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Università Commerciale Luigi Bocconi

Department of Decision Sciences

Statistics Seminar

On a Sharper Lower Bound for a t-Percentile with an Application in Sequential Estimation

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12:30pm Room 3-E4-SR03 Via Rontgen 1 Milano

Abstract. We set out to compare z_α and $t_{\nu,\alpha}$, the upper $100\alpha\%$ points of a standard normal distribution and a Student's t_ν distribution respectively. We will begin with a quick proof of a well-known result, namely, for every fixed $0 < \alpha < \frac{1}{2}$ and degree of freedom ν, one has $t_{\nu,\alpha} > z_\alpha$.

Next, we provide a new and explicit expression $b_\nu(> 1)$ such that for every fixed $0 < \alpha < \frac{1}{2}$ and ν, we have $t_{\nu,\alpha} > b_\nu z_\alpha$. Indeed we propose to show that whatever be the fixed positive integer ν and $0 < \alpha < \frac{1}{2}$, we have $t_{\nu,\alpha} > b_\nu z_\alpha$ where $b_\nu = \sqrt{\frac{1}{2}\nu}\Gamma\left(\frac{1}{2}\nu\right)\left\{\Gamma\left(\frac{1}{2}(\nu+1)\right)\right\}^{-1}$ which exceeds one. This is a significant improvement over the well-known result (namely, $t_{\nu,\alpha} > z_\alpha$) that is customarily quoted by nearly every source.

In the end, we will apply the new found inequality to draw attention to some interesting observations in a sequential fixed-width confidence interval estimation problem.

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