highly ranked than all men in the set $M_{\ell+1}$. So W_1 is the set of the s_m^* most highly ranked women, W_2 is the set of the next s_m^* most highly ranked women, and so on. Similarly, M_1 is the set of the s_w^* most highly ranked men, M_2 is the set of the next s_w^* most highly ranked men, and so on. It is easily shown that the unique pairwise stable network in this economy is the network in which all women in each set W_ℓ, $\ell \in I_{k+1}$, are matched with all men in the corresponding set M_ℓ. This matching is feasible because $k \leq k'$. If $r = 0$, meaning that W_{k+1} is empty, then the remaining men in the sets $M_{k+1}, .., M_{k'+1}$ (if not empty) are unmatched. If $r \neq 0$, meaning that W_{k+1} is not empty, then the remaining men in the sets $M_{k+2}, .., M_{k'+1}$ (if not empty) are unmatched.

2) *a*) If $|M| \leq s_w^*$, given that each man is matched to all women, the unique pairwise stable network g has only one component which contains all men and all women; so $\mathcal{F}(g) = \frac{2}{n^2} \sum_{i \in I_k} (m_i^2 - w_i^2) = \frac{2}{n^2}(|M|^2 - |W|^2) = 0$.

b) Assume that $|M| > s_w^*$. We shall distinguish two cases: $s_w^* < |M| \leq s_m^*$ and $|M| > s_m^*$.

b − 1) Suppose that $s_w^* < |M| \leq s_m^*$. The unique pairwise stable network g has only one non-isolated component in which the number of men (s_w^*) strictly exceeds the number of women ($|M|$), and $|M| - s_w^* > 0$ men are isolated. It therefore follows from Lemma 6 that $\mathcal{F}(g) < 0$.

b − 2) Suppose that $|M| > s_m^*$. Write $|M| = |W| = ks_m^* + r = k's_w^* + r'$ where k, r, k' and r' are integers such that $0 \leq r < s_m^*$ and $0 \leq r' < s_w^*$.

- If $r = 0$, then the unique pairwise stable network g described in part 1-c) is such that the number of women strictly exceeds the number of men in each non-isolated component. Resorting to Lemma 6, we have $\mathcal{F}(g) < 0$.

- If $r \neq 0$, it is easy to check that $k \leq k'$. Let us distinguish two cases: $k = k'$ and $k < k'$.

- Suppose that $k = k'$. This necessarily implies that there is no isolated man in the unique pairwise stable network described in part 1-c), and that $r < r' < s_w^* < s_m^*$. These inequalities in turn imply $s_w^* + s_m^* - r' - r > 0$. Also, $|M| = |W| = ks_m^* + r = k's_w^* + r'$ implies $r' - r = k(s_m^* - s_w^*)$. Note that there are s_m^* women and s_w^* men in each of the first k non-isolated components, and in the last component, we have $m_{k+1} = r'$ and $w_{k+1} = r$. It therefore follows that:

$$\begin{aligned}
\mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_{k+1}}(m_i^2 - w_i^2) \\
&= \frac{2}{n^2}\{k(s_w^{*2} - s_m^{*2}) + (r'^2 - r^2)\} \\
&= \frac{2}{n^2}\{k(s_w^* - s_m^*)(s_w^* + s_m^*) + (r' - r)(r' + r)\} \\
&= \frac{2}{n^2}\{k(s_w^* - s_m^*)(s_w^* + s_m^*) + k(s_m^* - s_w^*)(r' + r)\} \\
&= \frac{2}{n^2}\{k(s_w^* - s_m^*)(s_w^* + s_m^* - (r' + r)\} \\
&= \frac{2}{n^2}\{k(s_w^* - s_m^*)(s_w^* + s_m^* - r' - r)\} \\
&< 0
\end{aligned}$$

- Suppose that $k < k'$. This means that at least one man is isolated in the unique pairwise stable network g described in part 1-c), and that each of the first k non-isolated components of that network contains s_w^* men and s_m^* women, and the last non-isolated component contains $m_{k+1} = s_w^*$ men and $w_{k+1} = r$ women (note that we cannot resort to Lemma 6 in this case because there might be instances in which $r > s_w^*$). So g is such that $\forall g' \in \mathcal{J}(g)$, $|M(g')| > |W(g')| \Longrightarrow |M(g')| = s_w^*$, and $|M(g')| \leq |W(g')| \Longrightarrow |M(g')| \geq s_w^*$. It therefore follows from Lemma 8 that $\mathcal{F}(g) < 0$. This completes our proof. ■

Proof of Proposition 2

Proof. The proof is easy and left to the reader. ■

Proof of Proposition 3

Proof. The proof is easy and left to the reader. ■

Proof of Proposition 4

Proof. Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a mating economy of class societies, and i_v and j_v a man and a woman of rank v. To prove 1), 2) and 3), first remark that in any network (pairwise stable or not), for any individual i, $p(i) = \frac{n(i)}{n}$ where $n(i)$ is the size of the component to which i belongs in that network, and n the size of the total popualtion. We shall now compute $p(i_v)$ and $p(j_v)$ as a function of v in the unique pairwise stable network that arises in this economy. We recall that that network is described in the proof of Theorem 5.

a) If $|M| \leq s_w^*$, then the unique pairwise stable network is the one in which each woman is matched to all men. Therefore, $p(i_v) = p(j_v) = \frac{|M|+|W|}{n} = 1$, which implies (1) and (2).

b) If $s_w^* < |M| \leq s_m^*$, the unique pairwise stable network is the one in which all women are matched with the s_w^* most highly ranked men, and all other men are unmatched. Therefore:

- $v \leq s_w^* \Longrightarrow p(i_v) = \frac{|W|+s_w^*}{n}$ and $p(j_v) = \frac{|W|+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $v > s_w^* \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{|W|+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-|W|-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ weakly increases in v.

c) If $|M| > s_m^*$, write $|M| = |W| = ks_m^* + r$ such that $0 \leq r < s_m^*$. Pose $k_{\max} = \left\lceil \frac{|W|}{s_m^*} \right\rceil s_w^*$. Note that $k_{\max}$ is the rank below which all men are isolated in the unique pairwise stable network described in the proof of Theorem 5. We shall distinguish two cases: $r = 0$ and $r \neq 0$.

$c-1$) $r = 0$. In the unique pairwise stable network described in the proof of Theorem 5, j_v belongs to a component in which there are s_m^* women and s_w^* men, but this is true for i_v only if $v \leq k_{\max}$. Therefore, we have the following:

- $v \leq k_{\max} \Longrightarrow p(i_v) = \frac{s_m^*+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $v > k_{\max} \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-s_m^*-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ weakly increases in v.

$c-2$) $r \neq 0$. In the unique pairwise stable network described in the proof of Theorem 5, j_v belongs to the component in which there are r women and s_w^* men if $v > |W| - r$, and to a component in which there are s_m^* women and s_w^* men if $v \leq |W| - r$; i_v belongs to the component in which there are r women and s_w^* men if $k_{\max} - s_w^* + 1 \leq v \leq k_{\max}$, to a component in which there are s_m^* women and s_w^* men if $v < k_{\max} - s_w^* + 1$, and is isolated if $v > k_{\max}$. It is easy to check that $k_{\max} - s_w^* + 1 \leq |W| - r$. We shall distinguish two cases: (1) $k_{\max} - s_w^* + 1 = |W| - r$, and (2) $k_{\max} - s_w^* + 1 < |W| - r$.

$c-2-1$) Suppose that $k_{\max} - s_w^* + 1 = |W| - r$. We have the following:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(i_v) = \frac{s_m^*+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $v = k_{\max} - s_w^* + 1 \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{r-s_m^*}{n} < 0$.
- $k_{\max} - s_w^* + 1 < v \leq k_{\max} \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $k_{\max} < v \leq |M| \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-r-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ is non-monotonic in v.

$c-2-2$) Suppose that $k_{\max} - s_w^* + 1 < |W| - r$. We shall distinguish three cases: $|W| - r = k_{\max}$, $|W| - r < k_{\max}$ and $|W| - r > k_{\max}$. We have the following:

If $|W| - r = k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(i_v) = \frac{s_m^*+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $k_{\max} - s_w^* + 1 \leq v \leq k_{\max} \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{r-s_m^*}{n} < 0$.
- $k_{\max} < v \leq |M| \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-r-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ weakly increases in v.

If $|W| - r < k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(i_v) = \frac{s_m^*+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $k_{\max} - s_w^* + 1 \leq v \leq |W| - r \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{r-s_m^*}{n} < 0$.
- $|W| - r < v \leq k_{\max} \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.

- $k_{\max} < v \leq |M| \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-r-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ is non-monotonic in v.

If $|W| - r > k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(i_v) = \frac{s_m^*+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = 0$.
- $k_{\max} - s_w^* + 1 \leq v \leq k_{\max} \Longrightarrow p(i_v) = \frac{r+s_w^*}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{r-s_m^*}{n} < 0$.
- $k_{\max} < v \leq |W| - r \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-s_m^*-s_w^*}{n} < 0$.
- $|W| - r < v \leq |M| \Longrightarrow p(i_v) = \frac{1}{n}$ and $p(j_v) = \frac{r+s_w^*}{n} \Longrightarrow p(i_v) - p(j_v) = \frac{1-r-s_w^*}{n} < 0$.

We note that: (1) $p(i_v)$ and $p(j_v)$ weakly increase in v; (2) $p(i_v) - p(j_v) \leq 0$ for any v; and (3) $p(i_v) - p(j_v)$ is non-monotonic in v. ■

References

Aghion, P., C. Harris, P. Howitt, and J. Vickers (2001): "Competition, Imitation and Growth with Step-by-Step Innovation," *Review of Economic Studies*, LXVIII, 467–492.

Alós-Ferrer, C., and S. Weidenholzer (2007): "Contagion and Efficiency," *Journal of Economic Theory* 143, 251-274.

American Association of Blood Banks (2003): Annual Report Summary for Testing in 2003.

Aumann, R., and R. Myerson (1988): Endogenous Formation of Links Between Players and Coalitions: An Application of the Shapley Value, in A. Roth, Ed. "The Shapley Value" Cambridge University Press, Cambridge, 1988.

Australian Law Reform Commission/National and Health Medical Research Council (2003): Non-Confidential Submissions to the Inquiry into the Protection of Human Genetic Information, G245.

Baker, W., and A. Iyer (1992): "Information Networks and Market Behaviour," *Journal of Mathematical Sociology* 16, 305-332.

Bala, V., and S. Goyal (2000a): "A Non-cooperative Model of Network Formation," *Econometrica* 68, 1181-1230.

Bloch, F., and M. Jackson (2007): "The Formation of Networks with Transfers Among Players," *Journal of Economic Theory* 133, 83-110.

Boorman, S. (1975): "A Combinatorial Optimization Model for Transmission of Job Information through Contact Networks," *Bell Journal of Economics* 6, 216-249.

Calvo-Armengol, A. (2004): "Job Contact Networks," *Journal of Economic Theory* 115, 191-206.

Colman, J. (1966): Medical Innovation: A Diffusion Study, Second Edition. New York: Bobbs-Merrill.

Dare, O.O., and J.G. Cleland (1994): "Reliability and Validity of Survey Data on Sexual Behaviour," *Health Transition Review* 4 (Supplement), 93-110.

Dutta, B., S. Ghosal and D. Ray (2005): "Farsighted Network Formation," *Journal of Economic Theory* 122, 143-164.

Dutta, B., and S. Mutuswami (1997): "Stable Networks," *Journal of Economic Theory* 76, 322-344.

Dutta, B., A. van den Nouweland, and S. Tijs (1995): "Link Formation in Cooperative Situations," *International Journal of Game Theory* 27, 245-256.

Echenique, F., and J. Oviedo (2006): "A Theory of Stability in Many-to-Many Matching Markets," *Theoretical Economics* 1, 233-73.

Ellison, G. (1993): "Learning, Local Interaction and Coordination," *Econometrica* 61, 1047-1072.

Fafchamps, M., and S. Lund (2000): "Risk-Sharing Networks in Rural Philippines," mimeo, Stanford University.

Fenton, K.A., A.M. Johnson, S. McManus, and B. Erens (2001): "Measuring Sexual Behaviour: Methodological Challenges in Survey Research," *Sexually Transmitted Infections* 77, 84-92.

Fortson, J. (2008): "The Gradient in Sub-Saharan Africa: Socioeconomic Status and HIV/AIDS," *Demography* 45, 303-322.

Frenzen, J. K., and H. L. Davis (1990): "Purchasing Behavior in Embedded Markets," *Journal of Consumer Research* 17, 1-12.

Hall, P., (1935): "On representatives of subsets," *Journal of London Mathematical Society* 10, 26-30.

Gale, D., and L. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly* 69, 9-15.

Gray, R.H., M.J. Wawer, R. Brookmeyer, N.K. Sewankambo, D. Serwadda, F. Wabwire-Mangen, T. Lutalo, X. Li, T. vanCott, T.C. Quinn, and the Rakai Project Team (2001): "Probability of HIV-1 Transmission per Coital Act in Monogamous Heterosexual, HIV-1 Discordant Couples in Rakai, Uganda," *The Lancet* 357, 1149-1153.

Helpman, E. (1993): "Innovation, Imitation, and Intellectual Property Rights," *Econometrica* 61, 1246-1280.

Ioannides, Y.M., and L.D. Loury (2004): "Job Information Networks, Neighborhood Effects, and Inequality," *Journal of Economic Literature* 42, 1056-1093.

Kandori, M., G. Mailath, and R. Rob (1993): "Learning, Mutations and Long Run Equilibria in Games," *Econometrica* 61, 29-56.

Kohler, H., J.R. Behrman and S.C. Watkins (2007): "Social networks and HIV/AIDS risk perceptions." *Demography*, 44, 1–33.

Kojima, F., and P.A Pathak (2009): "Incentives and Stability in Large Two-Sided Matching Markets," *American Economic Review* 99, 608-627.

Jackson, M.O, and B.W. Rogers (2007): "Meeting Strangers and Friends of Friends: How Random are Social Networks?," *American Economic Review* 97, 890-915.

Jackson, M. O., and B.W. Rogers (2007): "Relating Network Structure to Diffusion Properties Through Stochastic Dominance," *The B.E. Press Journal of Theoretical Economics* 7, 1-13.

Jackson, M.O., and A. van den Nouweland (2005): "Strongly Stable Networks," *Games and Economic Behavior* 51, 420-444.

Jackson, M.O., and A. Watts (2002): "The Evolution of Social and Economic Networks," *Journal of Economic Theory* 106, 265-295.

Jackson, M.O., and A. Wolinsky (1996): "A Strategic Model of Economic and Social Networks," *Journal of Economic Theory* 71, 44 74.

Jackson, M.O., and L. Yariv (2007): "Diffusion of Behavior and Equilibrium Properties in Network Games," *American Economic Review* 42, 92-98.

Jaya, M., J. Hindin, and S. Ahmed (2008): "Differences in Young People's Reports of Sexual Behaviors According to Interview Methodology: A Randomized Trial in India," *American Journal of Public Health* 98, 169 - 174.

Kranton, R., and D. Minehart (2001): "A Theory of Buyer-Seller Networks," *American Economic Review* 91, 485-508

Lopez-Pintado, D. (2008): "Contagion in Complex Networks," *Games and Economic Behavior* 62, 573-590.

Mansfield, E. (1985): "How Rapidly Does New Technology Leak Out," *Journal of Industrial Economics* 34, 217-224.

Mansfield, E., A. Romeo, M. Schwartz, D. Teece, S. Wagner, and P. Brach (1982): Technology Transfer, Productivity, and Economic Policy, W. W. Norton, New York.

Mensch, B.S., P.C. Hewett, A.S. Erulkar (2003): "The Reporting of Sensitive Behaviour by Adolescents: A Methodological Experiment in Kenya," *Demography* 40, 247-268.

Mishra, V., A. Medley, R. Hong, Y. Gu, and B. Robey (2009): Levels and Spread of HIV Seroprevalence and Associated Factors: Evidence from National Household Surveys, DHS Comparative Reports No. 22. Calverton, Maryland, USA: Macro International Inc.

Mishra, V., S.B. Assche, R. Greener, M. Vaessen, R. Hong, P.D. Ghys, J.T. Boerma, A. Van Assche, S. Khan, S. Rutstein (2007) "HIV infection does not disproportionately affect the poorer in sub-Saharan Africa" *AIDS* 21, s17-s28.

Montgomery, J. (1991): "Social Networks and Labor Market Outcomes," *The American Economic Review* 81, 1408-1418.

Myerson, R. (1977): "Graphs and Cooperation in Games," *Mathematics and Operation Research* 2, 225-229.

Nussbaum, M.C., and J. Glover (eds.) (1995): Women, Culture, and Development: A Study of Human Capabilities, Oxford: Clarendon Press; New York: Oxford University Press.

Page Jr., F.H., M.H. Wooders, and S. Kamat (2005): "Networks and Farsighted Stability," *Journal of Economic Theory* 120, 257-269.

Pastor-Satorras, R., and A. Vespignani (2000): "Epidemic Spreading in Scale-Free Networks," *Physical Review Letters* 86, 3200-3203.

Pastor-Satorras, R., and A. Vespignani (2001): "Epidemic Dynamics and Endemic States in Complex Networks," *Physical Review E* 63, 066-117.

Pittman, F.S. (1990): Private Lies: Infidelity and Betrayal of Intimacy, W. W. Norton & Company.

Pittman, F.S. (1999): Grow Up!: How Taking Responsibility Can Make You a Happy Adult, Golden Guides from St. Martin's Press.

Pongou, R. (2008): "Fidelity to Groups," Mimeo. Department of Economics, Brown University.

Pongou, R. (2009b): "Anonymity and Infidelity: Ethnic Identity, Strategic Cross-Ethnic Sexual Network formation, and HIV/AIDS in Africa," Mimeo. Department of Economics, Brown University.

Pongou, R., and R. Serrano (2009): "A Dynamic Theory of Fidelity Networks with an Application to the Spread of HIV/AIDS," Department of Economics Working Paper 2009-2, Brown University.

Powers, K.A., C. Poole, A.E. Pettifor, and M.S. Cohen (2008): "Rethinking the heterosexual infectivity of HIV-1: a systematic review and meta-analysis," *The Lancet Infectious Diseases* 8, 553-563.

Quinn, T.C., M.J. Wawer, N. Sewankambo, D. Serwadda, C. Li, F. Wabwire-Mangen, M.O. Meehan, T. Lutalo, R.H. Gray (2000): "Viral Load and Heterosexual Transmission of Human Immunodeficiency Virus Type 1," *New England Journal of Medicine* 342, 921-929.

Reichheld, F.F. (1996): The Loyalty Effect: The Hidden Force Behind Growth, Profits and Lasting Value. Boston: Harvard Business School Press.

Roth, A., and M. Sotomayor (1989): Two Sided Matching, Econometric Society Monographs 18; Cambridge University Press, Cambridge.

Sen, A. (1999): Development as Freedom, Oxford: Oxford University Press.

Slikker, M., and A. van den Nouweland (2001a): Social and Economic Networks in Cooperative Game Theory, Kluwer Academic Publishers: Boston, MA.

Slikker, M., and A. van den Nouweland (2001b): "A One-stage Model of Link Formation and Payoff Division," *Games Economic Behavior* 34, 153-175.

Sotomayor, M. (2003): "A Labor Market with Heterogeneous Firms and Workers," *International Journal of Game Theory* 31, 269-283.

UNAIDS, 2008. The 2008 Report on the Global AIDS Epidemic. UNAIDS/08.25E / JC1510E.

Watts A. (2001): "A Dynamic Model of Network Formation," *Games and Economic Behavior* 34, 331-341.

Wollstonecraft, M. (1792): A Vindication of the Rights of Woman with Strictures on Moral and Political Subjects. London: Joseph Johnson.

World Health Organization (2003): Gender and HIV/AIDS. World Health Organization, Department of Gender and Women's Health, Geneva, Switzerland.

Zaba, B., E. Pisani, E. Slaymaker, and J.T. Boerma (2004): "Age at First Sex: 'Understanding Recent Trends in African Demographic Surveys," *Sexually Transmitted Diseases* 80 (Suppl. II), ii28-ii35.

Figure 1-1

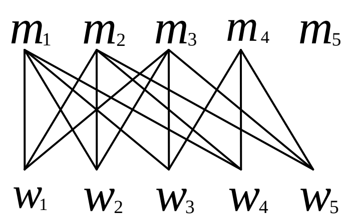

Figure 1-2

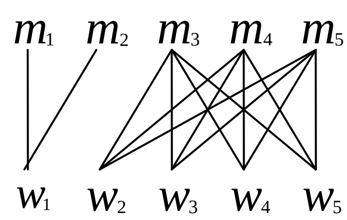

Figure 2-1

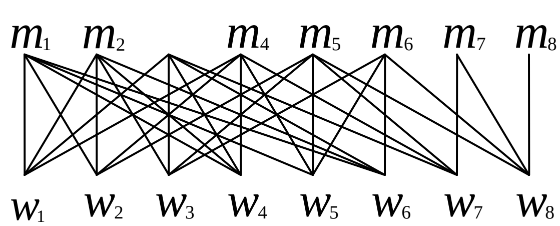

Figure 2-2

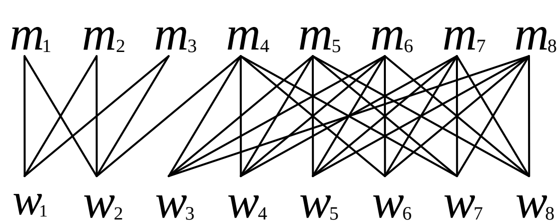

Figure 3-1

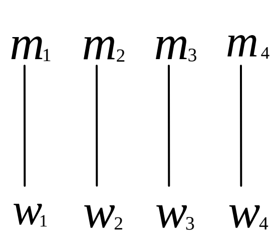

Figure 3-2

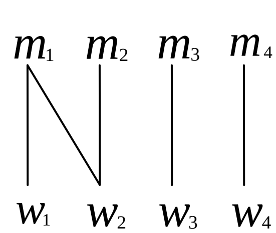

Figure 4-1

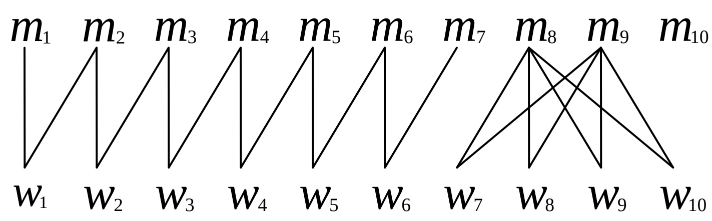

Figure 4-2

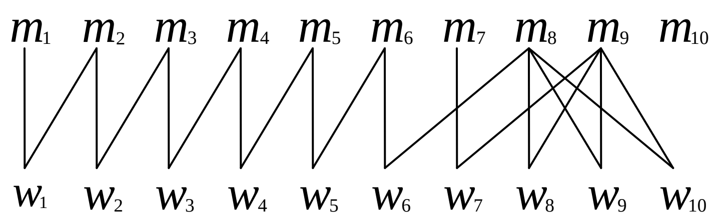

Figure 4-3

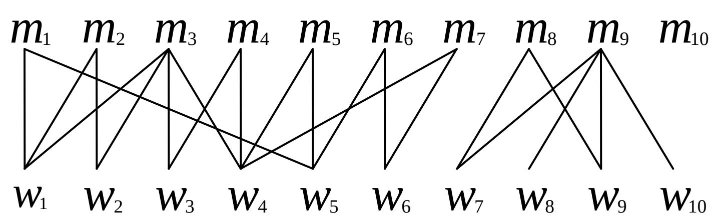

Figure 5-1

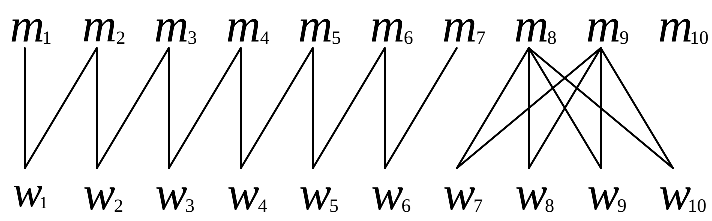

Figure 5-2

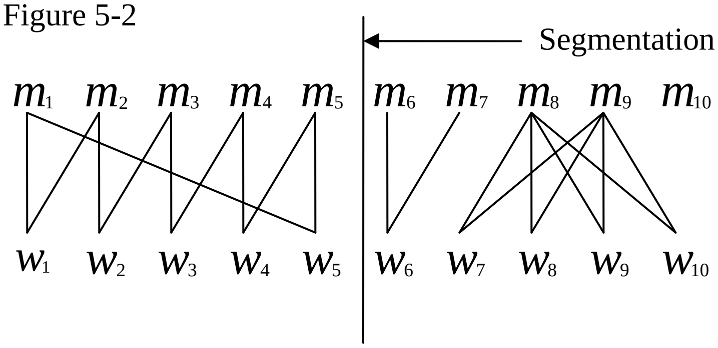

Figure 6: Relationship between the number of segmentation criteria and the maximal size of a female-information-biased economy

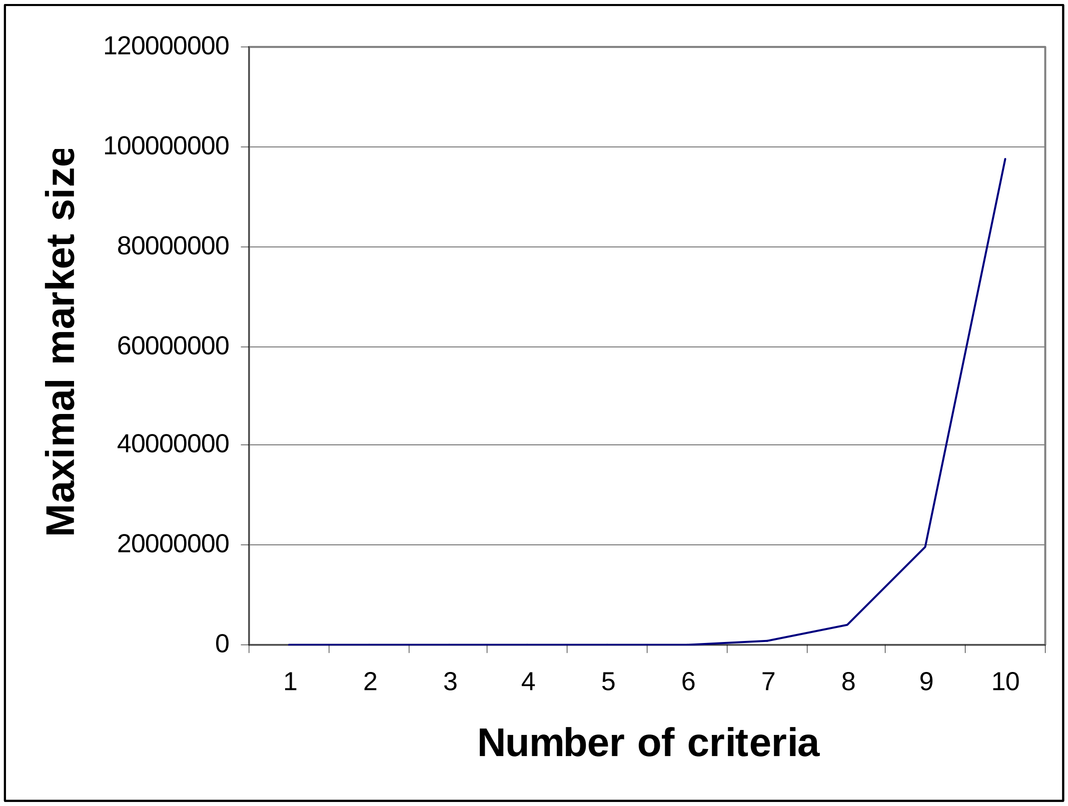

Figure 7-1

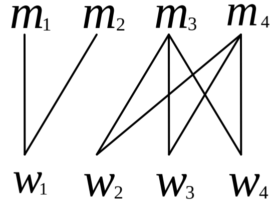

Figure 7-2

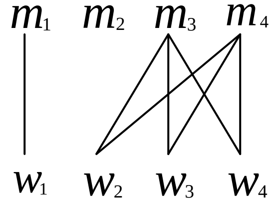

Figure 8-1

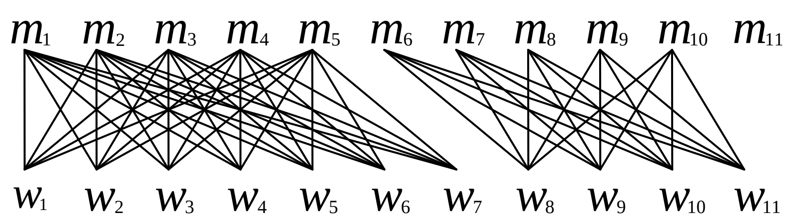

Figure 8-2

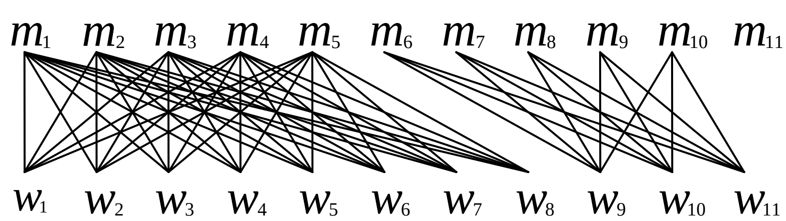

Figure 8-3

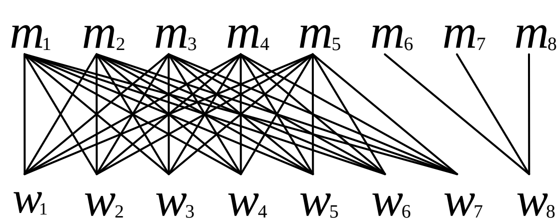

Figure 8-4