Dynamic Delegation: Specialization and Favoritism*

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Abstract

I study dynamic delegation of heterogeneous projects to agents with diverse capabilities. Each agent's (e.g., division managers, employees) true ability to carry out projects varies over time based on his expertise and private idiosyncratic compatibility with the specifics of the current project. The principal's (e.g., headquarters, management) ability to credibly provide incentives in order to delegate efficiently hinges on the degree of specialization across agents. Efficiency - where each project is assigned to the agent best suited for it - is attainable if and only if specialization does not exceed a threshold. If specialization is sufficiently high, communication breaks-down entirely. The derivation of a necessary and sufficient condition for efficiency, at fixed discounting, enables constructing a simple class of delegation rules that are efficient whenever any rule is, and deriving the key properties of such rules. These properties shed light on the potential benefits or drawbacks of certain management practices in the absence of monetary incentives. I establish an equivalence between ex-post equilibria – in which agents' ex-post incentive constraints are satisfied in each period – and a natural class of equilibria in which delegation is driven by past performance, but does not condition directly on past communication. The analysis also studies optimal delegation when the principal is unable to discriminate between the agents, and characterizes the cost associated with this inability.

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1 Introduction

Matching projects or tasks with those best suited for them is one of the most prominent goals in organizational decision making. A growing body of empirical evidence has documented a trend towards flexible workplace practices, which involve highly dynamic and less specialized job assignment, and in which divisions, teams or employees are able (and required) to perform a diverse and often overlapping range of tasks.[1] Since agents (e.g., division managers, employees) are typically better informed than the principal (e.g., headquarters, management) about how well suited they truly are given the specifics of each task, a principal's ability to efficiently delegate responsibilities and respond to a changing environment hinges on her ability to elicit such private information over time. In many economic environments, however, this ability is hindered by both a misalignment of interests (e.g., while headquarters aims to maximize overall profits, division managers are likely biased toward the profits of their own divisions) and a limited set of tools with which to reconcile this misalignment. In particular, in many environments of interest, the principal is limited in her ability to provide incentives through monetary compensation, or commit to future allocation decisions.[2]

Many environments share these features. A firm's headquarters may wish to choose a division to head up a new project or acquire a new client.[3,4] A district attorney chooses a prosecutor to lead a high exposure case. A military general selects an elite unit for a prestigious operation.[5] In such environments, (how) can the principal efficiently delegate tasks over time among her heterogeneously specialized but privately informed agents? What is the role of specialization in shaping dynamic incentive provision?

To study such questions, I introduce the following dynamic environment. A principal faces a stream of projects - whose types differ over time - to delegate to one of two agents. Each agent's true ability to successfully carry out a project is private and varies over time based on his specialization (known) in the current type of project and idiosyncratic compatibility (private) with its specific characteristics. The more specialized an agent is in a given type of project, the more likely he is to be well

[1]See, e.g., Osterman (1994), Brickley et al. (1996), OECD (1999), Caroli and Van Reenen (2001) (and references therein).

[2]The ability to exchange money within an organization is often hampered by various managerial and legal constraints. Indeed, it has long been observed (e.g., Cyert and March (1963)) that the promise of future influence is a prevalent method of payment within organizations. Such constraints also limit the principal's ability to contractually commit to future decisions. For example, courts typically do not enforce contracts between parties within the same organization (see, e.g., Bolton and Dewatripont (2013)).

[3]For example, within a consulting firm, divisions such as risk-management and process-improvement often share significant overlap in capabilities and vie for the same clients, as do cyber-security and business continuity management.

[4]As Ben-Porath et al. (2014) point out, in such a scenario, each division may be interested in the resources associated with the new project, independently of whether it is best equipped for it. Furthermore, using these resources as a means of providing monetary incentives appears to be wasteful.

[5]E.g., the UK's Parachute Regiment and the Royal Marines differ in their specialization, but share a significant overlap in capabilities, which has led to a decades-long competition for prestige. See Berbéri and Castro (2016).

suited for such projects and the less likely he is to be well suited for others. Indeed, gaining knowledge in a certain field often comes at the expense of another. In each period, the agents communicate with the principal, who subsequently makes her delegation decision. The outcome of each project is publicly observed but monitoring is imperfect - while an agent well equipped for a project is naturally more likely to be successful than one who is not, the latter may nevertheless carry out the project successfully; similarly, a well equipped agent may nevertheless fail. Furthermore, while the principal cares about the organization's performance as a whole, each agent is only concerned with his own outcomes. To provide incentives, the principal can rely only on credible promises; in particular, she cannot offer monetary incentives or formally commit to future decisions.[6]

The first results of the paper concern how the degree of specialization across agents shapes the principal's ability to delegate efficiently. I first derive a necessary and sufficient condition for *efficiency* (i.e., the principal's first-best), where each project is delegated to an agent best suited for it. Efficiency is attainable if and only if the degree of specialization does not exceed a certain threshold. Hence, specialization limits the scope for communication between the principal and the agents. Moreover, if agents are sufficiently specialized, communication is not only inefficient but breaks-down entirely. That is, agents cannot be incentivized to any extent and *decentralization* - where each agent is assigned the type of project he is specialized in regardless of whether he is truly suited for it or not - is inevitable.

The environment features a combination of adverse selection and imperfect monitoring. One of the contributions of the paper is to shed light on the interaction between these two components. In order to study this interaction and highlight particular features relevant for dynamic incentive provision, the analysis focuses on ex-post equilibria. Ex-post equilibria require that in each period, taking expectation over the future path of play, each agent's announcement remain optimal irrespective of his belief about the other agent's current type (see Athey and Miller (2007) and Bergemann and Välimäki (2010)). Such equilibria have the appeal of being robust to information leakage as well as certain model misspecifications.[7] Eliminating the need for stringent (simultaneous and private) communication protocols is particularly appealing in the context of organizations, in which it may be undesirable to restrict the way divisions or employees share information.

A key feature of the analysis is the derivation of a necessary and sufficient condition for efficiency (for fixed primitives, including patience). In particular, the approach differs from a folk-theorem analysis. Such a characterization is instrumental for the second component of the paper, which studies the design of rules for efficient delegation. In particular, I consider the following questions: What rules induce efficient communication *whenever possible* (i.e., over the entire region of primitives for which

[6] Note that the inability to provide monetary incentives does not imply the absence of monetary compensation altogether. Indeed, in many public organizations monetary compensation is fixed and nonnegotiable.

[7] Specifically, such equilibria require no knowledge of the distribution over potential payoff-irrelevant signals (see Bergemann and Morris (2005)). The presence of payoff-irrelevant signals which may be correlated with agents' types can generate arbitrary higher-order beliefs. While this complication can be avoided by assuming away the existence of payoff-irrelevant signals altogether, higher-order beliefs may have important equilibrium consequences (see Weinstein and Yildiz (2007)).

efficiency is attainable)? What are the key properties of such rules?

In order to delegate efficiently, the principal must use some form of *dynamic favoritism* – the credible promise (threat) of a future (dis)advantage. Such favoritism may take many forms. The key difficulty in designing efficient rules is that punishing one agent rewards another. A rule that is efficient whenever possible must perfectly balance the incentives of both agents, generating high variation in continuation payoffs while remaining credible. I characterize a simple class of rules that achieve this, and show that they are efficient whenever possible. For example, one such rule – *maximal-priority* – features the following dynamics. In each period, one of the agents is *favored*. If both agents request the project, or both do not, the project is assigned to this favored agent, who continues to be favored as long as he does not fail. If and only if a favored agent fails he immediately loses favor, and the other agent is favored instead. Importantly, note that a favored agent 'takes responsibility' for failing in a project even if he did not claim to be suited for it. While remarkably simple, this rule in fact incentivizes both agents to be truthful in all periods over the largest region of primitives for which efficiency is attainable (by any rule).

The results speak to the potential effectiveness of certain management styles in the absence of monetary incentives. First, dynamics under maximal-priority are *failure-driven* – priority is assigned based on solely the (most recent) failure of the favored agent; in particular, *success is not (directly) rewarded*. I show that this property is indeed essential for efficiency among a class of Markov-priority rules. Hence, in this sense, the stick is more effective than the carrot.[8] Second, as maximal-priority illustrates, efficiency does not require the principal's knowledge of project types or the agents' specialization. Indeed, in many circumstances, the principal may not posses the expertise required to assess the nature of incoming projects and their relation to the agents' specialization. The results show that such ignorance need not be costly for the principal.

The third main result sheds light on the interplay between adverse selection and imperfect monitoring and the role of each of these components in shaping dynamic incentives. In each period, efficiency requires the principal to condition her decision not only on current announcements by the agents but also on past (payoff-irrelevant) information, which may include past delegation decisions, communication, and performance. I introduce a class of *performance-based* equilibria, in which the ex-post requirement on equilibria is no longer imposed, but in each period the principal's decisions do not condition directly on past communication; that is, past communication shapes current decisions only through its impact on past delegation decisions and performance. This is the case, for example, under the maximal-priority rule mentioned above. I show that efficiency can be attained in an ex-post equilibrium if and only if it can be attained in a performance-based equilibrium. Furthermore, the sets of efficient equilibrium payoffs under the two coincide. On the one hand, a performance-based

[8] In "Inside Intel" (*Harvard Business Review*, Dec. 1996), Andrew Grove's (former Intel CEO) view on the effectiveness of a certain measure of fear within organizations is described as follows: "Fear can be a healthy antidote to the complacency that success often breeds". Such a view contrasts with the alternative view (see, e.g., Deming (1986)) that fear has no place within organizations.

equilibrium entails less flexibility in conditioning decisions on past information. On the other hand, ex-post incentive compatibility is relaxed. The analysis shows that the flexibility lost by the former restriction is precisely recovered by the relaxation of incentive constraints.[9]

In some economic environments favoring one agent over another, even temporarily, may be undesirable.[10] In such circumstances it may be in the interest of the principal to signal that she is not biased by avoiding asymmetric treatment. What is the cost of such a decision? To answer this question, I study *non-discriminatory* equilibria, where at the beginning of each period the expected continuation payoffs of the agents are required to be equal to one another. Efficiency is unattainable without discrimination. I characterize the highest payoff the principal can obtain without discriminating, and consequently derive the cost of restricting attention to non-discriminatory rules (alternatively, the benefit from favoritism). Furthermore, I construct a non-discriminatory rule that attains the principal's highest payoff without discrimination. Under this rule, each project is delegated by default to the agent more specialized in its type, unless the less specialized agent requests it. A decentralization phase is triggered if and only if an agent fails in a project in which he is less specialized. Interestingly, when the principal cannot discriminate, priority over a project is most effective when granted to the agent who is *less* specialized.

Finally, the model is extended to allow for correlation in agents' types over time in order to study the role of persistence in shaping dynamic incentive provision. Whether persistence is helpful or harmful for incentive provision is not immediate. On the one hand, the possibility of signaling may strengthen agents' incentives to be untruthful. On the other hand, persistence also permits the principal to infer information about the agents' future types based on past outcomes. I show that *persistence hinders efficiency*. In particular, efficiency is attainable if persistence is not too high, but not otherwise.

The rest of the paper is organized as follows. The remainder of this section includes a discussion of the related literature. The model is introduced in Section 2. Section 3 contains the results concerning the (im)possibility of efficiency, decentralization and the role of specialization. Section 4 studies the design of rules for efficient delegation. Section 5 establishes the equivalence between ex-post equilibria and their performance-based counterpart. Section 6 studies equilibria without discrimination and characterizes the principal's benefit from favoritism. Finally, Section 7 includes a discussion of possible extensions, while Section 8 concludes. The Appendix contains all proofs omitted from the main text.

[9]In particular, any improvement in the primitive region under which efficiency is attainable beyond the class of performance-based equilibria necessarily implies a loss of robustness.

[10]Asymmetric treatment may raise concerns that the principal is biased towards an agent. For example, if employees within a division believe that headquarters is biased toward another division, this may reduce motivation or distort incentives (e.g., by inducing counterproductive efforts to overcome such a bias).

1.1 Related literature

This paper is related to several strands of literature. First, it contributes to the literature on delegation, which studies the incentive problems in organizations due to the combination of misaligned preferences and hidden information.[11] In common with much of this literature is the assumption that the principal is unable to use monetary transfers as a means of aligning incentives. This assumption reflects the view that within organizations, various managerial and legal constraints often impede such monetary transfers.

While this literature has mostly focused on static environments, in recent years a growing literature has begun to study delegation and mechanism design without money in dynamic settings. Guo and Hörner (2017) study optimal dynamic mechanisms without money. An uninformed principal, who has full commitment power, decides in each period whether to provide a costly perishable good to an agent, whose valuation evolves over time. Lipnowski and Ramos (2016) and Li et al. (2017) both study a repeated game between a principal and a better informed agent, where the principal repeatedly relies on the agent to perform tasks, but cannot utilize monetary incentives or commit to future decisions. In common with these papers is the focus on non-monetary dynamic incentives, and (with respect to the latter two) the principal's inability to commit.[12] However, among other differences, while these papers consider environments with a single agent, I study dynamic delegation to multiple (and heterogeneous) agents. In particular, the focus is on how delegation should be organized among the agents over time, and how heterogeneity between the agents shapes dynamic incentive provision.

The environment studied in this paper is most closely related to Andrews and Barron (2016) and de Clippel et al. (2017). Andrews and Barron (2016) study relational contracting with multiple agents.[13] A firm repeatedly contracts with one of multiple suppliers, whose productivity is redrawn in each period and is observed by the principal, but whose effort is subject to moral hazard. Output is stochastic and non-contractible, and monitoring is private – suppliers observe only their own relationship with the principal. The principal can provide monetary compensation, but cannot commit to such compensation. In contrast, the environment in the current paper involves a combination of adverse selection and imperfect (public) monitoring. Furthermore, a key difference is the principal's inability to use monetary incentives. The results in the current paper show that the absence of monetary incentives leads to dynamics that contrast sharply with those that arise in their environment. In de Clippel et al. (2017), a principal with limited attention designs an idea-selection mechanism to repeatedly choose among multiple agents who wish to have their ideas implemented. As in the envi-

[11]The seminal work is Holmström (1977, 1984). See Armstrong and Vickers (2010), Amador and Bagwell (2013), Frankel (2014) and Ambrus and Egorov (2017) for more recent contributions.

[12]See also Alonso and Matouschek (2007), Guo (2016), Bird and Frug (2017) and Deb et al. (2017) for related contributions.

[13]Several other papers in the relational contracting literature, including Levin (2002), Board (2011) and Urgun (2017), consider environments with multiple agents. As in Andrews and Barron (2016), a key difference with respect to this paper is the principal's ability to provide incentives through monetary transfers.

ronment of the current paper, the principal can neither commit nor use transfers in order to incentivize the agents to suggest only good ideas. Among other differences, a key difference in the current paper is the derivation of a necessary and sufficient condition for efficiency, at fixed primitives (including patience). Such an approach enables identifying results that need not hold when focusing on asymptotically optimal rules (as discounting vanishes), and to study how effective different delegation rules are relative to one another. Section 7.3 discusses the relationship with these two papers in greater detail.

This paper is also related to the literature on "trading favors" studied in Möbius (2001) and Hauser and Hopenhayn (2008), in which players have private opportunities to do favors for one another. An important distinguishing feature is that in these papers agents benefit (in the stage game) at the expense of one another. See also Abdulkadiroğlu and Bagwell (2013) and Olszewski and Safronov (2017a,b) for more recent contributions. The analysis is also related to Athey and Bagwell (2001), in which colluding firms play a repeated Bertrand game and are privately informed about their respective costs. In a binary type model, they show how the firms can use future "market-share favors" in order to achieve first-best payoffs. Among other differences, the environment I consider features both adverse selection and imperfect monitoring. (In general, the literature on collusion has typically modeled these two issues separately.) Furthermore, the analysis imposes a robustness criterion in the form of a restriction to ex-post equilibria – where ex-post incentive constraints are imposed in each period (taking expectation over the future path of play) – which also sheds light on the interaction between these two components.[14] Another key difference with respect to these papers is the derivation of a condition both necessary and sufficient for efficiency. This condition plays an important role in the subsequent analysis in the paper, which focuses on equilibrium behavior rather than on payoffs.

A large literature on organizational decision making has argued that the advantages from specialization are constrained by the need to coordinate specialized activities (see, e.g., Becker and Murphy (1992), Bolton and Dewatripont (1994), Garicano (2000) and Dessein and Santos (2006)).[15] The more specialized employees or divisions within an organization are, the more communication is necessary to coordinate activities between them. The analysis in this paper sheds light on a different aspect of the tradeoff between specialization and communication by studying a dynamic environment and abstracting away from joint coordinated activity in order to focus on competition among agents within the organization. The analysis shows that when such competition is present, specialization hinders

[14]Such equilibria were separately introduced in Athey and Miller (2007) and in Bergemann and Välimäki (2010), and are often studied in the dynamic mechanism design literature (see also Athey and Segal (2013)). This concept is also related to belief-free equilibria in repeated games with imperfect private monitoring, as introduced in Piccione (2002) and Ely and Välimäki (2002) and extensively studied in Ely et al. (2005). Hörner and Lovo (2009) and Hörner et al. (2011) study belief-free equilibria in games with incomplete information. Fudenberg and Yamamoto (2010) study ex-post equilibria in repeated games with imperfect public monitoring in which the payoffs and monitoring structure are unknown.

[15]See also Dessein and Matouschek (2008), Fuchs et al. (2014) Alonso et al. (2015), Dessein et al. (2016) for related contributions.

dynamic incentive provision by creating endogenous communication costs. The more specialized agents are, the more costly communication becomes. When specialization is too high, communication breaks-down entirely.

In the environment of this paper, the principal uses dynamic favoritism – the promise (threat) of future (dis)advantage – as a means of aligning incentives. Such strategic use of favoritism also arises in static mechanism design environments in which monetary transfers are absent. In Ben-Porath et al. (2014), a principal allocates a good among multiple agents, each of which is privately informed about the principal's value from allocating the good to him. The principal can verify agents' private information at a cost, and transfers are not permitted. A favored-agent mechanism consists of a favored agent and a threshold value. If all other agents report values below the threshold, the good is allocated to the favored agent. Otherwise, the agent who reports the highest value is checked and receives the good if and only if his report is confirmed. Ben-Porath et al. (2014) show that all optimal mechanisms are essentially randomizations over optimal favored-agent mechanisms.[16]

The paper is also related to the literature on intrafirm resource allocation in finance. Harris et al. (1982) study the problem of how a firm should allocate a resource among divisions when its productivity in each division is privately known to the respective division manager. In a static environment in which transfers are permitted, they establish the optimality of certain transfer pricing schemes. The results in this paper shed light on the nature of such resource allocation in a dynamic environment, when the organization faces certain costs or constraints in utilizing such transfer schemes.

2 Model

Time is discrete, indexed by $t = 1, ..., \infty$. In each period, a risk-neutral principal, indexed by $i = 0$, receives a new project she may delegate to one of two risk-neutral agents, 1 and 2. Each period-t project may take one of two possible types, $\omega_t \in \{A, B\}$, with equal probability. The type of project is publicly observed.

Specialization

Agents are privately informed in each period about their suitability for the current type of project. Specifically, agent i's period-t type is denoted by $\theta_{it} \in \{\alpha, \beta\}$, where type α (β) is *suited* for a type A (B) project, and unsuited for a type B (A) project. If an agent is suited for a project, he is successful in carrying it out with probability $\overline{\mu} \in (0, 1)$. An unsuited agent may nevertheless succeed, but with a lower probability, $\underline{\mu} \in (0, \overline{\mu})$. Each agent i's type is drawn independently in each period $t \geq 1$,

[16] Antic and Steverson (2017) demonstrate how a principal can benefit by coordinating her actions when preferences exhibit complementarities, and show that such coordination may also result in strategic favoritism.

according to his *specialization*, which is captured by[17]

$$\phi^i := \Pr(\theta_{it} = \alpha) \in (0,1).$$

Hence, the higher is ϕ^i, the more likely agent i is to be suited for a type-A project, and the less likely he is to be suited for a type-B project.[18] For expositional purposes, we assume agents are heterogeneous in their specialization. Specifically, let $\phi \in [\frac{1}{2}, 1)$, $\phi^1 = \phi$ and $\phi^2 = 1 - \phi$. Hence, agent 1 is specialized in projects of type A, while agent 2 is specialized in projects of type B. The degree of specialization across agents is therefore measured by ϕ. The higher is ϕ, the more *specialized* the agents are; that is, the more likely they are to be suited for the projects in which they are specialized, and the less likely they are to be suited for projects in which they are not specialized.[19]

Timing

The timing of each stage-game, illustrated in Figure 1, is the following. First, a project arrives, its type $\omega \in \{A, B\}$ is publicly observed, and each agent $i = 1, 2$ privately observes his own type, $\theta_i \in \{\alpha, \beta\}$. Agents then simultaneously and publicly announce whether they are suited for the project or not, where $m_i = 1$ denotes i's announcement that he is suited for the project, and $m_i = 0$ his announcement that he is not. Next, the principal publicly makes an allocation decision $(x_1, x_2) \in X := \{(0,0), (0,1), (1,0)\}$. The principal's decision to allocate the current project to agent i is denoted by $x_i = 1$, whereas $x_i = 0$ denotes her choice to withhold the project from i. Each project can be delegated to at most a single agent, while $(x_1, x_2) = (0, 0)$ reflects the principal's decision to carry out the project herself, in which case her payoff is 0. The project is then carried out and its outcome $y \in \{0, S, F\}$ is publicly observed. Success and failure are represented by S and F, respectively, and 0 denotes the outcome in case the project has not been delegated. Finally, the principal and the agents observe the realization of a public randomization device.[20]

The principal cannot use monetary transfers in order to incentivize the agents to reveal their private information, nor can she commit to future allocation decisions. Monetary compensation may either be entirely infeasible, or it may be predetermined and fixed.

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[17] Section 7 considers an environment in which agents' private information evolves independently according to a two-state Markov chain.

[18] Note that the assumption that agents' types always realize either α or β is made only for convenience. It is irrelevant whether an agent who is not suited for the current type of project is indeed suited for the other. What matters is only whether agents are suited for the current type of project or not.

[19] The results extend to arbitrary specialization profiles $(\phi^1, \phi^2) \in (0,1)^2$. See Section 7 for a discussion.

[20] Randomization allows to convexify the set of equilibrium continuation payoffs. I do not explicitly model such randomization.

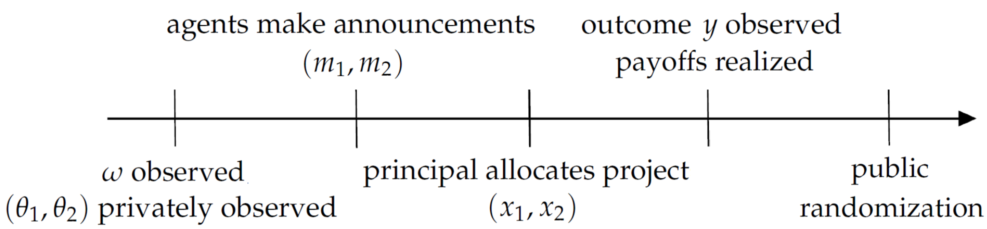


Figure 1: Timeline of the stage-game

Preferences

If an agent succeeds in a project, he receives a stage-game payoff of $\pi > 0$; otherwise, his payoff is 0. The payoff π may be interpreted as either a non-monetary reward, which may, for example, take the form of gain of professional experience, reputation or positive psychological reinforcement, or alternatively an exogenously fixed monetary payment.[21] Carrying out a project involves no cost for the agents (alternatively, this cost may be positive but small relative to $\underline{\mu}\pi$). Hence, the incentives of the agents and the principal are misaligned: While agents benefit only from success, they myopically prefer to be awarded a project regardless of whether they are truly suited for it or not.

The principal and agents share a common discount factor, $\delta \in (0, 1)$. The principal's period-t expected continuation payoff is given by

$$U_{0,t} := \mathbb{E} \sum_{r=t}^{\infty} \delta^{r-t} \mathbf{1}_{\{y_r = S\}},$$

the expected discounted number of successful projects. Each agent i's period-t expected continuation payoff is equal to the expected discounted sum of his payoffs,

$$U_{i,t} := \mathbb{E} \sum_{r=t}^{\infty} \delta^{r-t} x_{ir} \pi \mathbf{1}_{\{y_r = S\}}.$$

Throughout the paper, continuation payoffs are multiplied by $(1 - \delta)$ so as to be expressed as per-period averages. The term 'value' will often be used to refer to a player's expected average discounted continuation payoff.

Equilibrium and robustness

Following Fudenberg et al. (1994), we consider perfect public equilibria (PPE) – that is, sequential equilibria in which the players use public strategies. Denote by $h^t = \{\omega_r, m_r, x_r, y_r\}_{r=1}^{t}$ the period-t public history at the beginning of period $t + 1$, which includes the history of past project types, announcements, allocation decisions, and project outcomes until period t. h^0 denotes the null history.

[21]See Halac and Prat (2016) for a model of managerial attention in which a worker benefits from managerial attention, which may induce some form of psychological recognition of their performance.

Let $\mathcal{H}^t$ denote the set of period-t histories, and $\mathcal{H} := \bigcup_{t=1}^{\infty} \mathcal{H}^t$ the set of all histories. A public strategy for agent i is a sequence of functions $\{\mathcal{M}_{it}\}_{t=1}^{\infty}$, where each

$$\mathcal{M}_{it} : \mathcal{H}^{t-1} \times \{A, B\} \times \{\alpha, \beta\} \to \{0, 1\}$$

specifies an announcement $m_{it} \in \{0, 1\}$ as a function of the public history $h^{t-1} \in \mathcal{H}^{t-1}$, the current project type $\omega_t \in \{A, B\}$, and the agent's current private information $\theta_{it} \in \{\alpha, \beta\}$. An agent's announcement is truthful if $m_{it} = 1$ whenever $(\omega_t, \theta_{it}) \in \{(A, \alpha), (B, \beta)\}$, and $m_{it} = 0$ otherwise. Agent i's strategy is truthful if his announcement is truthful given any public history. The principal's public strategy is a sequence of functions $\{\chi_t\}_{t=1}^{\infty}$, where each

$$\chi_t : \mathcal{H}^{t-1} \times \{A, B\} \times \{0, 1\}^2 \to X$$

specifies an allocation decision $x \in X$ as a function of the public history $h^{t-1} \in \mathcal{H}^{t-1}$, the project type $\omega_t \in \{A, B\}$, and the period-t announcements $(m_{1t}, m_{2t}) \in \{0, 1\}^2$.

In the environment above, each of the agents has private information in each period. If an agent could delay the timing of his announcement, or learn about another agent's private information through other means, he might be led to change his announcement based on the additional information he obtains. A standard PPE breaks-down in such circumstances. We consider PPE that are robust in the following sense.

Definition 1 *An* ex-post PPE (XPPE) *is a PPE in which, in each period, taking expectation over the future path of play, each agent's announcement remains optimal irrespective of his belief about the other agent's past and current type.*

The above notion of ex-post equilibrium, imposing ex-post incentive compatibility in each period taking expectations over the future path of play, was introduced separately by Athey and Miller (2007) and Bergemann and Välimäki (2010).[22] Note that the 'ex-post' requirement applies to past and current signals, but not future ones.[23] An XPPE has the appeal of being robust to information leakage, as well as to the introduction of payoff-irrelevant signals and high-order beliefs (an XPPE can be constructed without any knowledge of the distribution over potential payoff-irrelevant signals; see Bergemann and Morris (2005)).

Such properties are particularly appealing in the context of organizations, in which it may be difficult or undesirable to restrict the way divisions or employees share information. In an XPPE, stringent (simultaneous and private) communication protocols are not necessary; division managers or employees can exchange information freely.[24]

[22]See also Miller (2012), who considers such ex-post equilibria in a model of collusion with adverse selection.

[23]For this reason, Bergemann and Välimäki (2010) use the term '*periodic* ex-post'.

[24]Even if it is indeed possible to enforce the communication protocols necessary for standard PPE, there may be other unmodelled costs or concerns associated with such protocols, which may be avoided by considering XPPE.

Efficiency and decentralization

An *efficient XPPE* is an XPPE in which at all periods $t \geq 1$ the project is allocated to an agent who is best suited for it. That is, in an efficient XPPE, any type-A or type-B project is assigned to an agent of type α or β, respectively, whenever such an agent exists. If an efficient XPPE exists, we say that efficiency is attainable. Furthermore, the principal's allocation rule is efficient if it is part of an efficient XPPE. Note that in an efficient XPPE the principal obtains her first-best value v^* (i.e., her first-best expected average payoff) equal to

$$\begin{aligned} v^* &= \phi(1-\phi)\underline{\mu} + (1-\phi(1-\phi))\,\overline{\mu} \\ &= \overline{\mu} - (\overline{\mu} - \underline{\mu})\phi(1-\phi). \end{aligned}$$

In any Nash equilibrium of the one-shot game the principal allocates each project to the agent who is specialized in it, regardless of the agents' announcements. An XPPE involving such *decentralization*, in which agent 1 receives all projects of type A and agent 2 all projects of type B, regardless of agents' announcements, will be referred to as a *communication-free* XPPE. The principal's value v^o in a communication-free XPPE is equal to

$$v^o = \phi\overline{\mu} + (1-\phi)\underline{\mu}.$$

3 Efficient Delegation

This section studies the role of specialization (as well as patience and the monitoring structure) in shaping incentive provision. Specifically, the following questions are considered: When can efficiency be attained? When is any form of 'informative' communication feasible?

3.1 Efficiency – a characterization

In this section, I identify a necessary and sufficient condition for the existence of an efficient XPPE, and characterize the set of efficient XPPE values. Importantly, the characterization applies for *any* set of primitives $\{\phi, \delta, \overline{\mu}, \underline{\mu}, \pi\}$; in particular, for a fixed discount factor δ, as opposed to studying the limit as $\delta \to 1$. That is, the approach differs from a folk-theorem analysis.

The set of equilibrium values is denoted by $\mathcal{E} \subseteq \mathbb{R}^3_+$, and the set of efficient XPPE values by

$$\mathcal{E}^* := \{(v_0, v_1, v_2) \in \mathcal{E} \mid v_0 = v^* \text{ and } v_1 + v_2 = \pi v^*\} \subseteq \mathbb{R}^3_+.$$

Proposition 1 identifies a necessary and sufficient condition for the set $\mathcal{E}^*$ to be non-empty, whereas Proposition 2 characterizes the set $\mathcal{E}^*$ completely.

Proposition 1 *For any $\delta, \underline{\mu}, \overline{\mu}$ there exists $\phi^*(\delta, \underline{\mu}, \overline{\mu}) < 1$ such that efficiency is attainable if and only if $\phi \in [\frac{1}{2}, \phi^*(\delta, \underline{\mu}, \overline{\mu})]$.*

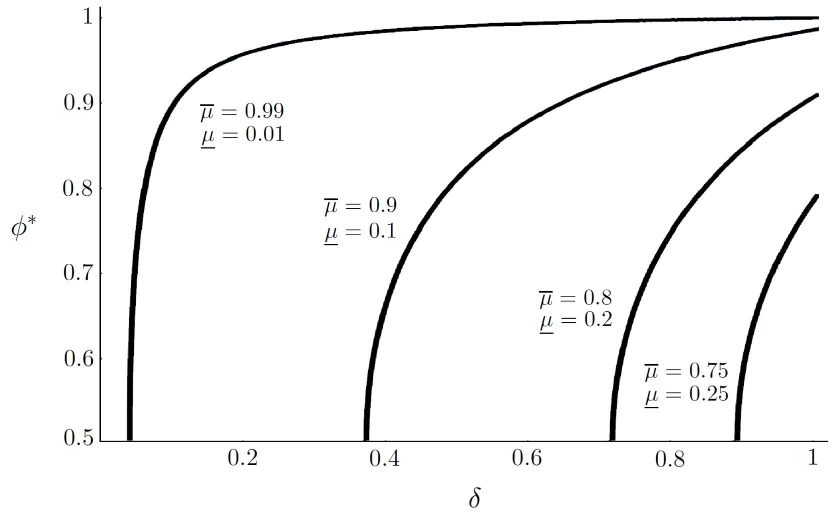


Figure 2: Threshold level of specialization ϕ^* below which efficiency is attainable, as a function of δ, given different success probabilities $(\overline{\mu}, \underline{\mu})$.

When agents' specialization exceeds a threshold, efficiency is unattainable. Figure 2 illustrates the threshold specialization level ϕ^* as a function of the discount factor δ, for different success probabilities $(\overline{\mu}, \underline{\mu})$. As Figure 2 illustrates, the region in which efficiency is attainable shrinks as agents become less patient. Note that if

$$\delta < \frac{4\underline{\mu}}{(\overline{\mu} - \underline{\mu})(1 - \underline{\mu}) + 4\underline{\mu}\overline{\mu}} \tag{1}$$

there cannot exists an efficient XPPE regardless of the level of specialization. However, for any $\delta, \underline{\mu}, \overline{\mu}$ the specialization threshold $\phi^*(\delta, \underline{\mu}, \overline{\mu})$ is strictly smaller than 1, hence high patience cannot fully compensate for high specialization. In other words, if agents are sufficiently specialized, efficiency cannot be attained regardless of how patient the agents are. The intuition for the results and the role of specialization are discussed in Section 3.2. We now describe the main steps of the proof of Proposition 1, the formal proof of which can be found in the Appendix.

Step 1. In order to describe $\mathcal{E}^*$, we first adapt the recursive methods of Abreu et al. (1990) and Fudenberg et al. (1994) to the current environment, which involves a combination of both adverse selection and imperfect monitoring, and in which XPPE are considered as opposed to standard PPE. XPPE payoffs are factored into two components: current-period payoffs and promised continuation payoffs, where the latter are themselves required to be XPPE payoffs. Observe that in an XPPE, the set of continuation payoffs can depend on the type of project, the profile of agents' announcements, the identity of the agent who carries out the project, and finally whether that agent was successful or not.

Denote by $\mathcal{Z} = (\chi, \mathcal{M}, \mathcal{V})$ a *policy*, which consists of (a) χ, a rule specifying the principal's allocation decision as a function of the agents' announcements and the type of project; (b) $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2)$, rules specifying the agents' announcements as a function of their own type and the current type of project;

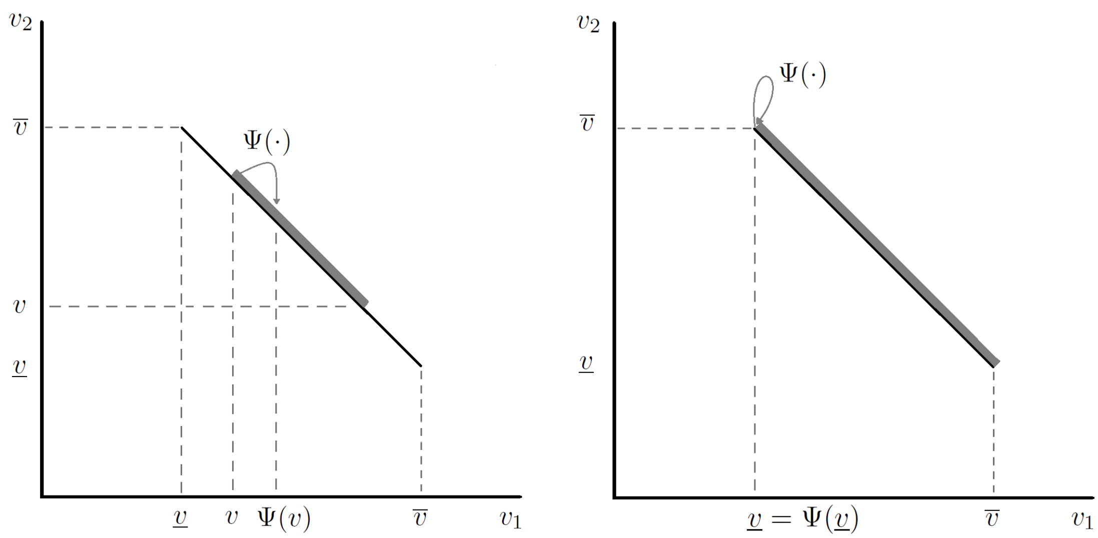


Figure 3: Illustration of Ψ

and (c) $\mathcal{V}$, a rule specifying the agents' promised continuation payoffs as a function of the outcomes described above. (See Appendix A for the formal definitions of concepts introduced in this section.)

A policy $\mathcal{Z}$ is *ex-post incentive compatible* (XIC) if agents' announcements are a best-response regardless of their belief about the other's type. Denote by $\Lambda_i(\mathcal{Z})$ the ex-ante expected payoff of player i under a policy $\mathcal{Z}$. A payoff vector $v \in \mathbb{R}^3_+$ is then *ex-post decomposable* on a set $V \subseteq \mathbb{R}^3_+$ if there exists a policy $\mathcal{Z}$ such that (i) $\mathcal{Z}$ is XIC; (ii) promised continuations are elements of V; and (iii) payoffs are dynamically consistent, i.e., $v_i = \Lambda_i(\mathcal{Z})$.[25]

In the Appendix, it is shown that the set of XPPE $\mathcal{E}$ can be characterized through *ex-post self-generation*. For any fixed set of primitives, $\{\phi, \delta, \overline{\mu}, \underline{\mu}, \pi\}$, if $v \in \mathcal{E}^*$, then v is ex-post decomposed by some $\mathcal{Z}$ on $\mathcal{E}^*$, where $\mathcal{Z}$ is such that allocation is efficient (i.e., the project is allocated to an agent suited for it whenever such an agent exists). In this case, $\mathcal{Z}$ is said to *ex-post E-decompose* v on $\mathcal{E}^*$. Moreover, either $\mathcal{E}^* = \emptyset$ or $\mathcal{E}^* = \text{co}\left(\{\hat{v}, \tilde{v}\}\right)$, for some $\hat{v}, \tilde{v} \in \left\{(v^*, v_1, v_2) \in \mathbb{R}^3_+ | v_1 + v_2 = \pi v^*\right\}$.

Step 2. Suppose $\mathcal{E}^* \neq \emptyset$. Then by compactness of $\mathcal{E}^*$ there exist minimal and maximal agent values that can be supported under an efficient XPPE,

$$\underline{v} := \min\left\{v \in \mathbb{R}_+ : (v^*, v, v_2) \in \mathcal{E}^*\right\}$$

and $\overline{v} := \pi v^* - \underline{v}$. Define a function Ψ that maps any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$ to the minimum of the set of values $\tilde{v} \in \mathbb{R}_+$ such that the payoff vector $(v^*, \tilde{v}, \pi v^* - \tilde{v})$ is ex-post E-decomposable on the interval $\text{co}\left\{(v^*, v, \pi v^* - v), (v^*, \pi v^* - v, v)\right\}$. See Figure 3 for an illustration. Observe that $\underline{v}$ is a fixed-point of Ψ. Denote by $\mathcal{J}(v)$ the set of policies that ex-post E-decompose $(v^*, \Psi(v), \pi v^* - \Psi(v))$ on

[25]Throughout the paper, whenever their is no confusion, we often use the term 'decomposable' rather then 'ex-post decomposable'.

co $\{(v^*, v, \pi v^* - v), (v^*, \pi v^* - v, v)\}$. For any value $v \in [\underline{v}, \frac{1}{2}\pi v^*]$ a key step of the proof is in solving for $\Psi(v)$ by characterizing completely the set $\mathcal{J}(v)$.

Step 3. The proof then identifies, using the above characterization, conditions on the underlying set of primitives necessary for the existence of a fixed-point of Ψ. These conditions boil down to the requirement that

$$\frac{1}{2} \leq \phi \leq \phi^* = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\left(\frac{\underline{\mu}\,(1-\delta\overline{\mu})}{\delta\left(\overline{\mu}-\underline{\mu}\right)\left(1-\underline{\mu}\right)}\right)}, \tag{2}$$

which is consequently necessary for the existence of an efficient XPPE.

Finally, this condition is shown to indeed be sufficient for the existence of efficient XPPE, which establishes the result.[26]

Remark 1 Section 5 shows that the set of efficient XPPE values coincides with the set of efficient equilibrium values under a natural class of 'performance-based' equilibria, in which dynamics are driven by past performance, rather than past communication (specifically, ex-post incentives are not imposed, but the principal's allocation procedure does not directly condition on past announcements), providing an alternative foundation for efficient XPPE.

3.2 The role of specialization

Proposition 1 highlights the effects of specialization, patience, and the success probabilities on efficient dynamic incentive provision.

When agents are highly specialized, efficiency is unattainable. The intuition is the following. In order to incentivize an agent, the principal must credibly promise more (or threaten with less) favorable treatment in the future. Efficiency means that the principal can only utilize such favoritism when it is unclear which of the agents is better suited for the project, hence such promises are only credible if agents are not too specialized. Since future delegation decisions are the only tool available to the principal, high specialization hinders incentive provision.

Proposition 1 also sheds light on the impact of the success probabilities, $(\underline{\mu}, \overline{\mu})$, on $\mathcal{E}^*$. As $\overline{\mu}$ decreases or $\underline{\mu}$ increases, it becomes more difficult to incentivize the agents, and the efficiency region shrinks (see Figure 2), as both the relative value of being suited for a project decreases and performance becomes less informative. In particular, note that as $\underline{\mu}$ approaches 0, agents' incentive to falsely claim to be suited for projects vanishes and an agent's success perfectly reveals that he was indeed suited for the project. Hence an efficient XPPE is guaranteed to exist for sufficiently small $\underline{\mu}$. On the

[26] Proposition 1 and the results below extend to the case of arbitrary specialization profiles $(\phi^1, \phi^2) \in (0,1)^2$. See Section 7 for further discussion.

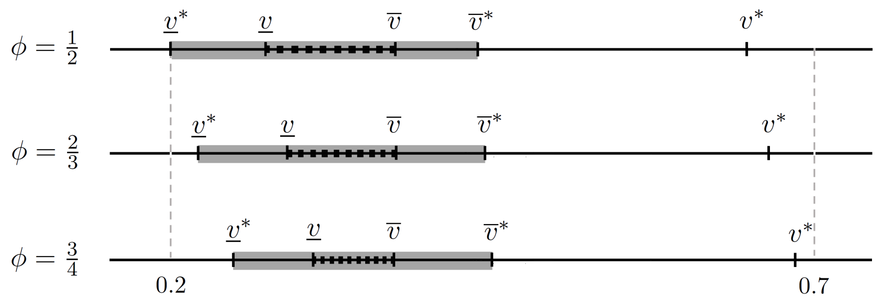


Figure 4: Feasible values under first-best delegation (shaded region) and efficient XPPE (dashed region) for specialization levels $\phi = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, given $(\delta, \underline{\mu}, \overline{\mu}, \pi) = (\frac{4}{5}, \frac{1}{5}, \frac{4}{5}, 1)$, for which $\phi^* = \frac{3}{4}$. The distortion in the set of efficient XPPE values due to the presence of private information, $\overline{v}^*(\phi) - \overline{v}(\phi)$, is equal to $\phi(1-\phi)\underline{\mu}\pi + \frac{\underline{\mu}(1-\overline{\mu})\pi}{2(1-\underline{\mu})}$.

other hand, as $\overline{\mu} \to 1$, although an agent suited for a project is almost guaranteed to succeed, an efficient XPPE need not exist if agents are too specialized or are not sufficiently patient.[27]

Denote the lowest and highest feasible (but not necessarily equilibrium) values for any given agent under first-best project delegation by $\underline{v}^*$ and $\overline{v}^*$, respectively. A simple calculation yields

$$\underline{v}^* = \frac{1}{2}\pi\overline{\mu}\left(\phi^2 + (1-\phi)^2\right), \qquad \overline{v}^* = \frac{1}{2}\pi\left(\overline{\mu} + 2\phi(1-\phi)\underline{\mu}\right).$$

Clearly, the value profile $(v^*, \underline{v}^*, \overline{v}^*)$ (and similarly $(v^*, \overline{v}^*, \underline{v}^*)$) cannot be sustained in equilibrium.

The set of efficient XPPE values can be explicitly solved for as a function of the underlying primitives of the model.

Proposition 2 *For any* $\phi \in [\frac{1}{2}, \phi^*]$, $\mathcal{E}^* = \text{co}\left(\{(v^*, \underline{v}, \overline{v}), (v^*, \overline{v}, \underline{v})\}\right)$, *where*

$$\underline{v} = \frac{1}{2}\pi\left(\overline{\mu} - 2\phi(1-\phi)\left(\overline{\mu} - \underline{\mu}\right) + \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right), \qquad \overline{v} = \frac{1}{2}\pi\left(\frac{\overline{\mu} - \underline{\mu}}{1-\underline{\mu}}\right). \tag{3}$$

In line with the intuition above, the set $\mathcal{E}^*$ is decreasing in the agents' specialization. The more specialized the agents are, the higher is the level of asymmetry between the agents (in terms of continuation values) that can be supported in an XPPE. Figure 4 contrasts the agents' set of efficient XPPE values with its counterpart in the complete information benchmark in which the principal perfectly observes agents' types.

3.3 High specialization and decentralization

The analysis above showed that efficiency is attainable if and only if agents are sufficiently non-specialized (namely, $\phi \in [\frac{1}{2}, \phi^*]$). The following result establishes that when agents' specialization

[27] Nevertheless, note that for $\overline{\mu}$ sufficiently close to 1, efficiency is always attainable if agents are sufficiently patient; that is, there always exists a threshold discount factor beyond which an efficient XPPE exists.

is sufficiently high, communication between the principal and the agents is not only inefficient, but breaks-down entirely, and decentralization becomes inevitable.

Proposition 3 *For any $\delta, \underline{\mu}, \overline{\mu}$ there exists $\overline{\phi}(\delta, \underline{\mu}, \overline{\mu}) < 1$ such that for all $\phi \geq \overline{\phi}$, any XPPE is communication -free.*

Proposition 3 shows that there exists a threshold level of specialization beyond which not only is efficiency unattainable, but the principal cannot sustain informative communication to any extent. In other words, the principal can do no better than allocating projects of type A to agent 1 and projects of type 2 to agent B, 'no questions asked'.

To understand this result, first note that the more specialized agents are, the less the principal stands to gain from informative communication in the first place, as agents are more likely to be suited for projects they are specialized in, and less likely to be suited for the other type of project. In particular, recalling that $v^o(\phi)$ denotes the principal's value under decentralization and $v^*(\phi)$ the principal's first-best value, as $\phi \to 1$, it must be the case that $v^*(\phi) - v^o(\phi) \to 0$. Since the principal cannot commit to future decisions, in equilibrium, at any history, the principal's continuation value $v(\phi)$ must satisfy

$$v^o(\phi) \leq v(\phi) \leq v^*(\phi). \tag{4}$$

On the other hand, as shown in Proposition 1, when specialization exceeds ϕ^*, if communication is to be informative, incentivizing an agent requires the principal to promise to distort future allocation decisions in a way that is costly for her.[28] Denoting by $\underline{v}^*(\phi)$ and $\overline{v}^*(\phi)$ the lowest and highest feasible values for an agent under a first-best allocation rule, note that as $\phi \to 1$,

$$\overline{v}^*(\phi) = \frac{1}{2}\left(\overline{\mu} + 2\phi(1-\phi)\underline{\mu}\right)\pi \to \frac{1}{2}\overline{\mu}\pi$$

and

$$\underline{v}^*(\phi) = \frac{1}{2}\overline{\mu}\left(\phi^2 + (1-\phi)^2\right)\pi \to \frac{1}{2}\overline{\mu}\pi;$$

in particular $\overline{v}^*(\phi) - \underline{v}^*(\phi) \to 0$. Since any increase in an agent's continuation value above $\overline{v}^*(\phi)$ (or decrease below $\underline{v}^*(\phi)$) is proportional to the principal's efficiency loss from such an increase, from (4), the principal's ability to credibly promise to inefficiently distort future allocation is weakened as specialization increases. That is, as specialization increases, the principal becomes more constrained in the extent to which she can credibly promise to increase an agent's continuation value above $\overline{v}^*(\phi)$, or decrease it below $\underline{v}^*(\phi)$. In particular, her ability to credibly generate differences in an agent's continuation values at any given history vanishes as $\phi \to 1$. Consequently, as agents' level of specialization becomes sufficiently large, the principal's ability to incentivize agents through the promise of distorting future decisions vanishes, and informative communication can no longer be sustained.

[28] In particular, this requires promising an agent, with positive probability, future projects for which he will not be suited while the other agent will be; alternatively, threatening the agent through the promise of withholding, with positive probability, projects for which he will be suited while the other agent will not be.

4 Rules for efficient delegation

The analysis until now has focused on the principal's ability to sustain efficient (or any informative) communication. However, it does not shed much light on what form such dynamic incentive provision should take. In each period $t \geq 1$, past allocation decisions x^{t-1}, performance outcomes y^{t-1}, announcements m^{t-1} and projects ω^{t-1} are all payoff-irrelevant with respect to the continuation game starting from period t. Nevertheless, in the absence of monetary incentives, it is necessary for the principal to make use of such payoff-irrelevant information and engage in *dynamic favoritism* - the promise of an advantage or threat of a disadvantage in future decisions - as a means of incentivizing agents to truthfully reveal whether they are suited for projects or not.[29] There are, however, many ways in which the principal might design such favoritism: A 'favored' agent may be granted a slight advantage or a significant one; the advantage may be constant or change over time; the identity of the agent with the advantage could shift frequently or infrequently, and such dynamics may be more or less history dependent, and may be driven by rewards for success or punishment for failure.

The key difficulty in the design of such favoritism is that punishing one agent often implies rewarding the other. To attain efficiency, a priority rule must balance the agents' incentives, generating high variation in their continuation payoffs while remaining credible.

This section introduces a simple (albeit non-stationary) family of *Markov priority rules* (MPR), and shows that whenever efficiency is attainable (i.e., whenever there exists *any* efficient XPPE), it is attainable using a rule within this family. Furthermore, we establish necessary properties that any MPR that indeed induces efficient delegation of projects over the entire region $\phi \in [\frac{1}{2}, \phi^*]$ must satisfy. To illustrate the results, I introduce one such simple MPR, referred to as *maximal-priority*, and show that under this rule allocation is efficient and agents are provided (robust) incentives to be truthful over the entire region $\phi \in [\frac{1}{2}, \phi^*]$.[30] Finally, Section 4.3 highlights additional robustness properties of such rules, which permit the principal to utilize a 'hands-off' approach in her delegation of projects at no cost.

4.1 Markov priority rules

We now introduce the following simple family of rules. A *Markov priority rule* consists of a 'favored agent' in the initial period, a fixed rule prescribing project allocation given the identity of the favored agent, and a fixed rule governing transitions in the identity of the favored agents. Formally:

Definition 2 *A* Markov priority rule (MPR) *is characterized by* $(f_1, \mathcal{X}, \psi)$, *with*

- $f_1 \in \{1, 2\}$ *denoting the identity of the 'favored agent' in period 1;*

[29]Clearly, if the principal assigns incoming projects according to a stationary policy, an agent could always announce that he is suited for the project without concern for future repercussions.

[30]Hence, whenever maximal-priority fails to incentivize the agents to be truthful, efficiency is unattainable.

- $\mathcal{X} : \{A, B\} \times \{1,2\} \times \{0,1\}^2 \to X$ *describing the allocation of the project as a function of its type* $\omega \in \{A, B\}$, *the current favored agent* $f \in \{1,2\}$, *and current announcements* $m \in \{0,1\}^2$;

- $\psi : \{A, B\} \times \{1,2\} \times \{1,2\} \times \{S, F\} \to \Delta\{1,2\}$ *describing the transitions in the identity of next period's favored agent, as a function the project type* $\omega \in \{A, B\}$, *the current favored agent* $f \in \{1,2\}$, *the agent allocated the project* $j \in \{1,2\}$, *and its outcome* $y \in \{S, F\}$.

Note that $\mathcal{X}$ and ψ are independent of t. That is, both what it means to be favored and the rule governing who is favored are fixed over time.

Given a transition rule ψ, it will be convenient to denote by $\psi_f(\omega, j, y)$ the probability with which the favored agent $f \in \{1,2\}$ remains as such in the following period, given (ω, j, y).

Definition 3 A MPR $(f_1, \mathcal{X}, \psi)$ is *failure-driven* if $\psi_f(\omega, j, y) = \mathbf{1}_{\{(j,y) \neq (f,F)\}}$.

A MPR is *failure-driven* if the identity of the favored agent shifts (immediately and deterministically) if and only if the favored agent fails. Dynamics under such rules are driven entirely by the (most recent) failure of a favored agent, which triggers a reversal of the agents' roles. In particular, *failure is punished but success is not rewarded*. That is, while each agent is clearly always better off being successful in a project rather than failing, an agent's success does not directly result in a strict improvement in his continuation value.

Example 1 (Maximal-priority) Consider the following example of an MPR, which will be referred to as *maximal-priority*.

Definition 4 *Under the* maximal-priority *rule, in each period* $t \geq 1$, *projects are allocated as follows:*

- *At the beginning of the period, one of the agents is selected to be the* favored agent. *Given the subsequent announcements, if* $i \in \{1,2\}$ *is the favored agent,* i *receives the project whenever* $(m_i, m_{-i}) \neq (0,1)$; *otherwise, agent* $j \in \{1,2\}$, $j \neq i$ *is assigned the project.*

- *A favored agent remains favored as long as he does not fail in a project. If such a failure occurs, however, the other agent becomes the favored agent instead. The identity of the first favored agent is determined arbitrarily in period 1.*

Figure 5 illustrates the dynamics governing the identity of the favored agent under maximal-priority. Clearly, maximal-priority is failure-driven. Furthermore, a key feature of this MPR is its severe use of punishment – the favored agent (deterministically and immediately) loses his status as favored upon failing in a project, *even if it is publicly known that he was not at fault*; that is, even if he did not claim to be suited for the project in the first place.

Note that when delegating according to this simple rule, the principal only needs to keep track of the identity of the favored agent in the previous period, and whether the agent failed in that period or not. □

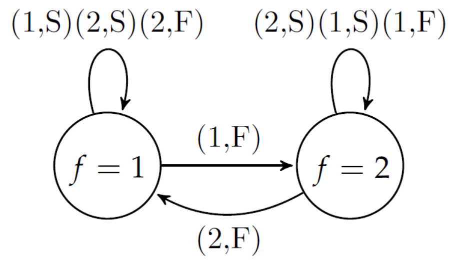


Figure 5: Transitions determining the identity of the favored agent, f, under maximal-priority.

4.2 Efficiency whenever possible

While MPR are relatively simple, the following result shows that whenever efficiency is attainable, it is in fact attainable using a MPR. Furthermore, any MPR that induces an allocation rule that is efficient over the widest region of primitives for which efficiency is attainable must be failure-driven, and must also satisfy the property that whenever both agents claim to be suited for a project, the favored agent is awarded the project.

Proposition 4

1. *If and only if* $\phi \in [\frac{1}{2}, \phi^*]$, *there exists a MPR that induces an allocation rule that is efficient.*

2. *If the allocation rule induced by the MPR* $(f_1, \mathcal{X}, \psi)$ *is efficient for any* $\phi \in [\frac{1}{2}, \phi^*]$ *then:*

   (a) $(f_1, \mathcal{X}, \psi)$ *is failure-driven; and*

   (b) if $m = (1, 1)$ *then the project is assigned to the current favored agent.*

The first part of Proposition 4 shows that in search of a rule that is efficient over the widest region of primitives for which efficiency is attainable, one can restrict attention to the simple class of MPR. The second part of Proposition 4 shows that punishing failure and ignoring success is necessary in order to attain efficiency over the widest possible primitive region. In particular, while failure of the favored agent is punished, failure of the non-favored agent is not. Attaining efficiency whenever possible is linked to the highest payoff asymmetry credibly sustainable between the agents. When a MPR is failure-driven, the non-favored agent is incentivized entirely through the favored agent's outcomes. The proof in the Appendix shows that incentivizing the non-favored agent directly through his own success or failure requires projects to be assigned to him too often in order for the principal to be able to generate the necessary asymmetry in continuation values required to induce efficiency over the entire region $\phi \in [\frac{1}{2}, \phi^*]$.

Example 1 (continued) If agents are truthful, delegating projects according to maximal-priority is clearly efficient. The question is then: When can agents be provided incentives to be truthful?

For each agent $i \in \{1,2\}$, denote by v_i^f the expected average continuation payoff at the beginning of a period in which he is chosen to be the favored agent. Similarly, denote by v_i^{-f} agent i's continuation value when agent $-i$ is the favored agent.

Suppose $\omega = A$ (analogous arguments apply for the case in which $\omega = B$). XIC requires considering the interim incentive constraints of an agent i to announce truthfully, given any belief the agent may hold about agent $-i$'s current type, when he expects $-i$ to be truthful in the continuation game. Consider first the incentives of agent i when he is the favored agent. Regardless of i's type, if he believes $\theta_{-i} = \beta$, i's ex-post incentive constraint (XIC) is satisfied, as he expects to receive the project regardless of his announcement. Suppose then that i believes $\theta_{-i} = \alpha$. If $\theta_i = \beta$, then XIC implies

$$\overbrace{\delta v_i^f}^{m_i=0,\text{ forgo project}} \geq \overbrace{\underbrace{\underline{\mu}\Big((1-\delta)\pi + \delta v_i^f\Big)}_{\text{succeed}} + \underbrace{(1-\underline{\mu})\delta v_i^{-f}}_{\text{fail, lose favor}}}^{m_i=1,\text{ get project}},$$

whereas if $\theta_i = \alpha$, XIC requires

$$\overbrace{\underbrace{\overline{\mu}\Big((1-\delta)\pi + \delta v_i^f\Big)}_{\text{succeed}} + \underbrace{(1-\overline{\mu})\delta v_i^{-f}}_{\text{fail, lose favor}}}^{m_i=1,\text{ get project}} \geq \overbrace{\delta v_i^f}^{m_i=0,\text{ forgo project}} .$$

Consider next i's incentives when $-i$ is the favored agent. Regardless of i's type, if he believes $\theta_i = \alpha$, i's XIC is satisfied, since he does not expect to receive the project regardless of his announcement. Suppose i believes $\theta_{-i} = \beta$. If $\theta_i = \beta$ then XIC requires

$$\overbrace{\underbrace{\underline{\mu}\delta v_i^{-f}}_{-i\text{ succeeds}} + \underbrace{(1-\underline{\mu})\delta v_i^f}_{-i\text{ fails, gain favor}}}^{m_i=0,\text{ forgo project}} \geq \overbrace{\underline{\mu}(1-\delta)\pi + \delta v_i^{-f}}^{m_i=1,\text{ get project}},$$

and finally if $\theta_i = \alpha$ then XIC implies

$$\overbrace{\overline{\mu}(1-\delta)\pi + \delta v_i^{-f}}^{m_i=1,\text{ get project}} \geq \overbrace{\underbrace{\underline{\mu}\delta v_i^{-f}}_{-i\text{ succeeds}} + \underbrace{(1-\underline{\mu})\delta v_i^f}_{-i\text{ fails, gain favor}}}^{m_i=0,\text{ forgo project}} .$$

Rearranging these condition, it is easily verified that XIC is equivalent to

$$v_i^{-f} + \frac{\underline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})} \leq v_i^f \leq v_i^{-f} + \frac{\overline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})}. \tag{5}$$

The proof, in the Appendix, derives an additional equilibrium consistency condition relating v_i^f and v_i^{-f}. Combined, this condition and the above XIC condition can be used to show that maximal-priority is XIC whenever an efficient XPPE exists.[31]

[31] Recall that the principal's allocation rules is said to be efficient if it is part of an efficient XPPE.

Proposition 5 *The allocation rule induced by maximal-priority is efficient if and only if an efficient XPPE exists.*

Proposition 5 shows that whenever efficiency can be attained, it can be attained using a remarkably simple rule. The favored agent, who has priority over the other agent whenever it is unclear which of the agents is better suited for the project (hence the term maximal-priority), is incentivized to request only those projects for which he is suited through the threat of punishment for failure. Interestingly, however, the favored agent may be punished (by losing favor) as a result of his failure in a project even if he did not claim to be suited for it. This harsh punishment serves the role of incentivizing the other agent to be truthful, through the hope of potentially becoming the favored agent in the next period. Interestingly, maximal-priority perfectly balances the agents' incentives, generating the highest possible variation in continuation payoffs while remaining credible.

Remark 2 In the equilibrium above, players do not use "review strategies," (see, for example, Radner (1985), Rubinstein (1979), and Rubinstein and Yaari (1983)) whereby they infer the likelihood of sequences of announcements of types; in particular, maximal-priority provides agents with incentives to announce truthfully regardless of the likelihood of false past announcements.

Remark 3 It can easily be verified that the robustness (in the sense of Definition 1) of the particular equilibrium above comes without a cost. That is, the strategy profile in which the principal's allocation of projects follows maximal-priority and agents are truthful constitutes an efficient PPE (without the 'ex-post' qualification) over precisely the same region (2). The strategy profile in fact constitutes an ex-post Perfect Bayesian Equilibrium (PBE), in which each agent finds it optimal to be truthful regardless of his belief about the other's past and current types, taking expectation over the future path of play.

□

4.3 Hands-off delegation

Maximal-priority does not require randomization, makes no use of information about the types of projects, and does not differentiate between agents based on their specialization in these projects.

In certain environments, the principal might be limited in her ability to continually learn whether projects are of one type or another (for example, this may require a certain level of expertise the principal does not posses). Even if the principal is capable of observing the types of projects that arrive and able to recognize how they relate to the agents' specialization, a strategy that is not sensitive to such information may eliminate certain (unmodelled) costs. An immediate consequence of Proposition 5 is that the principal's potential ignorance in this matter is not costly for her, in the following sense.

Corollary 1 *Whenever an efficient XPPE exists, there exists an efficient XPPE that does not require the principal's knowledge of project types or agents' specialization.*

The robustness of maximal-priority together with the fact that it is does not directly condition on past announcements (see Section 5 for further discussion of such rules) also allows the principal to dispense with the simultaneous announcement protocol and permits her to avoid potentially costly communication.[32]

5 Equivalence between ex-post and performance-based equilibria

Efficiency requires the principal to condition her decisions on past (payoff-irrelevant) information, which may include past allocation decisions, communication, and outcomes. In this section, we introduce a class of *performance-based* equilibria, in which the ex-post requirement is no longer imposed, but in each period the principal's decisions do not condition directly on past communication. The analysis provides an alternative foundation for efficient XPPE by showing that, fixing any set of primitives $\{\phi, \delta, \underline{\mu}, \overline{\mu}, \pi\}$, the set of equilibrium values in an efficient XPPE coincides with its counterpart under performance-based equilibria.

Definition 5 *The period-t histories* $h^t = (\omega^t, m^t, x^t, y^t)$ *and* $\hat{h}^t = (\hat{\omega}^t, \hat{m}^t, \hat{x}^t, \hat{y}^t)$ *are* performance equivalent *if* $(\omega^t, x^t, y^t) = (\hat{\omega}^t, \hat{x}^t, \hat{y}^t)$. *The principal's strategy is* performance-based *if for all* $t \geq 1$, *if* $h^{t-1}, \hat{h}^{t-1} \in \mathcal{H}^{t-1}$ *are performance-equivalent then*

$$\chi_t(h^{t-1}, \omega_t, m_t) = \chi_t(\hat{h}^{t-1}, \omega_t, m_t), \ \ \forall \omega \in \{A, B\}, m_t \in \{0, 1\}^2.$$

A PPE in which the principal's strategy is performance-based is a performance-based equilibrium.

Performance-based equilibria avoid any potential difficulties associated with keeping track of past communication. Note that a performance-based equilibrium does not require that announcements play no role in allocation decisions, or that past announcements do not shape future allocations. In each period, the principal's decision may arbitrarily depend on past and current project types and the entire history of allocations decisions and past performance. While the latter are clearly also a function of past announcements, the restriction imposed is that period-t allocation decisions do not *directly* condition on such past announcements. In other words, past announcements shape current allocations only through their effect on past allocations and performance.

[32] To see this, consider the following modification of maximal-priority: (i) In each period, the principal assigns the project to the favored agent, the identity of which is determined as in maximal-priority. (That is, arbitrarily in period 1 and in any subsequent period $t \geq 2$, the favored agent in the previous period remains favored unless he failed in the project in that period.) (ii) The favored agent is responsible for the project in the following sense: He can choose whether to retain it or to offer it to the other agent. If the favored agent chooses the latter, the non-favored agent may choose whether to accept the project – in which case it is assigned to him – or to reject it – in which case the favored agent retains the project.

Clearly, such a procedure is efficient whenever efficiency is attainable. Furthermore, under such a procedure, the principal simply delegates responsibility for each project to the favored agent in each period, leaving communication within the period up to the agents.

Example 2 Maximal-priority is performance-based. Recall that the only past information upon which period-t allocation decisions under maximal-priority condition are: (a) The identity of the period-$t-1$ favored agent and (b) whether that agent failed in period $t-1$ or not. In particular, while the identity of the previous period's favored agent was shaped by past announcements, the current allocation decision does not directly condition on such past announcements. Hence, maximal-priority is indeed performance-based. □

Example 3 Consider a rule under which the principal assigns each project to the agent who has accumulated the higher number of past successes in this type of project.[33] Such a rule is also performance-based. □

Example 4 Consider any rule according to which an agent is rewarded (through some form of future advantage over the other agent) for seldomly claiming to be suited for projects in the past, or punished for claiming to be suited too often. Such rules are *not* performance-based.

Alternatively, consider a variant of maximal-priority in which a favored agent is punished more severely (e.g., through a longer period of being unfavored) for failure in a project he announced he was suited for than for failure in a project he did not claim to be suited for. Such a rule is *not* performance-based. □

Denote the set of performance-based equilibrium values by $\mathcal{E}_P \subseteq \mathbb{R}^3_+$, and the set of efficient performance-based equilibrium values by (recall that robustness – as in Definition 1 – is not imposed here)

$$\mathcal{E}^*_P := \{(v_0, v_1, v_2) \in \mathcal{E}_P | \; v_0 = v^* \text{ and } \; v_1 + v_2 = \pi v^*\} \subseteq \mathbb{R}^3_+.$$

The following result shows that efficient ex-post and efficient performance-based equilibria are equivalent, in that they yield the same set efficient equilibrium values.

Proposition 6

1. *For any $\phi, \delta, \underline{\mu}, \overline{\mu}$, $\mathcal{E}^*_P = \mathcal{E}^*$. In particular, efficiency is attainable in a performance-based equilibrium if and only if it is attainable in an XPPE.*

2. *Efficiency is attainable over a strictly larger primitive region in a standard PPE (i.e., in the absence of the ex-post – equivalently, the performance-based – restriction).*

In a performance-based equilibrium there is less freedom in prescribing continuation values, as these can no longer be conditioned directly on past announcements. On the other hand, ex-post incentive compatibility is relaxed. Proposition 6 shows that the freedom lost by the former restriction is precisely recovered by the latter relaxation of incentive constraints. Conditioning directly on past

[33] If both agents have the same number of past successes, ties are broken arbitrarily.

announcements creates a stronger incentive to lie. In particular, any improvement in the efficiency region beyond performance-based equilibria necessarily implies a loss of robustness. The reason is that each of these two requirements restricts the principal from making the continuation values of the agents contingent on the failure of an agent differ depending on the profile of announcements. Such a rule would be in direct violation of the performance-based restriction, but also, as shown in the Appendix, must violate the ex-post restriction in order to improve upon the region in which efficiency can be attained. In particular, in the absence of the ex-post (equivalently, the performance-based) restriction, the principal can indeed strictly increase the region in which efficiency can be induced using precisely such a rule.

The proof of Proposition 6 follows a similar technique to that of Proposition 1. That is, the endpoints of the set of efficient performance-based equilibrium values, assuming it is non-empty, are shown to correspond to a fixed-point of a function $\tilde{\Psi}$, analogous to Ψ (see discussion in Section 3.1).[34] The proof shows that these two functions, despite being solutions to different linear programs, are in fact identical. This in turn guarantees the desired equivalence.

6 Delegation without favoritism

The analysis in the previous sections has illustrated how the principal can use dynamic favoritism in order to provide incentives. In some settings, however, treating the agents asymmetrically, even temporarily, may be either impossible, or undesirable. For instance, such favoritism might raise concerns that the principal is biased in favor of one of the agents. It may be important for the principal to signal that she is not biased by avoiding such asymmetric treatment. What is the cost (if any) of such a decision?

To study this question, this section considers XPPE in which agents' expected continuation payoffs at the beginning of each period may not differ from one another. In other words, agents can only be incentivized through *mutual* punishment or rewards.

Definition 6 *A* non-discriminatory XPPE *is an XPPE such that, for any* $t \geq 1$ *and any* $h^{t-1} \in \mathcal{H}^{t-1}$, $\mathbb{E}\left[U_{1,t}|h^{t-1}\right] = \mathbb{E}\left[U_{2,t}|h^{t-1}\right]$.

Let $\mathcal{E}^{\bullet}$ denote the set of non-discriminatory XPPE values. Note that decentralization yields the lowest non-discriminatory XPPE value for the principal, $v^o = \phi\overline{\mu} + (1-\phi)\underline{\mu}$, and $\frac{1}{2}\pi v^o$ for each of the agents. We then have the following result.

Proposition 7 *A non-discriminatory XPPE satisfies the following properties.*

[34] Note that the definition of policies, decomposability, etc. are different in each of these environments. Since the relevant incentive constraints differ and continuation values are restricted, solving for such fixed-points and identifying the conditions under which they may exist requires solving a linear program different than the one in the proof of Proposition 1.

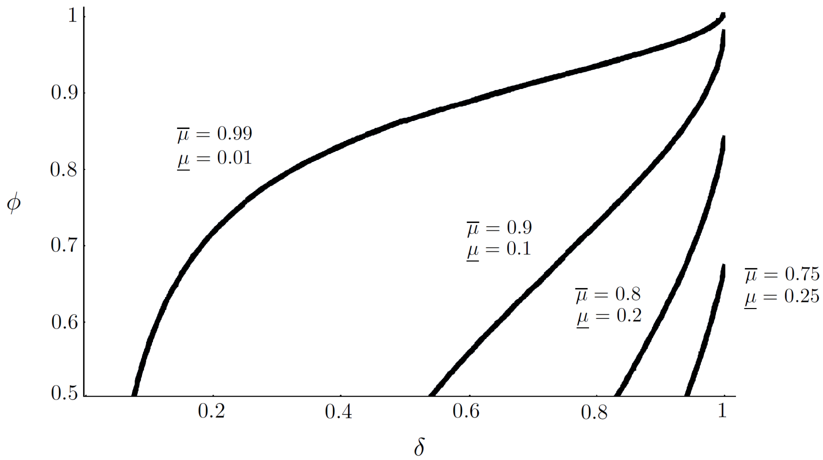


Figure 6: Threshold ϕ below which condition (6) holds, as a function of δ, for different success probabilities $(\underline{\mu}, \overline{\mu})$. In this region the principal's maximal equilibrium value is $v^o < v^\bullet < v^*$.

1. *For any $\{\phi, \delta, \overline{\mu}, \underline{\mu}, \pi\}$, efficiency is unattainable in an XPPE.*

2. *Suppose agents are highly patient and sufficiently non-specialized, such that*

$$\delta \geq \delta^\bullet := \frac{\underline{\mu}}{\frac{1}{2}(1-\phi)^2 \left(\overline{\mu} - \underline{\mu}\right)(1-\underline{\mu}) + \underline{\mu}\left(1 - (1-\phi)(1-\overline{\mu})\right)}. \tag{6}$$

   Then the principal's maximal equilibrium value is equal to

$$v^\bullet := v^* - \underbrace{2(1-\phi)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}}_{\text{cost of non-discrimination}}, \tag{7}$$

   which satisfies $v^o < v^\bullet < v^*$.

Non-discrimination is costly for the principal unless agents are highly specialized. When this is the case, from Proposition 3, the benefit from favoritism vanishes, and decentralization becomes inevitable. When agents are sufficiently patient and non-specialized (in the sense of (6); see Figure 6), however, the principal can indeed do better than decentralization, but efficiency is unattainable, and the highest value she can obtain is $v^\bullet < v^*$.

Furthermore, note that when the agents are sufficiently non-specialized, as $\overline{\mu} \to 1$, the benefit from favoritism vanishes. Specifically, if

$$\frac{1}{2} \leq \phi \leq 1 - \sqrt{(1-\delta)\underline{\mu} / \left(\delta \frac{1}{2}\left(1 - \underline{\mu}\right)^2\right)}$$

then the principal's highest XPPE value approaches her first-best, v^*.

What allocation procedure attains the principal's highest value, $v^\bullet$? Consider the following *joint responsibility* (JR) rule.

Definition 7 *Under JR, projects are allocated as follows:*

- Communication phase: in each period, the agent who is *not* specialized in the current type of project ω receives the project if he claims to be suited for it; otherwise, the agent who is specialized in ω is allocated the project.[35]

- Starting from $t = 1$, as long as decentralization has not been triggered, the communication phase continues. If and only if an agent who is not specialized in the current type of project receives it and fails, with probability

$$q := \frac{2\underline{\mu}(1-\delta)}{\delta(1-\phi)\left((1-\phi)\left(\overline{\mu}-\underline{\mu}\right)(1-\underline{\mu})-2\underline{\mu}(1-\overline{\mu})\right)} \tag{8}$$

  communication breaks-down immediately; that is, an indefinite decentralization phase is triggered.

We then have the following result.

Proposition 8 *Assume (6) holds. The strategy profile in which the principal delegates projects according to JR and agents are truthful constitutes a non-discriminatory XPPE in which the principal's value is $v^\bullet$.*

When a project arrives, the default in the communication phase is to delegate it to the agent who is specialized in it, unless the agent who is not specialized requests it. If this occurs, however, and the agent fails, then the agents share responsibility for this failure and communication breaks-down with probability $q(\phi, \delta, \overline{\mu}, \underline{\mu})$. As the above proposition shows, JR induces truthful announcements if and only if condition (6) holds, and the principal obtains the value $v^\bullet$.[36]

Incentive provision takes a very different form under JR than under the MPR studies in Section 4. Perhaps surprisingly, when both agents claim to be suited for a project, the project is allocated to the agent who is *not* specialized in it. That is, the non-specialized agent is granted priority. Such a rule provides incentives effectively while triggering an (inevitable) punishment with small probability. In fact, it can be shown that not only does the above procedure obtain $v^\bullet$ over the entire region (6), but alternative procedures in which priority is granted to the specialized agent, or punishments are triggered by specialized agents are less effective – they either cannot sustain a value of $v^\bullet$, or do so over a strictly smaller region of primitives.

[35] That is, if $\omega = A$ ($\omega = B$), agent 1 (2) receives the project unless agent 2 (1) claims to be suited for it, in which case agent 2 (1) receives the project.

[36] It is possible to construct alternative procedures in which the decentralization phase is temporary, which induce truthful announcements under the same condition (6) and generate the same value $v^\bullet$ for the principal.

7 Discussion and extensions

7.1 Persistent private-information

The analysis until now has considered a repeated game between the principal and the agents, in which each agent's private information is i.i.d. from one period to the next. This section relaxes this assumption and considers an environment in which agents' types are correlated over time. In particular, we study how persistence in agents' types affects the principal's ability to delegate efficiently.

When agents' types are correlated over time, they may use others' past or current announcement in order to predict their future types, and might wish to change their behavior in order to influence others' beliefs about their own future type. It is not immediate whether persistence helps the principal to provide agents with incentives to reveal their private information or not. While the possibility of signaling might strengthen an agent's incentive to announce untruthfully, persistence also allows the principal to infer information about future types based on past outcomes.

To examine this tradeoff in the context of our framework, agents' types are assumed to evolve according to the following simple first-order Markov process. For all $t > 1$, $i \in \{1, 2\}$, let

$$\theta_{it} \begin{cases} = \theta_{it-1} & \text{w.p. } \rho \\ \neq \theta_{it-1} & \text{w.p. } 1-\rho, \end{cases}$$

where $\rho \in [\frac{1}{2}, 1]$, and let each agents' period 1 type θ_{i1} be drawn from a commonly known distribution P_0 over $\{\alpha, \beta\}$.[37] Since the game is no longer a repeated one, we will use ex-post perfect public Bayesian equilibrium (XPPBE; see Athey and Bagwell (2008) for a formal definition of a perfect public Bayesian equilibrium) as the solution concept in this section.[38] An efficient XPPBE is an XPPBE in which, in each period, the project is allocated to a suited agent whenever one exists.

We then have the following result, which shows that *persistence hinders efficiency*. In particular, efficiency is attainable (in an XPPBE) if persistence is not too high, but not otherwise.[39]

Proposition 9 *1. Maximal-priority induces a first-best XPPBE if and only if*

$$\frac{1-2\rho(1-\rho)}{2\rho(1-\rho)} \leq \frac{\delta(1-\underline{\mu})\frac{1}{2}\left(\overline{\mu}+\underline{\mu}\right)-\underline{\mu}+\delta^2\overline{\mu}\underline{\mu}\left(\overline{\mu}+\underline{\mu}-1\right)}{\underline{\mu}-\delta\underline{\mu}\left(2\overline{\mu}+\underline{\mu}-1\right)+\delta^2\overline{\mu}\underline{\mu}\left(\overline{\mu}+\underline{\mu}-1\right)}. \tag{9}$$

[37] Similar results obtain if persistence may be negative as well, i.e., $\rho \in [0, 1]$. Note that $\rho = \frac{1}{2}$ corresponds to case in which $\phi = \frac{1}{2}$ in the i.i.d. setting.

[38] Perfect public Bayesian equilibrium (Athey and Bagwell (2008)) extends PPE to a dynamic Bayesian game, and consists of a profile of public strategies, initial beliefs, and a belief updating function that specifies beliefs about others' types at each period. An XPPBE additionally requires that in each period, taking expectation over the future path of play, each agent's strategy remains optimal irrespective of his belief about the other agent's past and current types.

[39] For this result it is assumed that $1-\overline{\mu} \leq \underline{\mu} \leq 2(1-\overline{\mu})$. This assumption is not required for the result but greatly simplifies the analysis.

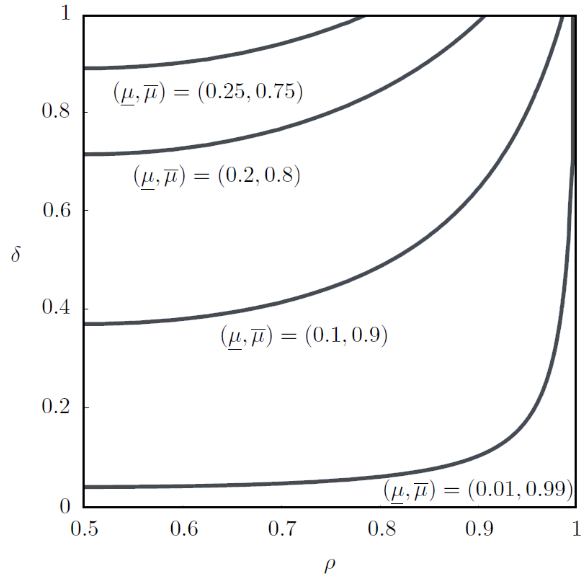


Figure 7: Threshold level of patience, δ, above which maximal-priority attains efficiency, as a function of persistence, ρ.

2. *There exists $\rho^* \in [\frac{1}{2}, 1)$ such that for all $\rho \in (\rho^*, 1]$ there does not exist an efficient XPPBE.*

Figure 7 illustrates condition (9), which highlights the tradeoff between persistence and patience in determining the (im)possibility of efficiency under maximal-priority. As the LHS of (9) is strictly increasing in the level of persistence, ρ, higher persistence makes it more difficult to attain efficiency. The intuition is that persistence preserves heterogeneity between the agents 'along the worst path'. That is, at histories in which agents differ in their specialization, high persistence increases the probability that this heterogeneity will be preserved the continuation game. Such future heterogeneity is detrimental to incentive provision for reasons similar to those discussed in the i.i.d. case studied in the previous sections. That is, future heterogeneity renders favoritism ineffective.

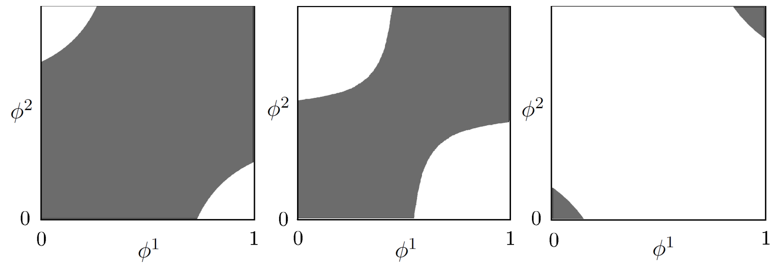
7.2 Homogeneous agents and general specialization profiles

For expositional purposes, the environment considered above has focused on the case $\phi = \phi^1 = 1 - \phi^2$ in which agents are heterogeneous in their specialization. Consider instead and environment in which agents are entirely symmetric; that is, both agents are equally specialized in either A or B, $\phi^1 = \phi^2 = \phi \in (0, 1)$.

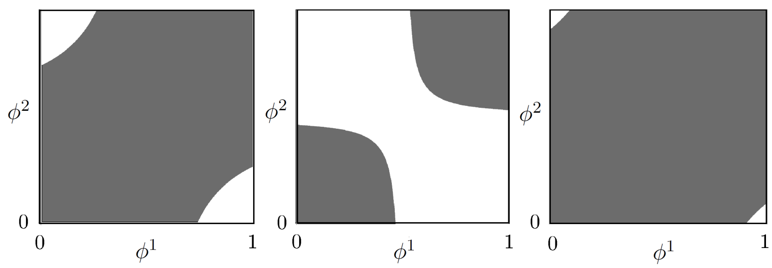
We then have the following result.

Proposition 10 *Assume $\phi^1 = \phi^2 = \phi$, where $\phi \in (0, 1)$. Then efficiency is attainable if and only if*

$$\delta \geq \frac{\underline{\mu}}{\left(\frac{1}{2} - \phi(1-\phi)\right)\left(\overline{\mu} - \underline{\mu}\right)\left(1 - \underline{\mu}\right) + \underline{\mu}\overline{\mu}} \tag{10}$$



(a) $\delta = 0.9$ (b) $\delta = 0.75$ (c) $\delta = 0.55$



(a) $\underline{\mu} = 0.2$, $\overline{\mu} = 0.99$ (b) $\underline{\mu} = 0.2$, $\overline{\mu} = 0.75$ (c) $\underline{\mu} = 0.05$, $\overline{\mu} = 0.75$

Figure 8: *Top panel*: Specialization profiles for which an efficient XPPE exists (shaded region) for different discount factors δ and given $\overline{\mu} = 0.8$, $\underline{\mu} = 0.2$. *Bottom panel*: Specialization profiles for which an efficient XPPE exists (shaded region) for different success probabilities $(\underline{\mu}, \overline{\mu})$ and $\delta = 0.75$. Homogeneity in agents' specialization is crucial for efficiency

Note that the RHS in (10) is decreasing in ϕ. Hence, given the homogeneity of the agents, high specialization is beneficial for incentives. Consistent with the intuition in the case of heterogeneous agents, when agents are highly specialized (in the same project), being unfavored means an agent is likely to miss out on precisely the type of projects he is more likely to be suited for. Hence, given that agents are homogneous, punishments (or rewards) are more effective when specialization is high.

More generally, the results above extend to the case of arbitrary specialization profiles $(\phi^1, \phi^2) \in (0,1)^2$. Figure 8 illustrates the specialization profiles $(\phi^1, \phi^2) \in (0,1)^2$ for which an efficient XPPE exists, for different discount factors fixing the probabilities $(\underline{\mu}, \overline{\mu})$ and the corresponding region for different probabilities $(\underline{\mu}, \overline{\mu})$ fixing the discount factor δ. As these figures illustrate, homogeneity in agents' specialization is crucial for efficiency.[40]

7.3 Relationship to Andrews and Barron (2016) and de Clippel et al. (2017)

In Andrews and Barron (2016), a firm repeatedly contracts with one of multiple suppliers, whose productivity (redrawn independently in each period) is observed by the principal but whose effort is subject to moral hazard. Output (stochastic) is non-contractible, and suppliers observe only their

[40]Note that the role of patience and the success probabilities in determining whether efficiency is attainable is similar to the one in the benchmark case $\phi = \phi^1 = 1 - \phi^2$.

own relationship with the principal (i.e., monitoring is private). The principal *can* provide monetary compensation, but cannot formally commit to such compensation. The paper constructs a dynamic allocation rule that attains the principal's first best whenever possible. Interestingly, in sharp contrast to maximal-priority, such a rule "favors past success and tolerates past failure". Monetary incentives allow the principal to punish by way of withholding compensation while rewarding through future promises. The results in the current paper show that, absent monetary incentives, such dynamics are reversed.

In de Clippel et al. (2017) (henceforth dCER), a principal limited in her attention designs an idea-selection mechanism to repeatedly choose among multiple agents who wish to have their ideas implemented. The principal can neither commit to future decisions nor use monetary transfers in order incentivize the agents to suggest only good ideas.[41] A particular strategy profile, the "silent treatment" (S-T) is studied: in each period an agent is designated as a last-resort, and his idea selected if all other agents refrain from suggesting ideas. This agent is replaced with another as last-resort if the latter suggests an idea which yields the principal a low profit.[42] dCER study when the principal's first-best is "achievable" - i.e., can be obtained in a PBE *when players are sufficiently patient*. Hence, a crucial difference in the approach of the current paper is the derivation of a necessary and sufficient condition for efficiency (over all primitives, including patience), which allows to study how effective certain allocation rules are relative to one another; in particular, to identify which allocation rules are efficient over the widest region of primitives. A consequence of Proposition 6 is that the principal's strategy under the S-T (adapted to the environment of this paper), like maximal-priority, is not efficient whenever possible unless the ex-post (or, equivalently, the performance-based) restriction is imposed.[43] Also, Proposition 1 can be used to provide an additional foundation for the S-T, which adapted to the current environment can be shown to be efficient whenever possible, under the ex-post (equivalently, performance-based) restriction. Proposition 4 sheds light on the properties of the S-T that permit this – the crucial feature being that dynamics are failure-driven – i.e., driven entirely by the failures of the agent who is granted priority.

8 Concluding remarks

This paper has studied dynamic delegation to multiple heterogeneously specialized agents. In order to delegate efficiently, the principal must incentivize the agents to reveal their private information

[41]In dCER, agents are symmetric in the benchmark model, and later are assumed to differ along a vertical dimension - their ability to generate good ideas may be different.

[42]Such a scheme (adapted to the current environment) differs from maximal-priority in the manner in which cases of indifference are resolved. In particular, under maximal-priority any indifference is broken in favor of the favored agent, and both agents are incentivized to be truthful.

[43]dCER also consider a notion of robustness in their analysis (similar to the one introduced in Fudenberg and Yamamoto (2010)) which differs from the one studied in the current paper.

over time. The analysis sheds light on the scope and shape of dynamic incentive provision when the principal cannot commit to future decisions and cannot use monetary incentives.

The environment considered in this paper permits a precise characterization of the set of efficient XPPE at fixed discounting, and in particular the derivation of a condition necessary and sufficient for the existence of such equilibria. Such a characterization, as well as the focus on robust equilibria, is not necessary in order to establish qualitatively similar results regarding the tradeoff between efficiency and specialization. Indeed such results would continue to hold in environments more general than the one considered here. However, such a characterization permits to study the design of delegation rules that are efficient over the maximal primitive region and the key properties of such rules (as well as to identify results that need not hold for other asymptotically efficient rules). It also permits to shed light on the interaction between adverse selection and imperfect monitoring in the form of an equivalence between robust and performance-based equilibria.

As discussed in Section 3.1, the observation that the set of equilibrium payoffs $\mathcal{E}(\delta)$ is the largest self-generating set does not provide a simple technique for its characterization.[44] While many of the qualitative results are expected to hold in more general settings, a necessary and sufficient condition for the (im)possibility of efficiency might be difficult to obtain. In such settings, however, it would be possible to adapt techniques as in Fudenberg et al. (1994) to the environment in order to bound the set of equilibrium payoffs and completely characterize it for high δ.[45]

In addition, the environment I study assumes specialization as well as the arrival of projects are both exogenous. It would be interesting to extend the environment to one in which agents' specialization is a function of their actions and may be influenced by the principal (e.g., through recruitment or training), or one in which the arrival of projects is endogenous and may be utilized by the principal as an additional tool for incentive provision. Such extensions are left for future work.

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[44]See, e.g., Mailath and Samuelson (2006) pp. 273.

[45]An alternative approach would be to further restrict the class of delegation rules under consideration in order to obtain optimality results at fixed primitives within that class.

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Appendix

A Preliminaries

In order to describe $\mathcal{E}^*$, we first adapt the recursive methods of Abreu et al. (1990) and Fudenberg et al. (1994) to the current environment, factoring agents' equilibrium payoffs into two components: current-period payoffs and promised continuation values, where the latter are themselves required to be XPPE values. Observe that in an XPPE, the set of continuation values can depend on (*i*) the type of project; (*ii*) the vector of agents' announcements; (*iii*) the identity of the agent who performs the project; and finally (*iv*) whether that agent was successful or not. The promised continuation values are therefore given by $\mathcal{V} := (\mathcal{V}_0^\omega, \mathcal{V}_1^\omega, \mathcal{V}_2^\omega)_{\omega\in\{A,B\}}$, where each function

$$\mathcal{V}_i^\omega : \{0,1\}^2 \times \{1,2\} \times \{S,F\} \to \mathbb{R}_+^3, \tag{11}$$

specifies the promised continuation value of player $i \in \{0,1,2\}$ given that the current project is of type $\omega \in \{A,B\}$, the announcements are $m \in \{0,1\}^2$, agent $j \in \{0,1,2\}$ is assigned the project and its outcome is $y \in \{0,S,F\}$.

With slight abuse of notation, denote by $\chi := (\chi^\omega)_{\omega=A,B}$ the principal's allocation policy, where $\chi^\omega : \{0,1\}^2 \to X$ prescribes the identity of the agent who receives the project as a function of the agents' announcements $m = (m_1, m_2)$. Denote by $\chi_i^\omega(m) = 1$ the decision to allocate the project of type ω to agent i given the announcements m, and similarly by $\chi_i^\omega(m) = 0$ the decision to withhold the project from i. Denote by $\mathcal{M}:=(\mathcal{M}_1^\omega, \mathcal{M}_2^\omega)_{\omega=A,B}$ the agents' announcement policies, where each function $\mathcal{M}_i^\omega : \{\alpha,\beta\} \to \{0,1\}$ specifies i's announcement $m_i \in \{0,1\}$ as a function of his type θ_i and the current type of project ω. Finally, let $\mathcal{Z} := (\chi, \mathcal{M}, \mathcal{V})$ denote the profile of the principal and the agents' policies.[46]

[46]Below, a collection $\mathcal{Z}$ is often referred to simply as a 'policy'.

Given $(\mathcal{M}_{-i}, \chi, \mathcal{V})$, agent i's interim payoff when the project type is ω, the agent announces m_i, his true type is θ_i and he believes the other agent's type is θ_{-i}, is given by[47]

$$\begin{aligned}
&U_i^{\omega}(m_i, \theta_i; \theta_{-i}, \mathcal{M}_{-i}, \chi, \mathcal{V}) \\
&:= \mathbb{E}_{\theta_{-i}} \chi_i^{\omega}(m_i, m_{-i}) \left(\mu_i^{\omega}(\theta_i) \left((1-\delta)\pi + \delta \mathcal{V}_i^{\omega}((m_i, m_{-i}), i, S) \right) + (1 - \mu_i^{\omega}(\theta_i))\, \delta \mathcal{V}_i^{\omega}((m_i, m_{-i}), i, F) \right) \\
&+ \mathbb{E}_{\theta_{-i}} \chi_{-i}^{\omega}(m_i, m_{-i}) \left(\mu_{-i}^{\omega}(\theta_{-i}) \delta \mathcal{V}_i^{\omega}((m_i, m_{-i}), -i, S) + \left(1 - \mu_{-i}^{\omega}(\theta_{-i})\right) \delta \mathcal{V}_i^{\omega}((m_i, m_{-i}), -i, F) \right)
\end{aligned}$$

where m_{-i} are the announcements generated by $\mathcal{M}_{-i}$ and

$$\mu_j^{\omega}(\theta_j) := \begin{cases} \overline{\mu} & , (\omega, \theta_j) \in \{(A, \alpha), (B, \beta)\} \\ \underline{\mu} & , \text{otherwise.} \end{cases}$$

Similarly, given $(\mathcal{M}, \mathcal{V})$ and a project of type $\omega \in \{A, B\}$, denote the principal's expected payoff when his allocation policy is χ by

$$\begin{aligned}
&U_0^{\omega}(\chi; \mathcal{M}, \mathcal{V}) \\
&:= \mathbb{E}_{\theta} \sum_{j=1,2} \chi_j^{\omega}(m) \left(\mu_j^{\omega}(\theta_j) \left((1-\delta) + \delta \mathcal{V}_0^{\omega}((m_i, m_{-i}) j, S) \right) + \left(1 - \mu_j^{\omega}(\theta_j)\right) \delta \mathcal{V}_0^{\omega}((m_i, m_{-i}), j, F) \right),
\end{aligned}$$

where the expectation is taken over both agents' types.

For each of the players $i = 0, 1, 2$, let $\Lambda_i(\mathcal{Z})$ denote i's ex-ante expected payoff under $\mathcal{Z}$.

In an XPPE, each agent's announcement must be optimal regardless of the other agent's type. The policy $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ is ex-post incentive compatible (XIC) if for all $\omega \in \{A, B\}$, $i = 1, 2$, $\theta_i, \theta_{-i} \in \{\alpha, \beta\}$ and $m_i \in \{0, 1\}$,

$$U_i^{\omega}(\mathcal{M}_i^{\omega}(\theta_i), \theta_i; \theta_{-i}, \mathcal{M}_{-i}, \chi, \mathcal{V}) \geq U_i^{\omega}(m_i, \theta_i; \theta_{-i}, \mathcal{M}_{-i}, \chi, \mathcal{V}) \tag{12}$$

and for all $\omega \in \{A, B\}$, $\tilde{\chi} : \{0, 1\}^2 \to X$,

$$U_0^{\omega}(\chi; \mathcal{M}, \mathcal{V}) \geq U_0^{\omega}(\tilde{\chi}; \mathcal{M}, \mathcal{V}). \tag{13}$$

The vector of values $(v_0, v_1, v_2) \in \mathbb{R}_+^3$ is *ex-post decomposable*[48] on $V \subseteq \mathbb{R}_+^3$ if there exists a policy $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ such that the following conditions are satisfied:

- $\mathcal{Z}$ is XIC;
- $(\mathcal{V}_0^{\omega}(m, j, y), \mathcal{V}_1^{\omega}(m, j, y), \mathcal{V}_2^{\omega}(m, j, y)) \in V, \forall \omega \in \{A, B\}, j \in \{0, 1, 2\}, y \in \{0, S, F\}, m \in \{0, 1\}^2$;
- For each $i \in \{0, 1, 2\}$, $v_i = \Lambda_i(\mathcal{Z})$.

[47]Throughout the paper, continuation values are multiplied by $(1-\delta)$ in order to express them as per-period averages.

[48]Throughout the paper, whenever there is no confusion, the 'ex-post' qualification will often be omitted when referring to ex-post decomposability (as well as ex-post E-decomposability, ex-post self-generation, etc.).

The second condition is a feasibility condition requiring the continuation values to be elements in the set V under consideration, whereas the third condition imposes dynamic consistency. If the above three conditions are satisfied, the vector $v \in \mathbb{R}^3_+$ is said to be decomposed by the policy $\mathcal{Z}$ on V.

Given any $(\omega, \theta_1, \theta_2) \in \{A, B\} \times \{\alpha, \beta\}^2$ denote by

$$\mathcal{Q}(\omega, \theta_1, \theta_2) := \{i \in \{1,2\} | (\omega, \theta_i) \in \{(A, \alpha), (B, \beta)\}$$

the set of agents suited for project ω.

A vector $v \in \mathbb{R}^3_+$ is *ex-post E-decomposed* by $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ on V if the following hold:

- v is decomposed by $\mathcal{Z}$ on V;
- For all $l \in \{1,2\}$, $y \in \{S, F\}$ and any $(\omega, \theta_1, \theta_2) \in \{A, B\} \times \{\alpha, \beta\}^2$, $\mathcal{V}$ satisfies

$$\mathcal{V}_1^{\omega}\left(\mathcal{M}_1^{\omega}(\theta_1), \mathcal{M}_2^{\omega}(\theta_2), l, y\right) + \mathcal{V}_2^{\omega}\left(\mathcal{M}_1^{\omega}(\theta_1), \mathcal{M}_2^{\omega}(\theta_2), l, y\right) = \pi v^*;$$

- For any $(\omega, \theta_1, \theta_2) \in \{A, B\} \times \{\alpha, \beta\}^2$, χ satisfies

$$\chi_1^{\omega}(\mathcal{M}_1^{\omega}(\theta_1), \mathcal{M}_2^{\omega}(\theta_2)) + \chi_2^{\omega}(\mathcal{M}_1^{\omega}(\theta_1), \mathcal{M}_2^{\omega}(\theta_2)) = 1;$$

  and if $\mathcal{Q}(\omega, \theta_1, \theta_2) \neq \emptyset$ then $\chi_j^{\omega}(\mathcal{M}_1^{\omega}(\theta_1), \mathcal{M}_2^{\omega}(\theta_2)) = 1$ for some $j \in \mathcal{Q}(\omega, \theta_1, \theta_2)$.

The following lemma establishes some properties of the set $\mathcal{E}^*$, which will be useful in its characterization.

Lemma 1 *For any $\phi, \delta, \overline{\mu}, \underline{\mu}, \pi$, the set $\mathcal{E}^*$ satisfies the following properties*

1. *If $v \in \mathcal{E}^*$ then there exists $\mathcal{Z}$ such that v is E-decomposed by $\mathcal{Z}$ on $\mathcal{E}^*$.*
2. *Either $\mathcal{E}^* = \emptyset$ or $\mathcal{E}^* = \mathrm{co}\left(\{\hat{v}, \tilde{v}\}\right)$, for some $\hat{v}, \tilde{v} \in \left\{(v^*, v_1, v_2) \in \mathbb{R}^3_+ | v_1 + v_2 = v^*\right\}$.*

PROOF. For any $V \subseteq \mathbb{R}^3_+$, let

$$\mathcal{W}(V) := \left\{v \in \mathbb{R}^3_+ : v \text{ is decomposable on } V\right\}.$$

The set $V \subseteq \mathbb{R}^3_+$ is *ex-post self-generating* if $V \subseteq \mathcal{W}(V)$.

Adapting the methodology of APS to the current environment, $\mathcal{E}$ can be characterized through self generation (see Section G for the proof):

Lemma 2 *$\mathcal{E}$ and $\mathcal{W}$ satisfy the following properties: (i) If $V \subset \mathbb{R}^3$ is bounded and ex-post self-generating then $\mathcal{W}(V) \subseteq \mathcal{E}$ (and in turn $V \subseteq \mathcal{E}$); (ii) $\mathcal{W}(\mathcal{E}) = \mathcal{E}$; (iii) If $V \subseteq V'$ then $\mathcal{W}(V) \subseteq \mathcal{W}(V')$; (iv) If V is compact then $\mathcal{W}(V)$ is compact; (v) Let $\mathcal{W}^k(\mathcal{O})$ denote the set obtained following k iterations of $\mathcal{W}$, starting with the feasible set $\mathcal{O} := \mathrm{co}\{\vec{0}, (v^*, \pi v^*, 0), (v^*, 0, \pi v^*)\}$. Then $\mathcal{E} = \mathcal{W}_\infty := \bigcap_k \mathcal{W}^k(\mathcal{O})$; in particular, $\mathcal{E}$ is compact.*

Let $v = (v_0, v_1, v_2) \in \mathcal{E}^*$. Since $\mathcal{E}^* \subseteq \mathcal{E} = \mathcal{W}(\mathcal{E})$, there exists a policy $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ such that $v \in \mathcal{E}^*$ is decomposed by $\mathcal{Z}$ on $\mathcal{E}$. By definition of v^* and feasibility, $\mathcal{V}_1^\omega(m,j,y) + \mathcal{V}_2^\omega(m,j,y) \leq \pi v^*$, all $\omega \in \{A,B\}$, $m \in \{0,1\}^2$, $j \in \{1,2\}$, $y \in \{S,F\}$. Suppose $\mathcal{V}_1^\omega(m,j,y) + \mathcal{V}_2^\omega(m,j,y) < \pi v^*$ and $\chi_j^\omega(m) > 0$ for some $m \in \{0,1\}^2$, $j \in \{1,2\}$, $\omega \in \{A,B\}$, $y \in \{S,F\}$. Then since $\underline{\mu}, \overline{\mu}, \phi \in (0,1)$, dynamic consistency implies

$$v_1 + v_2 = \Lambda_1(\mathcal{Z}) + \Lambda_2(\mathcal{Z}) < \pi v^*,$$

contradicting $v \in \mathcal{E}^*$. Therefore, without loss of generality, $\mathcal{V}$ can be chosen to satisfy $\mathcal{V}_1^\omega(m,j,y) + \mathcal{V}_2^\omega(m,j,y) = \pi v^*$ for all $\omega \in \{A,B\}, m \in \{0,1\}^2, y \in \{S,F\}, j \in \{1,2\}$, implying $v \in \mathcal{W}(\mathcal{E}^*)$. Similarly, for any $(\omega, \theta_1, \theta_2) \in \{A,B\} \times \{\alpha,\beta\}^2$, if $\sum_{i=1,2} \chi_i^\omega(\mathcal{M}_1^\omega(\theta_1), \mathcal{M}_2^\omega(\theta_2)) < 1$ or if $\mathcal{Q} = \{i | (\omega, \theta_i) \in \{(A,\alpha),(B,\beta)\} \neq \emptyset$ but $\chi_j^\omega(\mathcal{M}_1^\omega(\theta_1), \mathcal{M}_2^\omega(\theta_2)) = 0$ for all $j \in \mathcal{Q}$ then dynamic consistency must be violated.

Part 2 follows immediately from the fact that, given the presence of a randomization device, $\mathcal{E}$ is not only compact but also convex. ■

Lemma 1 shows that for any set of primitives the set $\mathcal{E}^*$ is either empty or a closed interval. Furthermore, if $\mathcal{E}^*$ is non-empty then any vector of payoffs in $\mathcal{E}^*$ can be decomposed using a policy in which allocations are efficient (i.e., each project is allocated to one of the agents, and is allocated to a suited one whenever one exists) and continuations values are themselves efficient XPPE values.

Fixing the primitives $\{\delta, \phi, \underline{\mu}, \overline{\mu}, \pi\}$, observe that if $\mathcal{E}^* \neq 0$ then by Lemma 1 there exist minimal and a maximal values that can be supported under an efficient XPPE:

$$\underline{v} := \min \{v \in \mathbb{R}_+ : (v^*, v, v_2) \in \mathcal{E}^*\}$$

and $\overline{v} := \pi v^* - \underline{v}$.

Define $\Psi : [\underline{v}, \frac{1}{2}\pi v^*] \to [\underline{v}, \overline{v}]$ as follows. For each $v \in [\underline{v}, \frac{1}{2}\pi v^*]$,

$$\Psi(v) := \inf \{\tilde{v} \in \mathbb{R}_+ : (v^*, \tilde{v}, \pi v^* - \tilde{v}) \in \mathcal{W}^e(\text{co}\{(v^*, v, \pi v^* - v), (v^*, \pi v^* - v, v)\})\}, \quad (14)$$

where for any $V \subseteq \mathbb{R}_+^3$,

$$\mathcal{W}^e(V) := \{v \in \mathbb{R}_+^3 : v \text{ is E-decomposable on } V\}.$$

That is, Ψ maps any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$ to the infimum of the set of values $\tilde{v} \in \mathbb{R}_+$ such that the payoff vector $(v^*, \tilde{v}, \pi v^* - \tilde{v})$ can be E-decomposed on $\text{co}\{(v^*, v, \pi v^* - v), (v^*, \pi v^* - v, v)\}$.

The following must then hold.

Lemma 3 *If $\mathcal{E}^* \neq \emptyset$ then $\underline{v}$ is a fixed-point of Ψ.*

PROOF. Suppose $\mathcal{E}^* \neq \emptyset$. Let $\mathcal{Z}$ be a policy that E-decomposes $(v^*, \underline{v}, \overline{v})$ on $\mathcal{E}$. By definition of $\underline{v}$ and $\overline{v}$, $\underline{v} \leq \mathcal{V}_1^\omega(m,j,y) \leq \overline{v}$. Therefore, $(v^*, \underline{v}, \overline{v})$ is E-decomposed by $\mathcal{Z}$ on $\text{co}\{(v^*, \underline{v}, \overline{v}), (v^*, \overline{v}, \underline{v})\}$, hence $\underline{v} \geq \Psi(\underline{v})$. Suppose that $\underline{v} > \Psi(\underline{v})$, then there exists $\tilde{v} < \underline{v}$ such that $(v^*, \tilde{v}, \pi v^* - \tilde{v})$ is E-decomposable on $\text{co}\{(v^*, \underline{v}, \overline{v}), (v^*, \overline{v}, \underline{v})\}$, and in particular on $\mathcal{E}$. By definition of $\underline{v}$, however, $(v^*, \tilde{v}, \pi v^* - \tilde{v}) \notin \mathcal{E}$. Thus, $\mathcal{W}(\mathcal{E}) \neq \mathcal{E}$, a contradiction. ■

B Proofs for Section 3 - Efficiency

Proof of Proposition 1. First, note that if $v \in \mathbb{R}^3$ is E-decomposed by a policy that satisfies XIC, feasibility and dynamic consistency for the agents $i = 1, 2$, then these conditions are trivially satisfied for the principal. Hence, we can ignore the principal's incentives in the analysis that follows.

For any $l \in \{\underline{\mu}, \overline{\mu}\}, i, j \in \{1,2\}, \omega \in \{A, B\}, m \in \{0,1\}^2$, denote

$$\mathbb{E}_l \mathcal{V}_i^\omega(m, j) := l\mathcal{V}_i^\omega(m, j, S) + (1 - l)\mathcal{V}_i^\omega(m, j, F).$$

The following lemma shows that it is without loss of generality to restrict attention to a particular set of policies.

Lemma 4 *Suppose $\mathcal{Z}$ E-decomposes $v \in \mathbb{R}^3$ on $V \subseteq \mathbb{R}^3$. Then there exists $\mathcal{Z}' = (\mathcal{M}, \chi, \mathcal{V})$ that also E-decomposes v on V such that for all $\omega \in \{A, B\}$ either*

1. *for all $i \in \{1,2\}, \theta_i \in \{\alpha, \beta\}, m \in \{0,1\}^2$, $\mathcal{M}_i^\omega(\theta_i) = 1 \Leftrightarrow (\omega, \theta_i) \in \{(A, \alpha), (B, \beta)\}$, $\chi_1^\omega(m) + \chi_2^\omega(m) = 1$, and $(m_i, m_{-i}) = (1, 0) \Rightarrow \chi_i^\omega(m) = 1$; or*
2. *for some $i \in \{1,2\}$: $\mathcal{M}_i^\omega(\theta_i) = 1 \Leftrightarrow (\omega, \theta_i) \in \{(A, \alpha), (B, \beta)\}$, $\mathcal{M}_{-i}^\omega(\alpha) = \mathcal{M}_{-i}^\omega(\beta) = 1$, $m_i = 1 \Rightarrow \chi_i^\omega = 1$ and $m_i = 0 \Rightarrow \chi_{-i}^\omega = 1$.*

Proof. Let $\theta(\omega)$ and $\theta(-\omega)$ denote the types suited and unsuited, respectively, for a project $\omega \in \{A, B\}$. If $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ E-decomposes v on V then each project must be allocated to one of the agents. Furthermore, at least one of the agents must reveal his type; in other words, for any $\omega \in \{A, B\}$, for at least one of the agents j, $\mathcal{M}_j^\omega(\alpha) \neq \mathcal{M}_j^\omega(\beta)$. Furthermore, if for some $\omega \in \{A, B\}$ one of the agents, say i, pools (i.e., $\mathcal{M}_i^\omega(\theta(\omega)) = \mathcal{M}_i^\omega(\theta(-\omega))$), then regardless of i's announcement $\mathcal{M}_i^\omega(\theta_i)$ agent $-i$ must be allocated the project whenever he reveals he is suited, $\chi_{-i}^\omega(\mathcal{M}_{-i}^\omega(\theta(\omega)), \mathcal{M}_i^\omega(\theta_i)) = 1$, and otherwise $\chi_i^\omega(\mathcal{M}_i^\omega(\theta_i), \mathcal{M}_{-i}^\omega(\theta(-\omega))) = 1$. Finally, that $\mathcal{Z}$ can be modified to obtain $\mathcal{Z}'$ in which the conditions stated in the lemma are satisfied is just a matter of relabling. ■

The lemma above therefore guarantees that there will be no loss of generality in restricting attention to policies such that either (a) both agents announce truthfully, or (b) only one agent announces truthfully, the other pools, and the project is allocated as described above.

In the analysis that follows, we consider policies $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ that, given $\omega \in \{A, B\}$, correspond to the first case in Lemma 4 above; that is, for all $i \in \{1,2\}, \theta_i \in \{\alpha, \beta\}, m \in \{0,1\}^2$, $\mathcal{M}_i^\omega(\theta_i) = 1 \Leftrightarrow (\omega, \theta_i) \in \{(A, \alpha), (B, \beta)\}$, $\chi_1^\omega(m) + \chi_2^\omega(m) = 1$, and $(m_i, m_{-i}) = (1, 0) \Rightarrow \chi_i^\omega(m) = 1$. This is without loss, as will be shown below.

For such policies $(\mathcal{M}, \chi, \mathcal{V})$, the expected payoff of agent 1 is given by

$$\Lambda_1(\mathcal{M}, \chi, \mathcal{V}) = \frac{1}{2}\Lambda_1^A(\mathcal{M}, \chi, \mathcal{V}) + \frac{1}{2}\Lambda_1^B(\mathcal{M}, \chi, \mathcal{V}),$$

where 1's expected payoff for $\omega = A$ is

$$
\begin{aligned}
&\Lambda_1^A(\mathcal{M},\chi,\mathcal{V}) \\
&:= (1-\phi^1)(1-\phi^2)\left(\chi_1^A(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^A((0,0),1)\right) + \left(1-\chi_1^A(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^A((0,0),2)\right) \\
&\quad + \phi^1(1-\phi^2)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A((1,0),1)\right) + (1-\phi^1)\phi^2\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A((0,1),2) \qquad (15)\\
&\quad + \phi^1\phi^2\left(\chi_1^A(1,1)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A((1,1),1)\right) + \left(1-\chi_1^A(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A((1,1),2)\right).
\end{aligned}
$$

and similarly for $\omega = B$,

$$
\begin{aligned}
&\Lambda_1^B(\mathcal{M},\chi,\mathcal{V}) \\
&:= \phi^1\phi^2\left(\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),1)\right) + \left(1-\chi_1^B(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2)\right) \\
&\quad + (1-\phi^1)\phi^2\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,0),1)\right) + \phi^1(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2) \qquad (16)\\
&\quad + (1-\phi^1)(1-\phi^2)\left(\chi_1^B(1,1)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),1)\right) + \left(1-\chi_1^B(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2)\right),
\end{aligned}
$$

XIC requires that agents do not have strict incentives to announce that they are suited for projects they are not truly suited for, regardless of their beliefs about the type of the other agent. Given $\omega = B$, the interim payoff of agent 1 of type $\theta_1 = \alpha$ must be weakly greater when he announces $m_1 = 0$ rather than $m_1 = 1$, regardless his belief about $\theta_2 \in \{\alpha, \beta\}$:

$$
\begin{aligned}
U_1^B(0,\alpha;\beta) &= \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2) \\
&\geq (1-\delta)\pi\underline{\mu}\chi_1^B(1,1) + \delta\chi_1^B(1,1)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),1) + \delta\left(1-\chi_1^B(1,1)\right)\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2) \\
&= U_1^B(1,\alpha;\beta), \qquad \text{(IC1)}
\end{aligned}
$$

$$
\begin{aligned}
U_1^B(0,\alpha;\alpha) &= (1-\delta)\pi\underline{\mu}\chi_1^B(0,0) + \delta\chi_1^B(0,0)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),1) + \delta\left(1-\chi_1^B(0,0)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2) \\
&\geq (1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,0),1) \\
&= U_1^B(1,\alpha;\alpha). \qquad \text{(IC2)}
\end{aligned}
$$

Analogous XIC conditions, $U_2^B(0,\alpha;\theta_1) \geq U_2^B(1,\alpha;\theta_1)$ for beliefs $\theta_1 = \alpha, \beta$, apply for agent 2. Imposing $\mathcal{V}_2^\omega(m,j,y) = \pi v^* - \mathcal{V}_1^\omega(m,j,y)$ and $\chi_2^\omega(m) = 1 - \chi_1^\omega(m)$, these conditions can be written as

$$
\begin{aligned}
U_2^B(0,\alpha;\beta) &= -\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,0),1) \\
&\geq (1-\delta)\pi\underline{\mu}\left(1-\chi_1^B(1,1)\right) - \delta\left(1-\chi_1^B(1,1)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),2) - \delta\chi_1^B(1,1)\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),1) \\
&= U_2^B(1,\alpha;\beta), \qquad \text{(IC3)}
\end{aligned}
$$

$$
\begin{aligned}
U_2^B(0,\alpha;\alpha) &= (1-\delta)\pi\underline{\mu}\left(1-\chi_1^B(0,0)\right) - \delta\left(1-\chi_1^B(0,0)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2) - \delta\chi_1^B(0,0)\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),1) \\
&\geq (1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2) \\
&= U_2^B(1,\alpha;\alpha). \qquad \text{(IC4)}
\end{aligned}
$$

Analogous XIC conditions can be derived for $\omega = A$; denote these by

$$U_i^A(0,\beta;\theta_{-i}) \geq U_i^A(1,\beta;\theta_{-i}) \quad ,\forall \theta_{-i} \in \{\alpha,\beta\}, i = 1,2. \tag{IC5}$$

The remaining XIC conditions guarantee that agents find it optimal to announce truthfully when they are suited for the project (again, regardless of their belief about the other agent's type):

$$U_i^\omega(1,\theta_i;\theta_{-i}) \geq U_i^\omega(0,\theta_i;\theta_{-i}) \quad \forall (\omega,\theta_i) \in \{(A,\alpha),(B,\beta)\}, \theta_{-i} \in \{\alpha,\beta\}, i = 1,2. \tag{IC6}$$

It can easily be verified that policies corresponding to the second case in Lemma 4 can be analyzed as special cases of the conditions above; without loss of generality, if agent 2 pools, setting $\chi_1^\omega(1,1) = 1$ and $\chi_1^\omega(0,0) = 0$ in (15), (16) and (IC1)-(IC6) yields the appropriate interim payoffs and XIC constraints for the agents.

Assuming $\mathcal{E}^* \neq \emptyset$, from Lemma 4, for any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$,

$$\begin{aligned} \Psi(v) = &\min_{(\chi,\mathcal{V})} \Lambda_1 \\ &\text{s.t.} \quad \text{(IC1)-(IC6)}, \\ &\qquad \chi_i^\omega(m) \in \{0,1\},\ \chi_1^\omega(m) + \chi_2^\omega(m) = 1, (m_l, m_{-l}) = (1,0) \Rightarrow \chi_i^\omega(m) = 1, \\ &\qquad \mathcal{V}_i^\omega(m,j,y) \in [v, \pi v^* - v], \mathcal{V}_1^\omega(m,j,y) + \mathcal{V}_2^\omega(m,j,y) = \pi v^*, \\ &\qquad \forall \omega \in \{A,B\}, m \in \{0,1\}^2, l,j \in \{1,2\}, y \in \{S,F\}. \end{aligned} \tag{17}$$

For any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$, the program on the RHS of (17) is referred to as $\mathcal{P}_v$. Denote by $\mathcal{P}_v^R$ the same program excluding the constraints (IC6). Fixing any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$, the following lemmas derive necessary properties that any $(\chi, \mathcal{V})$ minimizing Λ_1 must satisfy.

Lemma 5 *Suppose $\mathcal{E}^* \neq \emptyset$. Any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$ satisfies $\mathcal{V}_1^\omega((1,0),1,y) = v$, all $\omega \in \{A,B\}, y \in \{S,F\}$.*

PROOF. Suppose $\mathcal{V}_1^B((1,0),1,S) > v$ and consider the effect of a slight decrease in $\mathcal{V}_1^A((1,0),1,S)$. Such a decrease both reduces Λ_1 and relaxes the constraints (IC2) and (IC3), without affecting the remaining constraints; a contradiction. Similarly, $\mathcal{V}_1^B((1,0),1,F) = \mathcal{V}_1^A((1,0),1,S) = \mathcal{V}_1^A((1,0),1,F) = v$. ■

Lemma 6 *Suppose $\mathcal{E}^* \neq \emptyset$. For any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$, the constraints (IC2) and (IC3) (as well as the analogous constraints for $\omega = A$) must hold with equality.*

PROOF. Suppose (IC2) is slack. Assume first that $\chi_1^B(0,0) = 0$. Then (IC2) reduces to $\delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),2) > (1-\delta)\pi\underline{\mu} + \delta v$. However, $\mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),2)$ can then be decreased by a small amount without violating the strict inequality in (IC2) or feasibility. Furthermore, such a decrease reduces Λ_1, relaxes (IC4), and does not affect the remaining constraints; a contradiction. Similarly, if $\chi_1^B(0,0) = 1$ then (IC2) is equivalent to $\delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),1) > \delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,0),1) = \delta v$. But then $\mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),1)$ can be reduced by a small amount without violating the strict inequality in (IC2) or feasibility, which reduces Λ_1, relaxes (IC4), and does no affect the remaining constraints; a contradiction. Hence (IC2) must hold with equality.

Next, assume (IC3) is slack. Suppose $\chi_1^B(1,1) = 0$. Then (IC3) is equivalent to $\delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,1),2) > (1-\delta)\pi\underline{\mu} + \delta v$. Hence $\mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,1),2)$ can be reduced by a small amount without violating the strict inequality or feasibility, which decreases Λ_1 and relaxes (IC1) without affecting the remaining constraints; a contradiction. Suppose instead that $\chi_1^B(1,1) = 1$. Then $\delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,1),1) > \delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,0),1) = \delta v$, and again a contradiction is obtained by considering the effects of a small decrease in $\mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,1),1)$. Hence (IC3) also holds with equality.

Finally, note that a similar analysis applies for $\omega = A$. ■

Lemma 7 *Suppose $\mathcal{E}^* \neq \emptyset$. For any $(\chi, \mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$, $\chi_1^\omega(1,1) = 0$, $\omega \in \{A, B\}$.*

PROOF. First, note that the fact that (IC2) holds with equality (Lemma 6) together with Lemma 5 implies

$$\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),1)\right) + \delta\left(1 - \chi_1^B(0,0)\right) \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((0,0),2) = (1-\delta)\pi\underline{\mu} + \delta v. \quad (18)$$

Suppose $\chi_1^B(1,1) = 1$. The previous lemmas together with (IC3) imply $\mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,1),1) = v$, and hence $\mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,1),1) = v$. From (IC1),

$$\delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((0,1),2) \geq (1-\delta)\pi\underline{\mu} + \delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,1),1) = (1-\delta)\pi\underline{\mu} + \delta v. \quad (19)$$

Therefore, under $\chi_1^B(1,1) = 1$,

$$\begin{aligned}
\Lambda_1^B(\mathcal{M}, \chi, \mathcal{V}) &= \phi^1\phi^2\left((1-\delta)\pi\underline{\mu} + \delta v\right) + (1-\phi^1)\phi^2\left((1-\delta)\pi\overline{\mu} + \delta v\right) \\
&\quad + \phi^1(1-\phi^2)\delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((0,1),2) + (1-\phi^1)(1-\phi^2)\left((1-\delta)\pi\overline{\mu} + \delta v\right) \\
&\geq (1-\delta)\pi\left(\phi^1\underline{\mu} + (1-\phi^1)\overline{\mu}\right) + \delta v
\end{aligned} \quad (20)$$

where the first equality follows from (16), setting $\chi_1^B(1,1) = 1$, (18) and $\mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,1),1) = \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,0),1) = v$, and the inequality follows from (19).

Consider $(\chi, \mathcal{V})$ such that $\chi_1^B(1,1) = 0$, and furthermore

$$\delta \mathcal{V}_1^B((0,1),2,y) = \delta \mathcal{V}_1^B((1,1),2,y) = (1-\delta)\pi\underline{\mu} + \delta v,$$

and $\mathcal{V}_1^B((1,1),1,y) = v$, for all $y = S, F$. The policy is feasible (given v) only if $(1-\delta)\pi\underline{\mu} + \delta v \leq \delta\pi v^* - \delta v$. To see that this inequality must hold, note that from (IC3), which by Lemma 6 holds with equality,

$$\begin{aligned}
\delta v + (1-\delta)\pi\underline{\mu} &= \chi_1^B(1,1)\left((1-\delta)\pi\underline{\mu} + \delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,1),1)\right) + \left(1 - \chi_1^B(1,1)\right)\delta \mathbb{E}_{\underline{\mu}} \mathcal{V}_1^B((1,1),2) \\
&\leq \chi_1^B(1,1)\left((1-\delta)\pi\underline{\mu} + \delta\left(\pi v^* - v\right)\right) + \left(1 - \chi_1^B(1,1)\right)\delta\left(\pi v^* - v\right) \\
&\leq (1-\delta)\pi\underline{\mu} + \delta\left(\pi v^* - v\right),
\end{aligned}$$

where the first inequality follows from the fact that under $\mathcal{P}_v^R$, $\mathcal{V}_1^B(m,j,y) \leq \pi v^* - v$, all $m \in \{0,1\}^2, y \in \{S,F\}, j \in \{1,2\}$.

It can easily be verified that a policy with the above properties satisfies (IC1)-(IC4).[49] Furthermore, under such a policy,

$$\begin{aligned}
\Lambda_1^B &= \phi^1\phi^2\left((1-\delta)\pi\underline{\mu}+\delta v\right) + (1-\phi^1)\phi^2\left((1-\delta)\pi\overline{\mu}+\delta v\right) + \phi^1(1-\phi^2)\left((1-\delta)\pi\underline{\mu}+\delta v\right) \\
&\quad + (1-\phi^1)(1-\phi^2)\left((1-\delta)\pi\underline{\mu}+\delta v\right), \\
&= (1-\delta)\pi\left(\left(1-(1-\phi^1)\phi^2\right)\underline{\mu} + (1-\phi^1)\phi^2\overline{\mu}\right) + \delta v \\
&\leq (1-\delta)\pi\left(\phi^1\underline{\mu} + (1-\phi^1)\overline{\mu}\right) + \delta v
\end{aligned}$$

where the first equality follows from (16) (setting $\chi_1^B(1,1)=0$), (18), and the fact that $\delta\mathcal{V}_1^B((0,1),2,y) = \delta\mathcal{V}_1^B((1,1),2,y) = (1-\delta)\pi\underline{\mu}+\delta v$ under the proposed policy.

Thus, from (20), the proposed policy yields a lower Λ_1 than any policy for which $\chi_1^B(1,1)=1$, which yields a contradiction.

Finally, note that an analogous argument shows that $\chi_1^A(1,1)=0$ also holds. ■

Lemma 8 *Suppose $\mathcal{E}^* \neq \emptyset$. For any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$,*

$$\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega((1,1),2) = \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega((0,1),2) = (1-\delta)\underline{\mu}+\delta v, \quad \forall\omega\in\{A,B\}.$$

PROOF. Lemmas 5-7 combined with (IC3) imply $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega((1,1),2) = (1-\delta)\underline{\mu}+\delta v$. Furthermore, combining (18) with (IC4) gives $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2) \geq (1-\delta)\pi\underline{\mu}+\delta v$. Suppose $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2) > (1-\delta)\pi\underline{\mu}+\delta v$, then $\mathcal{V}_1^B((0,1),2,S)$ or $\mathcal{V}_1^B((0,1),2,F)$ can be decreased slightly (this can be done since it cannot be the case that $\mathcal{V}_1^B((0,1),2,S) = \mathcal{V}_1^B((0,1),2,F) = v$), such that the strict inequality is maintained. This decreases Λ_1 without violating any of the constraints, which gives a contradiction. Therefore, $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2) = (1-\delta)\pi\underline{\mu}+\delta v$.

An analogous argument applies for $\omega = A$. ■

Lemma 9 *Suppose $\mathcal{E}^* \neq \emptyset$. For any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$, either*

- *$\delta\mathcal{V}_1^\omega((1,1),2,S) = \delta\mathcal{V}_1^\omega((0,1),2,S) = \delta v$; and*
- *$\delta\mathcal{V}_1^\omega((1,1),2,F) = \delta\mathcal{V}_1^\omega((0,1),2,F) = \delta v + \frac{(1-\delta)\underline{\mu}\pi}{1-\underline{\mu}} \leq \delta(\pi v^* - v)$;*

or

- *$\delta\mathcal{V}_1^\omega((1,1),2,F) = \delta\mathcal{V}_1^\omega((0,1),2,F) = \delta(\pi v^* - v)$; and*
- *$\delta\mathcal{V}_1^\omega((1,1),2,S) = \delta\mathcal{V}_1^\omega((0,1),2,S) = (1-\delta)\pi + \frac{1}{\underline{\mu}}\delta v - \frac{1-\underline{\mu}}{\underline{\mu}}\delta(\pi v^* - v) \geq \delta v$.*

[49] It can also easily be verified that additionally setting $\chi_1^B(0,0)=0$, $\mathcal{V}_1^B((0,0),1,y)=v$, $\delta\mathcal{V}_1^B((0,0),2,y)=(1-\delta)\pi\underline{\mu}+\delta v$, all $y\in\{S,F\}$, the constraints relevant for $\omega=B$ in (IC6) are also satisfied.

PROOF. Consider $\omega = B$ (an analogous argument holds for $\omega = A$). Suppose neither of the conditions above hold. Then $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2)$ (or $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2)$) can be decreased by a small amount while keeping $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),2)$ (or $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2)$) fixed at $(1-\delta)\pi\underline{\mu}+\delta v$, as required by Lemma 8, without violating any of the constraints. In particular, as $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),2)$ and $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2)$ remain unchanged and the only constraint affected, (IC1), can be satisfied by setting $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2) = \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2)$. Such a decrease strictly reduces Λ_1, giving the contradiction. ∎

It can easily be verified that any $(\chi, \mathcal{V})$ that satisfies all of the constraints in $\mathcal{P}_v^R$ and furthermore $\mathcal{V}_1^\omega((1,0),1,y) = v$, $\chi_1^\omega(1,1) = 0$, $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega((1,1),2) = \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega((0,1),2) = (1-\delta)\underline{\mu}+\delta v$, (IC2) holds with equality, and finally $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega((1,1),2) = \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega((0,1),2)$, all $\omega \in \{A,B\}$, $y \in \{S,F\}$, also satisfies (IC6).[50] Therefore, the restriction to $\mathcal{P}_v^R$ is without loss of generality.

The following lemma completes the proof by using the properties established above to solve for $\underline{v}$ as a fixed-point of Ψ.

Lemma 10 *$\mathcal{E}^* \neq \emptyset$ if and only if*

$$\phi(1-\phi) \geq \frac{\underline{\mu}\,(1-\delta\overline{\mu})}{\delta\left(\overline{\mu}-\underline{\mu}\right)\left(1-\underline{\mu}\right)}. \tag{21}$$

PROOF. Suppose $\mathcal{E}^* \neq \emptyset$. From Lemma 3, $\underline{v} = \Psi(\underline{v})$. From Lemma 9, if $(\chi, \mathcal{V})$ yields a solution to $\mathcal{P}_{\underline{v}}$ then one of the two cases considered in Lemma 9 must hold. Below, each of these cases is considered.

Case 1. $(\chi, \mathcal{V})$ satisfies the following conditions: $\chi_1^\omega(1,1) = 0$, (IC2) holds with equality,

$$\delta\mathcal{V}_1^\omega((1,0),1,y) = \delta\mathcal{V}_1^\omega((1,1),2,S) = \delta\mathcal{V}_1^\omega((0,1),2,S) = \delta\underline{v},$$

and

$$\delta\mathcal{V}_1^\omega((1,1),2,F) = \delta\mathcal{V}_1^\omega((0,1),2,F) = \delta\underline{v} + \frac{(1-\delta)\underline{\mu}\pi}{1-\underline{\mu}} \leq \delta\overline{v}, \tag{22}$$

all $\omega \in \{A,B\}$, $y \in \{S,F\}$.

Such a policy $(\chi, \mathcal{V})$ satisfies,

$$\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega((1,1),2) = \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega((0,1),2) = \delta\underline{v} + \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}(1-\delta)\pi.$$

Therefore,

$$\begin{aligned}\Lambda_1^B &= \phi^1\phi^2\left((1-\delta)\pi\underline{\mu}+\delta\underline{v}\right) + (1-\phi^1)\phi^2\left((1-\delta)\pi\overline{\mu}+\delta\underline{v}\right) + \phi^1(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2) \\ &\quad + (1-\phi^1)(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2) \\ &= \delta\underline{v} + (1-\delta)\pi\left(\phi^1\phi^2\underline{\mu} + (1-\phi^1)\phi^2\overline{\mu} + (1-\phi^2)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right)\end{aligned}$$

[50]This is guaranteed by the condition (18).

and similarly

$$\Lambda_1^A = \delta \underline{v} + (1-\delta)\pi \left((1-\phi^1)(1-\phi^2)\underline{\mu} + \phi^1(1-\phi^2)\overline{\mu} + \phi^2 \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})} \right),$$

which implies

$$\underline{v} = \Psi(\underline{v}) = \Lambda_1 = \delta \underline{v} + \frac{1}{2}(1-\delta)\pi \left(2\phi(1-\phi)\underline{\mu} + ((1-\phi)^2 + \phi^2)\overline{\mu} + \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})} \right).$$

Hence,

$$\underline{v} = \frac{1}{2}\pi \left(\overline{\mu} - 2\phi(1-\phi)\left(\overline{\mu} - \underline{\mu}\right) + \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})} \right) \quad \text{and} \quad \overline{v} = \pi v^* - \underline{v} = \frac{1}{2}\pi \left(\frac{\overline{\mu} - \underline{\mu}}{1-\underline{\mu}} \right). \tag{23}$$

Now, feasibility requires that (22) hold; thus, it must be the case that

$$\delta \mathcal{V}_1^\omega((1,1),2,F) = \delta \mathcal{V}_1^\omega((0,1),2,F) = \delta \underline{v} + \frac{(1-\delta)\underline{\mu}\pi}{1-\underline{\mu}} \leq \delta \overline{v}.$$

It is easily verified that the latter condition holds if and only if (21) is satisfied.

Case 2. $(\chi, \mathcal{V})$ satisfies the following conditions: $\chi_1^\omega(1,1) = 0$, (IC2) holds with equality,

$$\delta \mathcal{V}_1^\omega((1,0),1,y) = \delta \underline{v}\ ,\ \delta \mathcal{V}_1^\omega((1,1),2,F) = \delta \mathcal{V}_1^\omega((0,1),2,F) = \delta \overline{v},$$

and

$$\delta \mathcal{V}_1^\omega((1,1),2,S) = \delta \mathcal{V}_1^\omega((0,1),2,S) = (1-\delta)\pi + \frac{1}{\underline{\mu}}\delta \underline{v} - \frac{1-\underline{\mu}}{\underline{\mu}}\delta \overline{v} \geq \delta \underline{v}, \tag{24}$$

all $\omega \in \{A, B\}$, $y \in \{S, F\}$.

Following steps similar to those for case 1, it can be shown that

$$\underline{v} = \pi v^* \left(\frac{2\underline{\mu} - \delta\left(\underline{\mu} + \overline{\mu}\right)}{2\left(\underline{\mu} - \delta \overline{\mu}\right)} \right)$$

which, after some algebra, in turn implies

$$\delta \mathcal{V}_1^\omega(m,2,S) = (1-\delta)\pi + \delta \underline{v} \left(\frac{2 - \delta\left(2 + \overline{\mu} - \underline{\mu}\right)}{2\underline{\mu} - \delta\left(\underline{\mu} + \overline{\mu}\right)} \right),$$

all $m \in \{(0,1),(1,1)\}$. Feasibility requires that the inequality (24) hold. It can easily be shown that the latter holds only if (21) is satisfied.

We have therefore shown that when (21) is violated, it cannot be the case that $\Psi(\underline{v}) = \underline{v}$. By Lemma 3, $\mathcal{E}^* \neq \emptyset$ implies (21). Furthermore, if (21) holds then the analysis above guarantees that $(v^*, \underline{v}, \overline{v})$ with $\underline{v}$ and $\overline{v}$ defined as in (23) is E-decomposable on co $\{(v^*, \underline{v}, \overline{v}), (v^*, \overline{v}, \underline{v})\}$. A policy $(\chi, \mathcal{V})$ analogous to the one decomposing $(v^*, \underline{v}, \overline{v})$, switching the roles of the two agents, decomposes $(v^*, \overline{v}, \underline{v})$

on co $\{(v^*,\underline{v},\overline{v}),(v^*,\overline{v},\underline{v})\}$. Since the constraints and payoffs are all linear in the continuation values, any $\tilde{v}\in$ co $\{(v^*,\underline{v},\overline{v}),(v^*,\overline{v},\underline{v})\}$ is E-decomposable on co $\{(v^*,\underline{v},\overline{v}),(v^*,\overline{v},\underline{v})\}$ using convex combinations of these two policies. In particular, the set co $\{(v^*,\underline{v},\overline{v}),(v^*,\overline{v},\underline{v})\}$ is self-generating, and hence $\mathcal{E}^*\neq\emptyset$. ■

Finally, Lemma 10 combined with the monotonicity of the LHS of (21) completes the proof of Proposition 1. *Q.E.D.*

Proof of Proposition 2. Follows immediately from the proof of Lemma 10. *Q.E.D.*

Proof of Proposition 3. For any $\phi\in[\frac{1}{2},1)$, recall that $v^o(\phi)$ denotes the principal's value under decentralization and $v^*(\phi)$ the principal's first-best value. Since the principal cannot commit to future allocation decisions, in equilibrium, at any history, the principal's continuation value $v(\phi)$ must satisfy

$$v^o(\phi)\leq v(\phi)\leq v^*(\phi).$$

Furthermore, as $\phi\to 1$,

$$v^o(\phi)=\phi\overline{\mu}+(1-\phi)\underline{\mu}\to\overline{\mu}\quad\text{and}\quad v^*(\phi)=\overline{\mu}-(\overline{\mu}-\underline{\mu})\phi(1-\phi)\to\overline{\mu}.$$

Hence, at any period and given any history, the efficiency loss $v^*(\phi)-v(\phi)$ becomes arbitrarily small, taking ϕ above sufficiently close to 1.

Suppose that there exists an XPPE in which, at some history, communication is fruitful: an agent who claims to be suited for the current type of project, although he is not specialized in it, is allocated the project given some profile of announcements. A necessary condition for the agent who receives the project he is not specialized in to have an (ex-post) incentive to be truthful about his suitability is that

$$(1-\delta)\underline{\mu}\pi\leq\delta\left(\overline{v}(\phi)-\underline{v}(\phi)\right),\tag{25}$$

where $\overline{v}(\phi)$ and $\underline{v}(\phi)$ denote the agent's highest and lowest feasible equilibrium values in the continuation game. For any $\phi\in[\frac{1}{2},1)$, denote by $\underline{v}^*(\phi)$ and $\overline{v}^*(\phi)$ the lowest and highest possible values for an agent under a first-best allocation rule. As $\phi\to 1$,

$$\overline{v}^*(\phi)=\frac{1}{2}\left(\overline{\mu}+2\phi(1-\phi)\underline{\mu}\right)\pi\to\frac{1}{2}\overline{\mu}\pi\quad\text{and}\quad\underline{v}^*(\phi)=\frac{1}{2}\overline{\mu}\left(\phi^2+(1-\phi)^2\right)\pi\to\frac{1}{2}\overline{\mu}\pi.$$

For any $\phi\in(\phi^*,1)$, by definition of $\overline{v}^*(\phi)$ and ϕ^*, $\overline{v}(\phi)>\overline{v}^*(\phi)$ implies a strict efficiency loss: $v^*(\phi)-v(\phi)>0$. As argued above, however, taking ϕ sufficiently close to 1, $v^*(\phi)-v(\phi)$ becomes arbitrarily small.

If $\overline{v}(\phi)\leq\overline{v}^*(\phi)$ and $\underline{v}(\phi)\geq\underline{v}^*(\phi)$ then, since $\overline{v}^*(\phi)-\underline{v}^*(\phi)\to 0$ as $\phi\to 1$, for ϕ sufficiently close to 1, $\overline{v}(\phi)-\underline{v}(\phi)$ becomes arbitrarily small. If $\overline{v}(\phi)>\overline{v}^*(\phi)$ then the difference $\overline{v}(\phi)-\overline{v}^*(\phi)$ must be generated by promising the agent, with positive probability, projects that he is not suited for even though the other agent is. While the gain in the agent's valuation for each such project is $\underline{\mu}$, the loss for the principal is $\overline{\mu} - \underline{\mu}$. Hence

$$v^*(\phi) - v(\phi) \geq \frac{\overline{\mu} - \underline{\mu}}{\underline{\mu}} \left(\overline{v}(\phi) - \overline{v}^*(\phi) \right).$$

In particular, for ϕ sufficiently close to 1, $\overline{v}(\phi) - \overline{v}^*(\phi)$ is made arbitrarily small. An analogous argument holds for $\underline{v}^*(\phi) - \underline{v}(\phi)$. Since $\overline{v}^*(\phi) - \underline{v}^*(\phi) \to 1$ as $\phi \to 1$, for ϕ sufficiently close to 1 the difference $\overline{v}(\phi) - \underline{v}(\phi)$ again becomes arbitrarily small.

Hence, there exists $\phi^o < 1$ such that for all $\phi \in [\phi^o, 1)$ the inequality (25) cannot be satisfied. *Q.E.D.*

C Proofs for Section 4 - Rules for efficient delegation

It is convenient to first prove Proposition 5, from which the proof of the first part of Proposition 4 immediately follows.

Proof of Proposition 5. By definition, in equilibrium, the values v_i^f, v_i^{-f} must satisfy the following consistency conditions

$$v_i^f = \frac{1}{2} \left(\begin{array}{ll} \omega{=}A & \left\{ \begin{array}{l} \phi^i \cdot \left(\overline{\mu} \left((1-\delta)\pi + \delta v_i^f \right) + (1-\overline{\mu})\delta v_i^{-f} \right) \\ +(1-\phi^i)\left(\phi^{-i}\delta v_i^f + (1-\phi^{-i})\left(\underline{\mu}\left((1-\delta)\pi + \delta v_i^f \right) + (1-\underline{\mu})\delta v_i^{-f} \right) \right) \end{array} \right. \\ \omega{=}B & \left\{ \begin{array}{l} +\phi^i\left((1-\phi^{-i})\delta v_i^f + \phi^{-i}\left(\underline{\mu}\left((1-\delta)\pi + \delta v_i^f \right) + (1-\underline{\mu})\delta v_i^{-f} \right) \right) \\ +(1-\phi^i)\left(\overline{\mu}\left((1-\delta)\pi + \delta v_i^f \right) + (1-\overline{\mu})\delta v_i^{-f} \right) \end{array} \right. \end{array} \right)$$

and similarly

$$v_i^{-f} = \frac{1}{2} \left(\begin{array}{ll} \omega{=}A & \left\{ \begin{array}{l} \phi^{-i}\left(\overline{\mu}\delta v_i^{-f} + (1-\overline{\mu})\delta v_i^{-f} \right) \\ +(1-\phi^{-i})\left(\phi^i\left(\overline{\mu}(1-\delta)\pi + \delta v_i^{-f} \right) + (1-\phi^i)\left(\underline{\mu}\delta v_i^{-f} + (1-\underline{\mu})\delta v_i^f \right) \right) \end{array} \right. \\ \omega{=}B & \left\{ \begin{array}{l} +(1-\phi^{-i})\left(\overline{\mu}\delta v_i^{-f} + (1-\overline{\mu})\delta v_i^f \right) \\ +\phi^{-i}\left((1-\phi^i)\left(\overline{\mu}(1-\delta)\pi + \delta v_i^{-f} \right) + \phi^i\left(\underline{\mu}\delta v_i^{-f} + (1-\underline{\mu})\delta v_i^f \right) \right) \end{array} \right. \end{array} \right)$$

Subtracting and rearranging yields

$$v_i^f = v_i^{-f} + \frac{(1-\delta)\pi(\overline{\mu} + \underline{\mu})\phi(1-\phi)}{1 - \delta\left(\overline{\mu} - 2(1-\underline{\mu})\phi(1-\phi) \right)}. \tag{26}$$

Combining (26) with (5) and rearranging, the left inequality in (5) is equivalent to

$$\phi \leq \phi^* = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\left(\frac{\underline{\mu}\,(1-\delta\overline{\mu})}{\delta\left(\overline{\mu} - \underline{\mu} \right)\left(1 - \underline{\mu} \right)} \right)},$$

whereas the right inequality in (5) holds for all $\phi \in [\frac{1}{2}, 1)$, which completes the proof. *Q.E.D.*

Proof of Proposition 4. *Part 1.* Follows immediately from the proof of Proposition 5. *Part 2.* First, observe that the efficiency in project allocation requires that for some agent $f \in \{1,2\}$ (without loss of generality, the favored agent), either (i) XIC holds for both agents, $(m_i, m_{-i}) = (1,0)$ implies i gets the project, and $m \in \{(1,1),(0,0)\}$ implies f gets the project; or (ii) XIC holds for the favored agent f (but not necessarily for the other agent, $-f$), $m_f = 1$ implies f gets the project, and $m_f = 0$ implies $-f$ gets the project. Note that maximal-priority corresponds to a special case of case (i). We now consider each of these two cases, and show that the desired properties in the proposition are necessary in order to attain efficiency whenever possible.[51]

Case 1. Consider the XIC constraints for the two agents, (abusing notation and) denoting by v^f and v^{-f} the expected average continuation payoff of an agent when he is and is not favored, respectively.

Consider first the XIC constraints for the favored agent, given that he is not suited for the current project and believes the other agent, $-f$, is suited for it:

$$\begin{aligned}&\overline{\mu}\left(\psi_{-f}(-f,S)\delta v^{-f} + \psi_f(-f,S)\delta v^f\right) + (1-\overline{\mu})\left(\psi_{-f}(-f,F)\delta v^{-f} + \psi_f(-f,F)\delta v^f\right)\\ &\geq \underline{\mu}(1-\delta)\pi + \underline{\mu}\left(\psi_{-f}(f,S)\delta v^{-f} + \psi_f(f,S)\delta v^f\right) + (1-\underline{\mu})\left(\psi_{-f}(f,F)\delta v^{-f} + \psi_f(f,F)\delta v^f\right).\end{aligned}$$

Rearranging, we have[52]

$$v^f - v^{-f} \geq \frac{\underline{\mu}(1-\delta)\pi}{\delta\left(\overline{\mu}\psi_f(-f,S) + (1-\overline{\mu})\psi_f(-f,F) - \underline{\mu}\psi_f(f,S) - (1-\underline{\mu})\psi_f(f,F)\right)}. \tag{27}$$

Next, consider the XIC of the non-favored agent, $-f$, when he is not suited for the current project and believes the favored agent is also not suited for it:

$$\begin{aligned}&\underline{\mu}\left(\psi_{-f}(f,S)\delta v^f + \psi_f(f,S)\delta v^{-f}\right) + (1-\underline{\mu})\left(\psi_{-f}(f,F)\delta v^f + \psi_f(f,F)\delta v^{-f}\right)\\ &\geq \underline{\mu}(1-\delta)\pi + \underline{\mu}\left(\psi_{-f}(-f,S)\delta v^f + \psi_f(-f,S)\delta v^{-f}\right) + (1-\underline{\mu})\left(\psi_{-f}(-f,F)\delta v^f + \psi_f(-f,F)\delta v^{-f}\right).\end{aligned}$$

Rearranging, we have

$$v^f - v^{-f} \geq \frac{\underline{\mu}(1-\delta)\pi}{\delta\left(\underline{\mu}\psi_f(-f,S) + (1-\underline{\mu})\psi_f(-f,F) - \underline{\mu}\psi_f(f,S) - (1-\underline{\mu})\psi_f(f,F)\right)}. \tag{28}$$

Dynamic consistency of v^f and v^{-f} in equilibrium, together with the properties of the allocation rule $\mathcal{X}$ for case (i), imply the following conditions:

$$\begin{aligned}v^f &= (1-\delta)\pi\frac{1}{2}\left(\overline{\mu} + 2\phi(1-\phi)\underline{\mu}\right) + \delta v^{-f}\\ &+ \delta(v^f - v^{-f})\frac{1}{2}\left(\begin{array}{l}\psi_f(-f,S)\left(1-2\phi(1-\phi)\right)\overline{\mu} + \psi_f(-f,F)\left(1-2\phi(1-\phi)\right)(1-\overline{\mu})\\ +\psi_f(f,S)\left(\overline{\mu} + 2\phi(1-\phi)\underline{\mu}\right) + \psi_f(f,F)\left((1-\overline{\mu}) + 2\phi(1-\phi)(1-\underline{\mu})\right)\end{array}\right)\end{aligned}$$

[51] Throughout the proof below, project types ω are omitted; that is, it is assumed that the same case applies for both types of projects. It is easy to verify that this is without loss.

[52] Below we show that in equilibrium $v^f - v^{-f} > 0$. XIC then implies that the denominator in (27) is also strictly positive.

and

$$v^{-f} = (1-\delta)\pi\frac{1}{2}\left(1-2\phi(1-\phi)\right)\overline{\mu} + \delta v^{-f}$$
$$+\,\delta(v^f - v^{-f})\frac{1}{2}\left(\begin{array}{c} \psi_{-f}(-f,S)\left(1-2\phi(1-\phi)\right)\overline{\mu} + \psi_{-f}(-f,F)\left(1-2\phi(1-\phi)\right)(1-\overline{\mu}) \\ +\psi_{-f}(f,S)\left(\overline{\mu}+2(1-\phi)\phi\underline{\mu}\right) + \psi_{-f}(f,F)\left((1-\overline{\mu})+2(1-\phi)\phi(1-\underline{\mu})\right) \end{array}\right)$$

Using $\psi_f(x,y) = 1 - \psi_{-f}(x,y)$, subtracting and rearranging, we have

$$v^f - v^{-f} = \frac{(1-\delta)\pi\phi(1-\phi)\left(\underline{\mu}+\overline{\mu}\right)}{1-\delta\left(\begin{array}{c} -1+\psi_f(-f,S)\left(1-2\phi(1-\phi)\right)\overline{\mu} + \psi_f(-f,F)\left(1-2\phi(1-\phi)\right)(1-\overline{\mu}) \\ +\psi_f(f,S)\left(\overline{\mu}+2\phi(1-\phi)\underline{\mu}\right) + \psi_f(f,F)\left((1-\overline{\mu})+2\phi(1-\phi)(1-\underline{\mu})\right) \end{array}\right)} \tag{29}$$

Note that the denominator in (29) is strictly positive, hence $v^f - v^{-f} > 0$.

Let $\kappa := \phi^*(1-\phi^*)\left(\overline{\mu}-\underline{\mu}\right)\left(1-\underline{\mu}\right) + \underline{\mu}\overline{\mu}$. From Proposition 1, if the MPR is to be efficient whenever possible, it must be the case that the difference in (29) satisfies (27)-(28) for

$$\delta^* = \frac{\underline{\mu}}{\kappa}.$$

After some algebra, it can be shown that this is the case if and only if the following conditions are satisfied

$$\left(\begin{array}{c} \psi_f(-f,S) \\ \psi_f(-f,F) \\ \psi_f(f,S) \\ \psi_f(f,F) \end{array}\right)^{\mathrm{T}} \left(\begin{array}{c} \underline{\mu}\overline{\mu} + \phi^*(1-\phi^*)\overline{\mu}\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}(1-\overline{\mu}) + \phi^*(1-\phi^*)(1-\overline{\mu})\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}\overline{\mu} - \phi^*(1-\phi^*)\underline{\mu}\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}(1-\overline{\mu}) - \phi^*(1-\phi^*)(1-\underline{\mu})\left(\overline{\mu}-\underline{\mu}\right) \end{array}\right) \geq \kappa + \underline{\mu}$$
$$\left(\begin{array}{c} \psi_f(-f,S) \\ \psi_f(-f,F) \\ \psi_f(f,S) \\ \psi_f(f,F) \end{array}\right)^{\mathrm{T}} \left(\begin{array}{c} \underline{\mu}\overline{\mu} - \phi^*(1-\phi^*)\underline{\mu}\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}(1-\overline{\mu}) + \phi^*(1-\phi^*)\left(1+\underline{\mu}\right)\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}\overline{\mu} - \phi^*(1-\phi^*)\underline{\mu}\left(\overline{\mu}-\underline{\mu}\right) \\ \underline{\mu}(1-\overline{\mu}) - \phi^*(1-\phi^*)(1-\underline{\mu})\left(\overline{\mu}-\underline{\mu}\right) \end{array}\right) \geq \kappa + \underline{\mu}. \tag{30}$$

Note that maximal-priority, for which

$$\psi_f(x,y) = \mathbf{1}_{\{(x,y)\neq(f,F)\}},$$

satisfies both of these conditions with equality. It is easy to check that each of the entries multiplying $\psi_f(-f,S), \psi_f(-f,F), \psi_f(f,S)$, are positive, and that $\underline{\mu}(1-\overline{\mu}) - \phi^*(1-\phi^*)(1-\underline{\mu})\left(\overline{\mu}-\underline{\mu}\right) < 0$ as

$$\phi^*(1-\phi^*) = \frac{\underline{\mu}\left(1-\delta^*\overline{\mu}\right)}{\delta^*\left(1-\underline{\mu}\right)\left(\overline{\mu}-\underline{\mu}\right)} > \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})\left(\overline{\mu}-\underline{\mu}\right)}.$$

Hence, the conditions (30) are only satisfied when $\psi_f(x,y) = \mathbf{1}_{\{(x,y)\neq(f,F)\}}$.

Case 2. The XIC condition for the favored agent, when he is not suited for the current project and believes the other agent, $-f$, is suited for it, is

$$\begin{aligned}&\overline{\mu}\left(\psi_{-f}(-f,S)\delta v^{-f}+\psi_f(-f,S)\delta v^f\right)+(1-\overline{\mu})\left(\psi_{-f}(-f,F)\delta v^{-f}+\psi_f(-f,F)\delta v^f\right)\\ &\geq \underline{\mu}(1-\delta)\pi+\underline{\mu}\left(\psi_{-f}(f,S)\delta v^{-f}+\psi_f(f,S)\delta v^f\right)+(1-\underline{\mu})\left(\psi_{-f}(f,F)\delta v^{-f}+\psi_f(f,F)\delta v^f\right),\end{aligned}$$

which gives

$$v^f - v^{-f} \geq \frac{\underline{\mu}(1-\delta)\pi}{\delta\left(\overline{\mu}\psi_f(-f,S)+(1-\overline{\mu})\psi_f(-f,F)-\underline{\mu}\psi_f(f,S)-(1-\underline{\mu})\psi_f(f,F)\right)}. \tag{31}$$

Following arguments similar to those in case (i), we obtain

$$v^f - v^{-f} = \frac{(1-\delta)\pi\phi(1-\phi)\left(\overline{\mu}-\underline{\mu}\right)}{1-\delta\left(\begin{array}{c}-1+\psi_f(-f,S)\left(\overline{\mu}-2\phi(1-\phi)\left(\overline{\mu}-\underline{\mu}\right)\right)+\psi_f(f,S)\overline{\mu}\\ +\psi_f(-f,F)\left((1-\overline{\mu})+2\phi(1-\phi)\left(\overline{\mu}-\underline{\mu}\right)\right)+\psi_f(f,F)(1-\overline{\mu})\end{array}\right)}. \tag{32}$$

As in case (i), it can now be shown that the difference in (32) satisfies (31) for $\delta^* = \frac{\mu}{\kappa}$ if and only if

$$\left(\begin{array}{c}\psi_f(-f,S)\\ \psi_f(-f,F)\\ \psi_f(f,S)\\ \psi_f(f,F)\end{array}\right)^{\mathrm{T}}\left(\begin{array}{c}\overline{\mu}\underline{\mu}+\phi^*(1-\phi^*)\left(\overline{\mu}-\underline{\mu}\right)\left(\overline{\mu}-2\underline{\mu}\right)\\ \underline{\mu}(1-\overline{\mu})+\phi^*(1-\phi^*)\left(\overline{\mu}-\underline{\mu}\right)\left(1-\overline{\mu}+2\underline{\mu}\right)\\ \overline{\mu}\underline{\mu}-\underline{\mu}\phi^*(1-\phi^*)\left(\overline{\mu}-\underline{\mu}\right)\\ (1-\overline{\mu})\underline{\mu}-(1-\underline{\mu})\phi^*(1-\phi^*)\left(\overline{\mu}-\underline{\mu}\right)\end{array}\right)\geq \kappa+\underline{\mu}.$$

The above condition is satisfied with equality when $\psi_f(x,y) = \mathbf{1}_{\{(x,y)\neq(f,F)\}}$. (It can also be verified that with such ψ, the remaining XIC for the favored agent is also satisfied.) Furthermore, it can easily be verified that the entries multiplying $\psi_f(-f,S), \psi_f(-f,F), \psi_f(f,S)$ are positive. The fact that $\underline{\mu}(1-\overline{\mu})-\phi^*(1-\phi^*)(1-\underline{\mu})\left(\overline{\mu}-\underline{\mu}\right)<0$ then implies that the condition above can only be satisfied when $\psi_f(x,y) = \mathbf{1}_{\{(x,y)\neq(f,F)\}}$, which concludes the proof. *Q.E.D.*

Proof of Corollary 1. Follows immediately from Proposition 5. *Q.E.D.*

D Proofs for Section 5 - Performance-based equilibria

Proof of *part 1* of Proposition 6. In a performance-based equilibrium, the set of promised continuation values can depend on (*i*) the project type, (*ii*) the identity of the agent who receives the project, and (*iii*) whether that agent was successful. Promised continuation values are therefore given by $\mathcal{V} := (\mathcal{V}_0^\omega, \mathcal{V}_1^\omega, \mathcal{V}_2^\omega)_{\omega\in\{A,B\}}$, where each function

$$\mathcal{V}_i^\omega : \{1,2\}\times\{S,F\}\to\mathbb{R}_+^3 \tag{33}$$

specifies the promised continuation value of player $i \in \{0,1,2\}$ given that the current project is of type $\omega \in \{A, B\}$, agent $j \in \{0,1,2\}$ is assigned the project and its outcome is $y \in \{0, S, F\}$.

With this modified definition of $\mathcal{V}$, the agents' and principal's interim and ex-ante payoffs can be defined analogously to the definitions of Section A.[53] Moreover, The principal's incentive constraint remains the same as in Section A. However, since we are no longer considering XPPE, the relevant incentive compatibility requirement for the agents is the following. The policy $\mathcal{Z} = (\mathcal{M}, \chi, \mathcal{V})$ is *incentive compatible* (IC) if for all $\omega \in \{A, B\}$, $i = 1, 2$, $\theta_i \in \{\alpha, \beta\}$ and $m_i \in \{0, 1\}$,

$$U_i^{\omega}(\mathcal{M}_i^{\omega}(\theta_i), \theta_i; \mathcal{M}_{-i}, \chi, \mathcal{V}) \geq U_i^{\omega}(m_i, \theta_i; \mathcal{M}_{-i}, \chi, \mathcal{V}). \tag{34}$$

The relevant notions of decomposability and E-decomposability can now be defined analogously to the ones in Section A, with the above modifications of the promised continuation values and incentive compatibility. The analogs of Lemmas 1-4 can be obtained using similar arguments. If $\mathcal{E}_P^* \neq 0$ then there exist minimal and maximal values that can be supported under an efficient performance-based equilibrium:

$$\underline{v}_P := \min\{v \in \mathbb{R}_+ : (v^*, v, v_2) \in \mathcal{E}_P^*\}$$

and $\overline{v}_P := \pi v^* - \underline{v}_P$. We can then define $\tilde{\Psi}(\cdot)$ analogously to Ψ (i.e., with respect to the new definition of E-decomposability).

As in the proof of Proposition 1, we can restrict attention to policies in which the agents are truthful. For such policies, the expected payoff of agent 1 is given by

$$\Lambda_1(\mathcal{M}, \chi, \mathcal{V}) = \frac{1}{2}\Lambda_1^A(\mathcal{M}, \chi, \mathcal{V}) + \frac{1}{2}\Lambda_1^B(\mathcal{M}, \chi, \mathcal{V}),$$

where 1's expected payoff for $\omega = A$ is

$$\begin{aligned}
\Lambda_1^A(\mathcal{M}, \chi, \mathcal{V}) := &(1-\phi^1)(1-\phi^2)\left(\chi_1^A(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^A(1)\right) + \left(1-\chi_1^A(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^A(2)\right) \\
&+ \phi^1(1-\phi^2)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A(1)\right) + (1-\phi^1)\phi^2\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A(2) \\
&+ \phi^1\phi^2\left(\chi_1^A(1,1)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A(1)\right) + \left(1-\chi_1^A(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^A(2)\right).
\end{aligned} \tag{35}$$

and similarly for $\omega = B$,

$$\begin{aligned}
\Lambda_1^B(\mathcal{M}, \chi, \mathcal{V}) := &\phi^1\phi^2\left(\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(1)\right) + \left(1-\chi_1^B(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right) \\
&+ (1-\phi^1)\phi^2\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(1)\right) + \phi^1(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) \\
&+ (1-\phi^1)(1-\phi^2)\left(\chi_1^B(1,1)\left((1-\delta)\pi\overline{\mu} + \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(1)\right) + \left(1-\chi_1^B(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2)\right),
\end{aligned} \tag{36}$$

where for any $l \in \{\underline{\mu}, \overline{\mu}\}, i, j \in \{1, 2\}, \omega \in \{A, B\}$,

$$\mathbb{E}_l\mathcal{V}_i^{\omega}(m, j) := l\mathcal{V}_i^{\omega}(j, S) + (1-l)\mathcal{V}_i^{\omega}(j, F).$$

[53]For convenience, the notation from Section A is maintained throughout this this proof.

Given $\omega = B$, the interim payoff of agent 1 of type $\theta_1 = \alpha$ must be (weakly) greater when he announces $m_1 = 0$ rather than $m_1 = 1$, that is,

$$\begin{aligned}
U_1^B(0,\alpha) &= (1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) + \phi^2\left(\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(1)\right) + \left(1-\chi_1^B(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right),\\
&\geq (1-\phi^2)\left(\chi_1^B(1,1)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(1)\right) + \left(1-\chi_1^B(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2)\right)\\
&+ \phi^2\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(1)\right) \qquad \text{(ICA)}\\
&= U_1^B(1,\alpha).
\end{aligned}$$

An analogous condition, $U_2^B(0,\alpha) \geq U_2^B(1,\alpha)$ apply for agent 2. Imposing $\mathcal{V}_2^\omega(j,y) = \pi v^* - \mathcal{V}_1^\omega(j,y)$ and $\chi_2^\omega(m) = 1 - \chi_1^\omega(m)$, this condition can be written as

$$\begin{aligned}
U_2^B(0,\alpha) &= -(1-\phi^1)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(1) + \phi^1\left(\left(1-\chi_1^B(0,0)\right)\left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right) - \chi_1^B(0,0)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(1)\right)\\
&\geq (1-\phi^1)\left(\left(1-\chi_1^B(1,1)\right)\left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right) - \chi_1^B(1,1)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(1)\right)\\
&+ \phi^1\left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right) \qquad \text{(ICB)}\\
&= U_2^B(1,\alpha).
\end{aligned}$$

Analogous constraints can be derived for the case of $\omega = A$ and $\theta_1 = \beta$,

$$U_i^A(0,\beta) \geq U_i^A(1,\beta) \quad i = 1,2, \qquad \text{(ICC)}$$

and the remaining IC constraints guarantee that agents find it optimal to truthfully announce that they are suited for the project:

$$U_i^\omega(1,\theta_i) \geq U_i^\omega(0,\theta_i) \quad (\omega,\theta_i) \in \{(A,\alpha),(B,\beta)\}, i = 1,2. \qquad \text{(ICD)}$$

Assume $\mathcal{E}_P^* \neq \emptyset$. For any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$,

$$\begin{aligned}
\tilde{\Psi}(v) = &\min_{(\chi,\mathcal{V})} \Lambda_1\\
&\text{s.t.} \quad \text{(ICA)-(ICD)},\\
&\qquad \chi_i^\omega(m) \in \{0,1\},\ \chi_1^\omega(m) + \chi_2^\omega(m) = 1, (m_l, m_{-l}) = (1,0) \Rightarrow \chi_i^\omega(m) = 1,\\
&\qquad \mathcal{V}_i^\omega(j,y) \in [v, \pi v^* - v], \mathcal{V}_1^\omega(j,y) + \mathcal{V}_2^\omega(j,y) = \pi v^*,\\
&\qquad \forall\omega \in \{A,B\}, m \in \{0,1\}^2, l,j \in \{1,2\}, y \in \{S,F\}. \qquad (37)
\end{aligned}$$

Denote for any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$ the program on the RHS of (37) by $\mathcal{P}_{P,v}$ and by $\mathcal{P}_{P,v}^R$ the same program excluding the constraints (ICD). Fixing any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$, the following lemmas derive necessary properties that any $(\chi, \mathcal{V})$ minimizing Λ_1 must satisfy.

Lemma 11 *Suppose $\mathcal{E}_P^* \neq \emptyset$. For any $(\chi, \mathcal{V})$ that yields a solution to $\mathcal{P}_{P,v}^R$, $(1-\delta)\pi\underline{\mu} + \delta v \leq \delta\pi v^* - \delta v$, $\omega \in \{A, B\}$.*

PROOF. First note that

$$\begin{aligned}
&\left((1-\delta)\pi\underline{\mu}+\delta v\right)\left(\phi^1\chi_1^\omega(0,0)+(1-\phi^1)\left(1-\chi_1^\omega(1,1)\right)\right)\\
&\leq(1-\delta)\pi\underline{\mu}\left(\phi^1\chi_1^\omega(0,0)+(1-\phi^1)\left(1-\chi_1^\omega(1,1)\right)\right)\\
&+\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega(1)\left(\phi^1\chi_1^\omega(0,0)\right)+\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega(1)\left((1-\phi^1)\left(1-\chi_1^\omega(1,1)\right)\right)\\
&\leq\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega(2)\left(\phi^1\left(\chi_1^\omega(0,0)\right)+(1-\phi^1)\left(1-\chi_1^\omega(1,1)\right)\right)\\
&\leq\delta(\pi v^*-v)\left(\phi^1\left(\chi_1^\omega(0,0)\right)+(1-\phi^1)\left(1-\chi_1^\omega(1,1)\right)\right),
\end{aligned}$$

where the first and third inequalities follow from the fact that $v\leq\mathcal{V}_1^\omega(j,y)\leq\pi v^*-v$ and the second from (ICB). If it is not the case that both $\chi_1^\omega(1,1)=1$ and $\chi_1^\omega(0,0)=0$ then clearly $(1-\delta)\pi\underline{\mu}+\delta v\leq\delta(\pi v^*-v)$. Hence suppose $\chi_1^\omega(1,1)=1$ and $\chi_1^\omega(0,0)=0$.

$$(1-\delta)\pi\underline{\mu}+\delta v\leq(1-\delta)\pi\underline{\mu}+\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega(1)\leq\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega(2)\phi^2+\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^\omega(2)(1-\phi^2)\leq\delta(\pi v^*-v),$$

where the first and third inequalities again follow from $v\leq\mathcal{V}_1^\omega(j,y)\leq\pi v^*-v$ and the second from (ICA). ■

Lemma 12 *Suppose $\mathcal{E}_P^*\neq\emptyset$. Any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_{P,v}^R$ satisfies $\mathcal{V}_1^\omega(1,y)=v$, all $\omega\in\{A,B\}$, $y\in\{S,F\}$.*

PROOF. Suppose $\mathcal{V}_1^B(1,S)>v$. A slight decrease in $\mathcal{V}_1^A(1,S)$ both reduces Λ_1 and relaxes the constraints (ICA) and (ICB), without affecting the remaining constraints; a contradiction. Similarly, $\mathcal{V}_1^B(1,F)=\mathcal{V}_1^A(1,S)=\mathcal{V}_1^A(1,F)=v$. ■

Lemma 13 *Suppose $\mathcal{E}_P^*\neq\emptyset$. For any $(\chi,\mathcal{V})$ that yields a solution to $\mathcal{P}_{P,v}^R$, $\chi_1^\omega(1,1)=0$, $\omega\in\{A,B\}$.*

PROOF. Consider a policy with $(\chi,\mathcal{V})$ such that $\chi_1^\omega(1,1)=\chi_1^\omega(0,0)=0$, and furthermore $\delta\mathcal{V}_1^\omega(2,y)=(1-\delta)\pi\underline{\mu}+\delta v$, and $\mathcal{V}_1^\omega(1,y)=v$, for all $y=S,F$. From Lemma 11, such a policy is feasible; furthermore, (ICA)-(ICC) are clearly satisfied and

$$\Lambda_1^B=\delta v+(1-\delta)\pi\left(\left(\phi+\phi(1-\phi)\right)\underline{\mu}+(1-\phi)^2\overline{\mu}\right).$$

Suppose $\chi_1^B(1,1)=1$. It must be that $\mathcal{V}_1^B(2,y)>v$ for some $y\in\{S,F\}$.[54] Hence either (ICA) or (ICB) must hold with equality, as otherwise Λ_1^B could be decreased without violating any of the constraints by slightly lowering $\mathcal{V}_1^B(2,y)$ for some $y\in\{S,F\}$. Suppose first that (ICA) holds with equality and (ICB) does not. The latter implies

$$-\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu}+\delta v-\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right)>0,$$

[54]Recall that by Lemma 12, $\mathcal{V}_1^B(1,y)=v$, $y=S,F$.

which gives $\chi_1^B(0,0) = 1$. Hence from (ICA), $\delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B(2) = (1-\delta)\pi\underline{\mu} + \delta v$. We then have

$$\begin{aligned}
\Lambda_1^B &= \phi^1\phi^2 \left((1-\delta)\pi\underline{\mu} + \delta v\right) + (1-\phi^1)\left((1-\delta)\pi\overline{\mu} + \delta v\right) + \phi^1(1-\phi^2)\delta\left((1-\delta)\pi\underline{\mu} + \delta v\right) \\
&= \delta v + (1-\delta)\pi\left(\phi\underline{\mu} + (1-\phi)\overline{\mu}\right) \\
&> \delta v + (1-\delta)\pi\left(\left(\phi + \phi(1-\phi)\right)\underline{\mu} + (1-\phi)^2\overline{\mu}\right).
\end{aligned}$$

Therefore, a policy such as the one considered above yields strictly lower Λ_1^B. Applying an analogous argument for $\omega = A$ gives the contradiction. Next suppose (ICA) does not hold with equality but (ICB) does. Then the latter implies

$$\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta v - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2)\right) = 0.$$

If $\chi_1^B(0,0) = 0$ then (ICB) holds regardless of $\mathcal{V}_1^B(2,y)$. Since (ICA) holds with strict inequality, $\mathcal{V}_1^B(2,y)$ for some $y \in \{S,F\}$ can be decreased by a small amount, decreasing Λ_1^B without violating any of the constraints; a contradiction. Hence suppose $\chi_1^B(0,0) = 1$. Then, from (ICA), $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) > (1-\delta)\pi\underline{\mu} + \delta v$. Hence

$$\begin{aligned}
\Lambda_1^B &= \phi^1\phi^2 \left((1-\delta)\pi\underline{\mu} + \delta v\right) + (1-\phi^1)\left((1-\delta)\pi\overline{\mu} + \delta v\right) + \phi^1(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) \\
&> \phi^1\phi^2 \left((1-\delta)\pi\underline{\mu} + \delta v\right) + (1-\phi^1)\left((1-\delta)\pi\overline{\mu} + \delta v\right) + \phi^1(1-\phi^2)\left((1-\delta)\pi\underline{\mu} + \delta v\right) \\
&= \delta v + (1-\delta)\pi\left(\phi\underline{\mu} + (1-\phi)\overline{\mu}\right) \\
&> \delta v + (1-\delta)\pi\left(\left(\phi + \phi(1-\phi)\right)\underline{\mu} + (1-\phi)^2\overline{\mu}\right),
\end{aligned}$$

which as in the previous case gives the contradiction. Finally, suppose both constraints (ICA) and (ICB) hold in equality. If $\chi_1^B(0,0) = 1$ then $\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) = (1-\delta)\pi\underline{\mu} + \delta v$ and hence

$$\Lambda_1^B = \delta v + (1-\delta)\pi\left(\phi\underline{\mu} + (1-\phi)\overline{\mu}\right) > \delta v + (1-\delta)\pi\left(\left(\phi + \phi(1-\phi)\right)\underline{\mu} + (1-\phi)^2\overline{\mu}\right),$$

which again gives the contradiction. If $\chi_1^B(0,0) = 0$ then (ICA) gives

$$(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) + \phi^2\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) = (1-\delta)\pi\underline{\mu} + \delta v,$$

hence

$$\begin{aligned}
\Lambda_1^B &= \phi^1\phi^2\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) + (1-\phi^1)\left((1-\delta)\pi\overline{\mu} + \delta v\right) + \phi^1(1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B(2) \\
&= \delta v + (1-\delta)\pi\left(\phi\underline{\mu} + (1-\phi)\overline{\mu}\right) \\
&> \delta v + (1-\delta)\pi\left(\left(\phi + \phi(1-\phi)\right)\underline{\mu} + (1-\phi)^2\overline{\mu}\right),
\end{aligned}$$

giving the contradiction.

Therefore, it must be that $\chi_1^B(1,1) = 0$. Analogous arguments imply $\chi_1^A(1,1) = 0$ as well. ■

Lemma 14 *Suppose $\mathcal{E}_P^* \neq \emptyset$. For any $(\chi, \mathcal{V})$ that yields a solution to $\mathcal{P}_{P,v}^R$, $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^\omega(2) = (1-\delta)\pi\underline{\mu} + \delta v$, $\omega = A, B$.*

PROOF. We consider $\omega = B$, analogous arguments apply for $\omega = A$. From the previous lemma, (ICA) and (ICB) are equivalent to

$$\begin{aligned}\left(1-\chi_1^B(0,0)\right)\left(\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) - (1-\delta)\pi\underline{\mu} - \delta v\right) &\geq 0\\ \left((1-\phi^1)+\phi^1\chi_1^B(0,0)\right)\left(\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) - (1-\delta)\pi\underline{\mu} - \delta v\right) &\geq 0,\end{aligned}$$

respectively. Hence $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) \geq (1-\delta)\pi\underline{\mu} + \delta v$. Suppose $\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B(2) > (1-\delta)\pi\underline{\mu} + \delta v$. Then (ICB) does not bind, which means (ICB) must.[55] This can only be the case if $\chi_1^B(0,0) = 1$. However, if $\chi_1^B(0,0) = 1$ then (ICA) holds regardless of $\mathcal{V}_1^B(2,y)$, and given that (ICB) does not bind at least one of $\mathcal{V}_1^B(2,y)$, $y \in \{S,F\}$, can be reduced slightly, reducing Λ_1^B without violating any of the constraints, giving the contradiction. ■

Arguments analogous to those in Lemma 9 now imply the following.

Lemma 15 *Suppose $\mathcal{E}^* \neq \emptyset$. For any $(\chi, \mathcal{V})$ that yields a solution to $\mathcal{P}_v^R$, one of the following holds*

- $\delta\mathcal{V}_1^\omega(2,S) = \delta v$ *and* $\delta\mathcal{V}_1^\omega(2,F) = \delta v + \frac{(1-\delta)\underline{\mu}\pi}{1-\underline{\mu}} \leq \delta(\pi v^* - v)$; *or*
- $\delta\mathcal{V}_1^\omega(2,F) = \delta(\pi v^* - v)$ *and* $\delta\mathcal{V}_1^\omega(2,S) = (1-\delta)\pi + \frac{1}{\underline{\mu}}\delta v - \frac{1-\underline{\mu}}{\underline{\mu}}\delta(\pi v^* - v) \geq \delta v$.

Given the results in Lemmas 12-15, a comparison with the analysis in the proof of Proposition 1 yields that for any v the solutions to the programs (17) and (37) are identical, and hence Ψ and $\tilde{\Psi}(\cdot)$ coincide. Since the sets $\mathcal{E}^*$ and $\mathcal{E}_P^*$ are entirely characterized through fixed-points of Ψ and $\tilde{\Psi}(\cdot)$, respectively, the sets $\mathcal{E}^*$ and $\mathcal{E}_P^*$ coincide given any set of primitives $\{\phi, \delta, \overline{\mu}, \underline{\mu}, \pi\}$. *Q.E.D.*

Proof of *part 2* of Proposition 6. Without the ex-post (or the performance-based) restriction, the IC constraints imposed on a policy $\mathcal{Z}$ are the following. First, given $\omega = B$, the interim payoff of agent 1 of type $\theta_1 = \alpha$ must be greater when he announces $m_1 = 0$ rather than $m_1 = 1$, that is,

$$\begin{aligned}U_1^B(0,\alpha) &= (1-\phi^2)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2)\\ &\quad + \phi^2\left(\chi_1^B(0,0)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),1)\right) + \left(1-\chi_1^B(0,0)\right)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2)\right)\\ &\geq (1-\phi^2)\left(\chi_1^B(1,1)\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),1)\right) + \left(1-\chi_1^B(1,1)\right)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2)\right)\\ &\quad + \phi^2\left((1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,0),1)\right) \qquad \text{(ICi)}\\ &= U_1^B(1,\alpha).\end{aligned}$$

[55] Otherwise some $\mathcal{V}_1^B(2,y)$ can be decreased slightly, reducing Λ_1^B (and hence Λ_1) without violating any of the constraints.

An analogous condition, $U_2^B(0,\alpha) \geq U_2^B(1,\alpha)$ applies for agent 2. Imposing $\mathcal{V}_2^{\omega}(j,y) = \pi v^* - \mathcal{V}_1^{\omega}(j,y)$ and $\chi_2^{\omega}(m) = 1 - \chi_1^{\omega}(m)$, this condition can be written as

$$\begin{aligned}
U_2^B(0,\alpha) &= -(1-\phi^1)\delta \mathbb{E}_{\overline{\mu}} \mathcal{V}_1^B((1,0),1) \\
&\quad + \phi^1 \left(\left(1 - \chi_1^B(0,0)\right) \left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2)\right) - \chi_1^B(0,0)\delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),1)\right) \\
&\geq (1-\phi^1)\left(\left(1 - \chi_1^B(1,1)\right) \left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((1,1),2)\right) - \chi_1^B(1,1)\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),1)\right) \\
&\quad + \phi^1 \left((1-\delta)\pi\underline{\mu} - \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,1),2)\right) \qquad \text{(ICii)} \\
&= U_2^B(1,\alpha).
\end{aligned}$$

Similar conditions apply for $\omega = A$, and the remaining IC conditions guarantee that agents find it optimal to announce truthfully when they are suited for the project:

$$U_i^{\omega}(1,\theta_i) \geq U_i^{\omega}(0,\theta_i) \quad \forall(\omega,\theta_i) \in \{(A,\alpha),(B,\beta)\}, i = 1,2.$$

Define decomposability, E-decomposability and self-generation analogously to the definitions in Section A, replacing IC with XIC, and denote the set of efficient PPE values by $\mathcal{E}^{\dagger}$ (clearly, $\mathcal{E}^* \subseteq \mathcal{E}^{\dagger}$). Results analogous to those in Lemmas 1 and Lemma 2 then apply.

Below, we construct a subset of $\mathcal{E}^{\dagger}$, which is self-generating outside of the region of primitives for which $\mathcal{E}^* \neq \emptyset$.

Consider a policy $\mathcal{Z}$ with: $\mathcal{M}_i^{\omega}(\theta_i) = 1 \Leftrightarrow (\omega,\theta_i) \in \{(A,\alpha),(B,\beta)\}$, $(m_i, m_{-i}) = (1,0) \Rightarrow \chi_i^{\omega}(m) = 1$, $\chi_1^{\omega}(0,0) = \chi_1^{\omega}(1,1) = 0$ and $v^{\dagger} := \mathcal{V}_1^{\omega}((1,0),1,y)$. Furthermore, let

$$\delta\mathcal{V}_1^{\omega}((0,0),2,y) = (1-\delta)\pi\underline{\mu} + \delta v^{\dagger} - \varepsilon,$$

$y = S, F$, for some $0 < \varepsilon < (1-\delta)\pi\underline{\mu}$, $\mathcal{V}_1^{\omega}((0,1),2,S) = \mathcal{V}_1^{\omega}((1,1),2,S) = v^{\dagger}$,

$$\delta\mathcal{V}_1^B((0,1),2,F) = \frac{(1-\delta)\pi\underline{\mu}}{(1-\underline{\mu})} + \delta v^{\dagger} - \varepsilon\left(\frac{\phi^2(1-\overline{\mu}) - (1-\phi)^2(1-\underline{\mu})}{\phi(1-\underline{\mu})(1-\overline{\mu})}\right)$$

and

$$\delta\mathcal{V}_1^B((1,1),2,F) = \frac{(1-\delta)\pi\underline{\mu}}{(1-\underline{\mu})} + \delta v^{\dagger} - \varepsilon\left(\frac{\phi(1-\overline{\mu}) + (1-\phi)(1-\underline{\mu})}{(1-\underline{\mu})(1-\overline{\mu})}\right).$$

One can verify that given $\mathcal{Z}$ above, (ICi) and (ICii) hold with equality (as do the corresponding IC's for $\omega = A$) and the remaining IC's also hold (but are slack).

We now consider when $(v^*, v^{\dagger}, \pi v^* - v^{\dagger})$ is E-decomposable on $\text{co}\{(v^*, v^{\dagger}, \pi v^* - v^{\dagger}), (v^*, \pi v^* - v^{\dagger}, v^{\dagger})\}$. For this to hold, it must be the case that both (i) $\Lambda_1 = v^{\dagger}$ and (ii) the promised continuations specified by $\mathcal{V}$ are elements of $\text{co}\{(v^*, v^{\dagger}, \pi v^* - v^{\dagger}), (v^*, \pi v^* - v^{\dagger}, v^{\dagger})\}$.

From (16), under $\mathcal{Z}$ above,

$$\begin{aligned}
\Lambda_1^B &= \phi(1-\phi)\left(\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((1,1),2) + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}_1^B((0,0),2)\right) + (1-\phi)^2\left((1-\delta)\pi\overline{\mu} + \delta v^{\dagger}\right) + \phi^2\delta\mathbb{E}_{\overline{\mu}}\mathcal{V}_1^B((0,1),2) \\
&= \delta v^{\dagger} + (1-\delta)\pi\left(\phi(1-\phi)\underline{\mu} + (1-\phi)^2\overline{\mu} + \phi\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right) - \varepsilon\left(\frac{\phi^2(1-\overline{\mu})}{(1-\underline{\mu})} + \phi(1-\phi)\right)
\end{aligned}$$

and similarly

$$\Lambda_1^A = \delta v^\dagger + (1-\delta)\pi\left(\phi(1-\phi)\underline{\mu} + \phi^2\overline{\mu} + (1-\phi)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right) - \varepsilon\left(\frac{(1-\phi)^2(1-\overline{\mu})}{(1-\underline{\mu})} + \phi(1-\phi)\right)$$

Solving $v^\dagger = \Lambda_1 = \frac{1}{2}\Lambda_1^A + \frac{1}{2}\Lambda_1^B$, a straightforward calculation yields[56]

$$v^\dagger = \underbrace{\frac{1}{2}\pi\left(\overline{\mu} - 2\phi(1-\phi)(\overline{\mu}-\underline{\mu}) + \frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right)}_{\underline{v}} - \frac{\varepsilon}{2(1-\underline{\mu})(1-\delta)}\Big((1-\overline{\mu}) + 2\phi(1-\phi)(\overline{\mu}-\underline{\mu})\Big). \tag{38}$$

With (38) in hand, it remains to check the conditions guaranteeing that the promised continuation values indeed fall within the set $\mathrm{co}\{(v^*, v^\dagger, \pi v^* - v^\dagger), (v^*, \pi v^* - v^\dagger, v^\dagger)\}$. In particular, it must be the case that $\delta\mathcal{V}_1^\omega((0,0),2,y), \delta\mathcal{V}_1^B((0,1),2,F), \delta\mathcal{V}_1^B((1,1),2,F) \in [\delta v^\dagger, \delta\pi v^* - \delta v^\dagger]$. Consider the following primitives: $(\delta, \overline{\mu}, \underline{\mu}, \pi) = (\frac{4}{5}, \frac{4}{5}, \frac{1}{5}, 1)$. The threshold level of specialization beyond which $\mathcal{E}^* = \emptyset$ is equal to $\phi^* = \frac{3}{4}$. However, taking $\varepsilon = \frac{1}{100} < \frac{4}{100} = (1-\delta)\pi\underline{\mu}$, the above feasibility conditions are satisfied for $\phi = \frac{4}{5} > \phi^*$. Hence, we have shown that, given $(\phi, \delta, \overline{\mu}, \underline{\mu}, \pi) = (\frac{4}{5}, \frac{4}{5}, \frac{4}{5}, \frac{1}{5}, 1)$, $(v^*, v^\dagger, \pi v^* - v^\dagger)$ is E-decomposable on $\mathrm{co}\{(v^*, v^\dagger, \pi v^* - v^\dagger), (v^*, \pi v^* - v^\dagger, v^\dagger)\}$. Analogous arguments show that $(v^*, \pi v^* - v^\dagger, v^\dagger)$ is also E-decomposable on $\mathrm{co}\{(v^*, v^\dagger, \pi v^* - v^\dagger), (v^*, \pi v^* - v^\dagger, v^\dagger)\}$. Using public randomization, the remainder of the set $\mathrm{co}\{(v^*, v^\dagger, \pi v^* - v^\dagger), (v^*, \pi v^* - v^\dagger, v^\dagger)\}$ is therefore E-decomposable on $\mathrm{co}\{(v^*, v^\dagger, \pi v^* - v^\dagger), (v^*, \pi v^* - v^\dagger, v^\dagger)\}$, and hence the latter is self-generating. In particular, we have shown that $\mathcal{E}^\dagger \neq \emptyset$ for primitives such that $\mathcal{E}^* = \emptyset$. *Q.E.D.*

E Proofs for Section 6 - Non-discriminatory equilibria

Proof of Proposition 7. A non-discriminatory XPPE, relative to a standard XPPE, is additionally constrained by the requirement that $\Lambda_1 = \Lambda_2$ and $\mathcal{V}_1^\omega(m,j,y) = \mathcal{V}_2^\omega(m,j,y)$, all (ω, m, j, y).

Suppose there exists a policy $(\mathcal{M}, \chi, \mathcal{V})$ that E-decomposes $(v^*, \frac{1}{2}\pi v^*, \frac{1}{2}\pi v^*)$ on $(v^*, \frac{1}{2}\pi v^*, \frac{1}{2}\pi v^*)$. Then, without loss of generality, one of the two conditions in Lemma 4 is satisfied. XIC then requires that agents do not have incentives to claim to be suited when they are not, for any belief they may hold about the type of the other agent. Specifically, given $\omega = B$, and imposing symmetry in the agents' promised continuation values,

$$\begin{aligned} U_1^B(0,\alpha;\beta) &= \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}^B((0,1),2) \\ &\geq (1-\delta)\pi\underline{\mu}\chi_1^B(1,1) + \delta\chi_1^B(1,1)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((1,1),1) + \delta\left(1-\chi_1^B(1,1)\right)\mathbb{E}_{\overline{\mu}}\mathcal{V}^B((1,1),2) \qquad \text{(ICI)} \\ &= U_1^B(1,\alpha;\beta), \end{aligned}$$

[56]Recall that $\underline{v}$ denotes that minimal equilibrium value in an XPPE: $\underline{v} = \min\{v \in \mathbb{R}_+ : (v^*, v, v_2) \in \mathcal{E}^*\}$.

$$\begin{aligned} U_1^B(0,\alpha;\alpha) &= (1-\delta)\pi\underline{\mu}\chi_1^B(0,0) + \delta\chi_1^B(0,0)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((0,0),1) + \delta\left(1-\chi_1^B(0,0)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((0,0),2) \\ &\geq (1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((1,0),1) \qquad \text{(ICII)} \\ &= U_1^B(1,\alpha;\alpha). \end{aligned}$$

and similarly for agent 2, imposing $\chi_2^\omega(m) = 1 - \chi_1^\omega(m)$,

$$\begin{aligned} U_2^B(0,\alpha;\beta) &= \delta\mathbb{E}_{\overline{\mu}}\mathcal{V}^B((1,0),1) \\ &\geq (1-\delta)\pi\underline{\mu}\left(1-\chi_1^B(1,1)\right) + \delta\left(1-\chi_1^B(1,1)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((1,1),2) + \delta\chi_1^B(1,1)\mathbb{E}_{\overline{\mu}}\mathcal{V}^B((1,1),1) \\ &= U_2^B(1,\alpha;\beta), \qquad \text{(ICIII)} \end{aligned}$$

$$\begin{aligned} U_2^B(0,\alpha;\alpha) &= (1-\delta)\pi\underline{\mu}\left(1-\chi_1^B(0,0)\right) + \delta\left(1-\chi_1^B(0,0)\right)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((0,0),2) + \delta\chi_1^B(0,0)\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((0,0),1) \\ &\geq (1-\delta)\pi\underline{\mu} + \delta\mathbb{E}_{\underline{\mu}}\mathcal{V}^B((0,1),2) \qquad \text{(ICIV)} \\ &= U_2^B(1,\alpha;\alpha). \end{aligned}$$

Analogous constraints hold for $\omega = A$.[57]

Clearly, the above conditions cannot jointly be satisfied when $\mathcal{V}^\omega(m,j,y)$ is the same for all (ω,m,j,y). In particular, this implies $v^\bullet < v^*$ for any set of primitives $\{\phi,\delta,\overline{\mu},\underline{\mu},\pi\}$, which proves part 1.

We now prove part 2. Fixing primitives $\{\phi,\delta,\overline{\mu},\underline{\mu},\pi\}$, we have

$$\min\left\{v \in \mathbb{R}_+ : (\frac{2}{\pi}v, v, v) \in \mathcal{E}^\bullet\right\} = \pi\frac{1}{2}v^o,$$

as the principal's value cannot be below what she can obtain from decentralization. Denote the maximal value that can be supported under a non-discriminatory XPPE by:

$$\overline{v}^\bullet := \max\left\{v \in \mathbb{R}_+ : (\frac{2}{\pi}v, v, v) \in \mathcal{E}^\bullet\right\}.$$

Modifying the definition of decomposability to include the additional symmetry requirements $\Lambda_1 = \Lambda_2$ and $\mathcal{V}_1^\omega(m,j,y) = \mathcal{V}_2^\omega(m,j,y)$, define $\Psi^\bullet : [\pi\frac{1}{2}v^o, \overline{v}^\bullet] \to [\pi\frac{1}{2}v^o, \overline{v}^\bullet]$. For each $v \in [\pi\frac{1}{2}v^o, \overline{v}^\bullet]$, let

$$\Psi^\bullet(v) := \sup\left\{\tilde{v} \in \mathbb{R}_+ : (\frac{2}{\pi}\tilde{v}, \tilde{v}, \tilde{v}) \in \mathcal{W}^\bullet\left(\text{co}\left\{(v^o, \frac{1}{2}\pi v^o, \frac{1}{2}\pi v^o), (\frac{2}{\pi}v, v, v)\right\}\right)\right\},$$

where for any $V \subseteq \mathbb{R}_+^3$,

$$\mathcal{W}^\bullet(V) := \left\{v \in \mathbb{R}_+^3 : v \text{ is decomposable on } V\right\}.$$

We now characterize $\overline{v}^\bullet$ as the largest fixed-point of $\Psi^\bullet(\cdot)$.

[57] Additional XIC constraints require that agents do not have an incentive to announce that they are not suited when they are (again, given any belief about the other agent's type). We henceforth ignore these constraints, which will be satisfied given any policy that yields $\Psi^\bullet(V)$.

Assume (6), and let $v \in [\pi\frac{1}{2}v^o, \overline{v}^\bullet]$. It can easily be verified that given (6), it is sufficient in search of $\Psi^\bullet(v)$ to consider policies $(\mathcal{M},\chi,\mathcal{V})$ in which one of the two conditions in Lemma 4 is satisfied. Let[58]

$$\Lambda(\mathcal{M},\chi,\mathcal{V}) = \frac{1}{2}\Lambda^A(\mathcal{M},\chi,\mathcal{V}) + \frac{1}{2}\Lambda^B(\mathcal{M},\chi,\mathcal{V}),$$

where Λ^A and Λ^B are defined as in (15) and (16). Without loss of generality, we consider policies for which the above XIC constraints hold.

Consider the following policy:

$$\chi_1^\omega(m) = \begin{cases} 1 & , m = (1,0) \vee (\omega, m) \in \{(B,(1,1)),(A,(0,0))\} \\ 0 & , \text{otherwise.} \end{cases}$$

$$\mathcal{V}^\omega(m,j,y) = \begin{cases} v & , (\omega,j) \in \{(A,1),(B,2)\} \vee (\omega,j,y) \in \{(A,2,S),(B,1,S)\} \\ \delta v - \frac{(1-\delta)\pi\underline{\mu}}{\delta(1-\underline{\mu})} & , \text{otherwise.} \end{cases}$$

In words, if only one of the agents claims to be suited, then the project is allocated to that agent. If both agents claim to be suited, indifference is broken in favor of the agent specialized in the *other* project. If both agents announce they are not suited, then the project is allocated to the agent specialized in the project. If an agent gets a project he is not suited for and fails, then the agents' continuation drops to $\delta v - \frac{(1-\delta)\pi\underline{\mu}}{\delta(1-\underline{\mu})}$; otherwise, agents continue to obtain the highest possible continuation v (taking v to be exogenous).

It can be checked that if such a policy is feasible, i.e., if $\delta v - \frac{(1-\delta)\pi\underline{\mu}}{(1-\underline{\mu})} \geq \frac{1}{2}\delta\pi v^o$, then it satisfies the above XIC constraints and furthermore yields $\Psi^\bullet(v)$; moreover, in such a case,

$$\Psi^\bullet(v) = \delta v + \frac{1}{2}(1-\delta)\pi\left(\underline{\mu}\phi(1-\phi) + \overline{\mu}\left(1-\phi(1-\phi)\right) - 2(1-\phi)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right).$$

Fixing $v = \overline{v}^\bullet$ and solving $\overline{v}^\bullet = \Psi^\bullet(\overline{v}^\bullet)$, we obtain

$$\overline{v}^\bullet = \frac{1}{2}\pi\left(\overline{\mu} - \phi(1-\phi)\left(\overline{\mu}-\underline{\mu}\right) - 2(1-\phi)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right) = \frac{1}{2}\pi\left(v^* - 2(1-\phi)\frac{\underline{\mu}(1-\overline{\mu})}{(1-\underline{\mu})}\right) = \frac{1}{2}\pi v^\bullet.$$

Finally, it can be verified that $\delta v^\bullet - \frac{(1-\delta)\pi\underline{\mu}}{(1-\underline{\mu})} \geq \frac{1}{2}\delta\pi v^o$ if and only if (6) holds.[59] *Q.E.D.*

Proof of Proposition 8. First, note that (6) guarantees that $q \in [0,1]$. Denote by v_i^c agent i's expected average continuation payoff at the beginning of a period in the communication phase. Similarly, denote by v_i^d agent i's expected continuation value under decentralization. Consider the incentives of agent 1 (the constraints for agent 2 are analogous). If the current type of project is $\omega = A$ then whether

[58]The subscripts are omitted given the restriction $\Lambda_1 = \Lambda_2$.

[59]Also note that, given (6), it must be the case that $v^o < v^\bullet < v^*$.

or not agent 1 receives the project is independent of his announcement. Suppose $\omega = B$. Regardless of agent 1's belief about θ_2, if $\theta_1 = \beta$ then XIC implies

$$\overbrace{\delta v_1^c}^{m_1=0,\text{ forgo project}} \geq \overbrace{(1-\delta)\pi\underline{\mu} + \left(\underline{\mu} + (1-\underline{\mu})(1-q)\right)\delta v_1^c + \underbrace{(1-\underline{\mu})q\delta v_1^d}_{\text{decentralization}}}^{m_1=1,\text{ get project}}.$$

If $\theta_1 = \alpha$, we have

$$\overbrace{(1-\delta)\pi\overline{\mu} + (\overline{\mu} + (1-\overline{\mu})(1-q))\,\delta v_1^c + \underbrace{(1-\overline{\mu})q\delta v^d}_{\text{decentralization}}}^{m_1=1,\text{ get project}} \geq \overbrace{\delta v_1^c}^{m_1=0,\text{ forgo project}}.$$

XIC is therefore equivalent to imposing that

$$v_1^d + \frac{(1-\delta)\pi\underline{\mu}}{\delta q(1-\underline{\mu})} \leq v_1^c \leq v_1^d + \frac{(1-\delta)\pi\overline{\mu}}{\delta q(1-\overline{\mu})} \tag{39}$$

Next, by definition the values v_i^c, v_i^d must satisfy the following consistency conditions

$$v_1^c = \frac{1}{2}\left(\begin{array}{l} \phi^2\left((\overline{\mu} + (1-\overline{\mu})(1-q))\,\delta v_1^c + (1-\overline{\mu})q\delta v_1^d\right) \\ +(1-\phi^2)\left(\left(\phi^1\overline{\mu} + (1-\phi^1)\underline{\mu}\right)(1-\delta)\pi + \delta v_1^c\right) \\ +(1-\phi^1)\left(\overline{\mu}(1-\delta)\pi + (\overline{\mu} + (1-\overline{\mu})(1-q))\,\delta v_1^c + (1-\overline{\mu})q\delta v_1^d\right) \\ +\phi^1\delta v_1^c \end{array}\right)$$

$$v_i^d = (1-\delta)\pi\frac{1}{2}v_i^o + \delta v_i^d = (1-\delta)\pi\frac{1}{2}\left(\phi\overline{\mu} + (1-\phi)\underline{\mu}\right) + \delta v_i^d.$$

Subtracting and setting $\phi^1 = 1 - \phi^2 = \phi$ we get

$$v_1^c = v_1^d + \frac{(1-\delta)\pi\frac{1}{2}(1-\phi)^2\left(\overline{\mu} - \underline{\mu}\right)}{1 - \delta\left(\phi + (1-\phi)\left(\overline{\mu} + (1-\overline{\mu})(1-q)\right)\right)}. \tag{40}$$

Substituting the expression for q in (8), simple calculations confirm that the RHS of (40) is equal to the LHS of (39); hence XIC is satisfied.

Finally, note that the principal's value v^{JR} under JR is

$$v^{JR} = (1-\delta)v^* + q(1-\phi)(1-\overline{\mu})\,\delta v^o + (1 - q(1-\phi)(1-\overline{\mu}))\,\delta v^{JR},$$

which gives, using (8),

$$v^{JR} = \frac{v^*(1-\delta) + \delta\left(q(1-\phi)(1-\overline{\mu})\,v^o\right)}{1 - \delta\left(1 - q(1-\phi)(1-\overline{\mu})\right)} = v^{\bullet}.$$

Q.E.D.

F Proofs for Section 7 - Extensions

Proof of Proposition 9. Suppose that projects are allocated according to maximal-priority. For each agent i, denote by $v^f(+)$ ($v^f(-)$) the continuation value at the beginning of a period in which he is chosen to be the favored agent, and given that the *previous* period announcements of the two agents were the same (differed). Similarly, denote by $v^{-f}(+)$ and $v^{-f}(-)$ agent i's continuation value when agent $-i$ is the favored agent. It is easy to verify that these values are equal to

$$v^f(+) = \frac{1}{2}\overline{\mu}(1-\delta)\pi + \left(\frac{1}{2} - \rho(1-\rho)\right)\left(\underline{\mu}(1-\delta)\pi + \left(\overline{\mu}+\underline{\mu}\right)\delta v^f(+) + \left(2 - \left(\overline{\mu}+\underline{\mu}\right)\right)\delta v^{-f}(+)\right) + \rho(1-\rho)\left((1+\overline{\mu})\,\delta v^f(-) + (1-\overline{\mu})\,\delta v^{-f}(-)\right) \tag{41}$$

$$v^f(-) = \frac{1}{2}\overline{\mu}(1-\delta)\pi + \rho(1-\rho)\left(\underline{\mu}(1-\delta)\pi + \left(\overline{\mu}+\underline{\mu}\right)\delta v^f(+) + \left(2 - \left(\overline{\mu}+\underline{\mu}\right)\right)\delta v^{-f}(+)\right) + \left(\frac{1}{2} - \rho(1-\rho)\right)\left((1+\overline{\mu})\,\delta v^f(-) + (1-\overline{\mu})\,\delta v^{-f}(-)\right) \tag{42}$$

and similarly

$$v^{-f}(+) = \rho(1-\rho)\left(\overline{\mu}(1-\delta)\pi + (1-\overline{\mu})\,\delta v^f(-) + (1+\overline{\mu})\,\delta v^{-f}(-)\right) + \left(\frac{1}{2} - \rho(1-\rho)\right)\left(\left(2 - \left(\overline{\mu}+\underline{\mu}\right)\right)\delta v^f(+) + \left(\overline{\mu}+\underline{\mu}\right)\delta v^{-f}(+)\right) \tag{43}$$

$$v^{-f}(-) = \left(\frac{1}{2} - \rho(1-\rho)\right)\left(\overline{\mu}(1-\delta)\pi + (1-\overline{\mu})\,\delta v^f(-) + (1+\overline{\mu})\,\delta v^{-f}(-)\right) + \rho(1-\rho)\left(\left(2 - \left(\overline{\mu}+\underline{\mu}\right)\right)\delta v^f(+) + \left(\overline{\mu}+\underline{\mu}\right)\delta v^{-f}(+)\right) \tag{44}$$

Assume $\omega = A$. We consider the interim incentive constraints of an agent i to announce truthfully, given any belief the agent may hold about agent $-i$'s current type, when he expects $-i$ to be truthful in the continuation game. Consider first the incentives of agent i when he is the favored agent. Regardless of i's type, if he believes $\theta_{-i} = \beta$, i's XIC is satisfied, as he expects to receive the project regardless of his announcement.[60] Suppose then that i believes $\theta_{-i} = \alpha$. If $\theta_i = \beta$, then XIC implies that

$$\delta v^f(-) \geq \underline{\mu}\Big((1-\delta)\pi + \delta v^f(-)\Big) + (1-\underline{\mu})\delta v^{-f}(-),$$

whereas if $\theta_i = \alpha$, XIC requires

$$\overline{\mu}\Big((1-\delta)\pi + \delta v_i^f(+)\Big) + (1-\overline{\mu})\delta v_i^{-f}(+) \geq \delta v_i^f(+).$$

[60]In an ex-post equilibrium, each agent believes the other is truthful in the continuation game, regardless of past announcements. Hence, i's announcement in this case is irrelevant as it affects neither the current period allocation nor the continuation game.

Consider next i's incentives when $-i$ is the favored agent. Regardless of i's type, if he believes $\theta_i = \alpha$, i's XIC is satisfied, since he does not expect to receive the project regardless of his announcement. Suppose i believes $\theta_{-i} = \beta$. If $\theta_i = \beta$ then XIC requires

$$\underline{\mu}\delta v_i^{-f}(+) + (1-\underline{\mu})\delta v_i^{f}(+) \geq \underline{\mu}(1-\delta)\pi + \delta v_i^{-f}(+),$$

and finally if $\theta_i = \alpha$ then XIC implies

$$\overline{\mu}(1-\delta)\pi + \delta v_i^{-f}(-) \geq \underline{\mu}\delta v_i^{-f}(-) + (1-\underline{\mu})\delta v_i^{f}(-).$$

Denote $v(\xi) := v^f(\xi) - v^{-f}(\xi)$ for each $\xi \in \{-,+\}$. Rearranging these condition, it is easily verified that XIC is equivalent to

$$v(\xi) \geq \frac{\underline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})}, \quad \forall \xi \in \{-,+\} \tag{45}$$

$$v(-) \leq \frac{\overline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})} \quad \text{and} \quad v(+) \leq \frac{\overline{\mu}(1-\delta)\pi}{\delta(1-\overline{\mu})}. \tag{46}$$

We now derive conditions under which, given (41)-(44), (45), (46) are satisfied. Subtracting (44) from (42) and (43) from (41) gives

$$v(-) = (1-\delta)\pi\rho(1-\rho)\left(\underline{\mu}+\overline{\mu}\right) + \delta v(+)2\rho(1-\rho)\left(\overline{\mu}+\underline{\mu}-1\right) + \delta v(-)\left(1-2\rho(1-\rho)\right)\overline{\mu},$$

$$v(+) = (1-\delta)\pi\left(\frac{1}{2}-\rho(1-\rho)\right)\left(\overline{\mu}+\underline{\mu}\right) + \delta v(-)2\rho(1-\rho)\overline{\mu} + \delta v(+)\left(1-2\rho(1-\rho)\right)\left(\overline{\mu}+\underline{\mu}-1\right).$$

Solving for $v(-)$ and $v(+)$ we have

$$v(-) = \frac{\rho(1-\rho)(1-\delta)\pi\left(\overline{\mu}+\underline{\mu}\right)}{1-\delta\left(2\overline{\mu}+\underline{\mu}-1\right)\left(1-2\rho(1-\rho)\right)+\delta^2\overline{\mu}\left(1-4\rho(1-\rho)\right)\left(\overline{\mu}+\underline{\mu}-1\right)}, \tag{47}$$

$$v(+) = \frac{(1-\delta)\pi\left(\overline{\mu}+\underline{\mu}\right)\frac{1}{2}\left[\left(1-2\rho(1-\rho)\right)-\delta\overline{\mu}\left(1-4\rho(1-\rho)\right)\right]}{1-\delta\left(2\overline{\mu}+\underline{\mu}-1\right)\left(1-2\rho(1-\rho)\right)+\delta^2\overline{\mu}\left(1-4\rho(1-\rho)\right)\left(\overline{\mu}+\underline{\mu}-1\right)}. \tag{48}$$

After some algebra, it can be verified that $v(+) \leq \frac{\overline{\mu}(1-\delta)\pi}{\delta(1-\overline{\mu})}$ and $v(-) \leq \frac{\overline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})}$ are satisfied without further assumption. Furthermore, note that $v(+) \geq v(-)$. Hence (45)-(46) are equivalent to

$$v(-) \geq \frac{\underline{\mu}(1-\delta)\pi}{\delta(1-\underline{\mu})}. \tag{49}$$

Combining (47) with (49) and rearranging yields

$$\frac{1-2\rho(1-\rho)}{2\rho(1-\rho)} \leq \frac{\delta(1-\underline{\mu})\frac{1}{2}\left(\overline{\mu}+\underline{\mu}\right)-\underline{\mu}+\delta^2\overline{\mu}\underline{\mu}\left(\overline{\mu}+\underline{\mu}-1\right)}{\underline{\mu}-\delta\underline{\mu}\left(2\overline{\mu}+\underline{\mu}-1\right)+\delta^2\overline{\mu}\underline{\mu}\left(\overline{\mu}+\underline{\mu}-1\right)}, \tag{50}$$

which gives the result.

Part 2. Consider a history in which each of the agents has a type that differs from his specialization. XIC requires that for at least one of the agents,

$$(1-\delta)\underline{\mu} \leq \delta\left(\overline{v}^{FB}(-,\rho) - \underline{v}^{FB}(-,\rho)\right), \tag{51}$$

where $\overline{v}^{FB}(-,\rho)$ and $\underline{v}^{FB}(-,\rho)$ denote an agent's highest and lowest feasible continuation values under a first-best delegation rule (beginning at the current history in which the agents' types differ). However, for ρ sufficiently close to 1, $\overline{v}^{FB}(-,\rho) - \underline{v}^{FB}(-,\rho)$ becomes arbitrarily small. Hence, for sufficiently large ρ, (51) cannot be satisfied. *Q.E.D.*

Proof of Proposition 10. Follows from arguments analogous to those in the proof of Proposition 1. *Q.E.D.*

G Ex-post self-generation and factorization

In this section, we show that the set of XPPE $\mathcal{E}$ can be characterized through self-generation using the methodology of APS. The key is that when $V \subset \mathbb{R}^3_+$ is *ex-post* self-generating,[61] the continuation payoffs used in the decomposition of $v \in V$ have the property that for any outcome (ω, m, j, y), $(\mathcal{V}^\omega(m,j,y))$ can in turn be generated using actions in which announcements are XIC. In this way, the strategy profile that is constructed by using the generation conditions is guaranteed to be XIC in each period. In addition, note that $\mathcal{E} \neq \emptyset$ for any set of primitives, as a communication-free XPPE exists for any set of primitives.

Lemma 16 *(i) If $V \subset \mathbb{R}^3$ is bounded and ex-post self-generating then $\mathcal{W}(V) \subseteq \mathcal{E}$ (and in turn $V \subseteq \mathcal{E}$); (ii) $\mathcal{W}(\mathcal{E}) = \mathcal{E}$; (iii) If $V \subseteq V'$ then $\mathcal{W}(V) \subseteq \mathcal{W}(V')$; (iv) If V is compact then $\mathcal{W}(V)$ is compact; (v) Let $\mathcal{W}^k(\mathcal{O})$ denote the set obtained following k iterations of $\mathcal{W}$, starting with the feasible set $\mathcal{O} :=$ $\text{co}\{\vec{0}, (v^*, \pi v^*, 0), (v^*, 0, \pi v^*)\}$. Then $\mathcal{E} = \mathcal{W}_\infty := \bigcap_k \mathcal{W}^k(\mathcal{O})$; in particular, $\mathcal{E}$ is compact.*

Proof. Part (i). Let $v \in V$. We construct an XPPE that yields payoffs v. Since $v \in \mathcal{W}(V)$, there exist $\mathcal{Z} = (\tilde{\mathcal{M}}, \tilde{\chi}, \tilde{\mathcal{V}})$ that decomposes v on V. Fix the allocation and announcement rules in the first period as $\chi|_{h^0} = \tilde{\chi}$, $\mathcal{M}|_{h^0} = \tilde{\mathcal{M}}$ and for each $h^1 = (\omega^1, m^1, x^1, y^1)$ fix $v|_{h^1} = \tilde{\mathcal{V}}^{\omega_1}((m_{11}, m_{21}), x_1, y_1) \in V$. Subsequent play can then be prescribed recursively using $v|_{h^t}$ as the state variable. The fact that V is bounded and $\delta \in (0,1)$ guarantees that payoffs are continuous at infinity, hence the strategy profile constructed above (and continuation strategies) yields v as the average payoff (and continuation payoffs $v|_{h^t}$). That the strategy profile is indeed an XPPE then follows since (i) by construction, at each period agents do not benefit from deviating from their announcements regardless of their belief about

[61] As in the previous sections, whenever there is no confusion we omit the 'ex-post' qualification when referring to ex-post decomposability and ex-post self generation.

the other agent's current type, and (ii) the one-shot deviation principal can be applied since payoffs are continuous at infinity.

Part (ii). Since $\mathcal{E}$ is bounded, if it is self-generating then $\mathcal{W}(\mathcal{E}) \subseteq \mathcal{E}$, thus $\mathcal{W}(\mathcal{E}) = \mathcal{E}$. Hence it suffices to show $\mathcal{E} \subseteq \mathcal{W}(\mathcal{E})$. Let $v \in \mathcal{E}$, and denote by $\sigma = (\mathcal{M}, \chi)$ (where $\mathcal{M}$ and χ denote the agents' and the principal's strategies) an XPPE that yields the value profile $v = U(\sigma)$. Let $\tilde{\mathcal{M}} = \mathcal{M}_1$, $\tilde{\chi} = \chi_1$ and $\tilde{\mathcal{V}}^\omega(m, x, y) = U(\sigma|_{h^1})$, for any $h^1 \in \mathcal{H}^1$. Consider the policy $\mathcal{Z} = (\tilde{\mathcal{M}}, \tilde{\chi}, \tilde{\mathcal{V}})$. Since σ is an XPPE, $\sigma|_{h^1}$ is also an XPPE, hence each $\tilde{\mathcal{V}}^\omega(m, x, y) \in \mathcal{E}$. Furthermore, $\Lambda(\mathcal{Z}) = v$. Finally, since σ is an XPPE, agents have no profitable deviation from their period-1 announcements, regardless of their beliefs about the other agent's type. Hence $\mathcal{Z}$ is XIC. Thus, $\mathcal{Z}$ decomposes v on $\mathcal{E}$ and $v \subseteq \mathcal{W}(\mathcal{E})$.

Let $v \in \mathcal{W}(V)$. Then v is decomposed by some $\mathcal{Z}$ on V, and so also on V'; hence $v \in \mathcal{W}(V')$, proving part (iii). Part (iv) follows from the observation that the constraints involve weak inequalities, the set of continuation payoffs is compact, and utility functions as well as constraint functions are all bounded and continuous.

Part (v). The set of feasible payoffs $\mathcal{O} = \mathrm{co}\{\vec{0}, (v^*, \pi v^*, 0), (v^*, 0, \pi v^*)\}$ is compact. Any payoff vector that is decomposable on $\mathcal{O}$ is also feasible, implying $\mathcal{W}(\mathcal{O}) \subseteq \mathcal{O}$. Since $\mathcal{E} = \mathcal{W}(\mathcal{E})$, and using parts (iii) and (iv), we have

$$\mathcal{E} \subseteq \mathcal{W}_\infty \subseteq ... \subseteq \mathcal{W}(\mathcal{O}) \subseteq \mathcal{O},$$

where each $\mathcal{W}^k(\mathcal{O})$, $k \in \mathbb{N}$, is non-empty and compact, as is $\mathcal{W}_\infty$ (that $\mathcal{W}_\infty$ is nonempty follows from the fact that (i) $\mathcal{E} \neq \emptyset$, as a communication-free XPPE always exists, and (ii) $\mathcal{E} \subseteq \mathcal{W}_\infty$). It remains to show that $\mathcal{W}_\infty$ is self-generating (which together with the fact that it is bounded and a superset of $\mathcal{E}$ implies $\mathcal{W}_\infty = \mathcal{E}$). Let $v \in \mathcal{W}_\infty$. Then $v \in \mathcal{W}^k(\mathcal{O})$ for any k. Hence, for any k there exists $\mathcal{Z}^k = (\mathcal{M}^k, \chi^k, \mathcal{V}^k)$ such that $\Lambda(\mathcal{Z}^k) = v$ and promised continuation values under $\mathcal{Z}^k$ are feasible with respect to $\mathcal{W}^{k-1}(\mathcal{O})$. Taking convergent subsequences if necessary, assume $\{\mathcal{Z}^k\}_k$ converges to $\mathcal{Z}^* = (\mathcal{M}^*, \chi^*, \mathcal{V}^*)$. We complete the proof by showing that $\mathcal{Z}^*$ decomposes v on $\mathcal{W}_\infty$. Suppose that there exists an outcome (ω, m, x, y) such that the promised continuation $\mathcal{V}^{*,\omega}(m, x, y) \notin \mathcal{W}_\infty$. Then, by the compactness of $\mathcal{W}_\infty$, there exists $\varepsilon > 0$ such that $B_\varepsilon(\mathcal{V}^{*,\omega}(m, x, y)) \cap \mathcal{W}_\infty = \emptyset$ (where $B_\varepsilon(\tilde{v}) \subset \mathbb{R}^3$ denotes the closed ball of radios ε with center $\tilde{v} \in \mathbb{R}^3$). On the other hand, there exists $\hat{k}$ such that for all $k > \hat{k}$, $\mathcal{V}^{k,\omega}(m, x, y) \in B_\varepsilon(\mathcal{V}^{*,\omega}(m, x, y))$, which in turn implies that $B_\varepsilon(\mathcal{V}^{*,\omega}(m, x, y)) \cap \mathcal{W}^k(\mathcal{O}) \neq \emptyset$ for any $k < \hat{k}$. Thus $\{B_\varepsilon(\mathcal{V}^{*,\omega}(m, x, y))\} \cup \{\mathcal{W}^k(\mathcal{O})\}_{k=1}^\infty$ is a collection of subsets of a compact topological space that satisfies the finite intersection property, which implies $B_\varepsilon(\mathcal{V}^{*,\omega}(m, x, y)) \cap \mathcal{W}_\infty \neq \emptyset$, giving the contradiction. Hence $\mathcal{V}^{*,\omega}(m, x, y) \in \mathcal{W}_\infty$ for all outcomes (ω, m, x, y), and $\mathcal{W}_\infty$ is self generating, completing the proof. ■