

Department of Decision Sciences

De Finetti Risk Seminar

Minkowski's theorem for quasi-concave functions

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Abstract

Minkowski's theorem on the existence of mixed volumes is arguably the most fundamental theorem in convex geometry. Many geometric quantities, such as volume, surface area and mean width, are all special examples of mixed volumes. Similarly, many classic theorems, such as the isoperimetric inequality, follow from general inequalities between mixed volumes.

Over the last decade, many results of convex geometry and asymptotic geometric analysis were extended to a functional setting. This new idea of treating functions as geometric objects creates a new bridge between geometry and analysis – tools from analysis and probability can be used to prove new results in geometry, and the intuition and notions of geometry may be used to prove new analytic results. In this talk we will present a functional extension of Minkowski's theorem. After a quick reminder of classical definitions and results, we will introduce several classes of functions, and discuss possible addition operations on these classes. We will see that using a "natural" notion of addition, it is possible to extend Minkowski's theorem to the case of general quasi-concave functions. This will allow us to introduce mixed integrals, which are functional extensions of mixed volumes. Finally, we will discuss several ways to state and prove inequalities between mixed integrals.