Financial reporting and market efficiency with extrapolative investors*

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Abstract

We consider a competitive financial market in which companies engage in strategic financial reporting knowing that investors only pay attention to a randomly drawn sample from firms' reports and extrapolate from their sample. We investigate the extent to which stock prices differ from the fundamental values, assuming that companies must report all their activities but are otherwise free to disaggregate their reports as they wish. We show that no matter how large the samples considered by investors are, a monopolist can induce a price of its stock bounded away from the fundamental. Besides, competition between companies may exacerbate the mispricing of stocks.

Keywords: Extrapolation, efficient market hypothesis, competition, sophistication, financial reporting.

JEL codes: C72, D53, G14.

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1 Introduction

The recent financial crisis as well as some famous accounting scandals have revealed that some firms (or banks) can deliberately obfuscate their financial statements, and that many investors may lack the sophistication needed to read through such opaqueness. As a result, financial markets may not be efficient in that stock prices may be far from the underlying fundamentals. A legitimate regulatory response would be to impose tighter disclosure requirements on firms while at the same time attempting to "educate" investors, if possible.[1] But, to the extent that anticipating all kinds of misperceptions is hard, a different response may instead rely on market forces, hoping that the competition to attract investors would discipline firms and lead to market efficiency.

In this paper, we develop a simple framework to investigate the impact of strategic financial reporting on whether the prices of stocks correctly reflect fundamental values. We focus on a setting in which investors are not fully sophisticated in the way they interpret the information provided by firms, and at the same time firms are required to meet (strong) regulatory standards insofar that all activities in the firm should be referred to in the financial report. We analyze how firms' reporting strategies and market prices vary as investors become more sophisticated and/or as the market becomes more competitive.

Specifically, we consider a stylized financial market in which each firm simultaneously chooses a reporting strategy with the goal of maximizing its trading price on the stock market.[2] A financial report in our setup can be thought of as a book which describes the profitability of the firm (how much investors can expect to receive for each dollar invested in the firm). Firms can choose a very simple book, a single page book with a single number summarizing the overall profitability of the firm. Or they can choose a more complicated book, with many pages and

[1] Forms of investor protection to enhance the reliability of financial reports were famously advocated by SEC Chairman Arthur Levitt (Levitt (1998)), and then incorporated in the Regulation for Fair Disclosure. Increasing the transparency of corporate disclosures lies at the heart of recent interventions such as Sarbanes–Oxley Act (adopted after Enron) and Dodd–Frank Reform (adopted after the subprime crisis). The latter has also created the Consumer Financial Protection Bureau with the intent of improving investors' sophistication.

[2] Managers' compensation is directly influenced by trading prices through stock options say. Evidence suggests a strong link between performance-related compensation and aggressive accounting practices, see Burns and Kedia (2006); Bergstresser and Philippon (2006); Efendi, Srivastava and Swanson (2007); Cornett, Marcus and Tehranian (2008).

many numbers describing the profitability of each single activity.[3] Importantly, irrespective of how complicated the firm wishes to make its report, we assume that nothing can be hidden from the book. As a result, the average reported profitability in the book must coincide with the true aggregate profitability of the firm. Thus, in our framework, regulatory standards are strong enough to prevent firms from systematically misreporting their overall profitability.[4]

In such a setting, if investors were able to read (and process) the entire book provided by a given firm, they would obtain a correct assessment of its value, irrespective of how activities are packaged in the firm. Yet, we have in mind that investors can pay attention only to a limited number of pages, and that they base their estimates of each firm's value by extrapolating from those few pages they pay attention to. Specifically, investors independently of each other sample at random K pages from the report of each firm, and assess the profitability of the firm as the average profitability observed on these pages.

Our assumed heuristic captures two tendencies frequently observed among many (less experienced) investors. First, a tendency to extrapolate from small samples considered to be more representative than they should (given the sample size).[5] Second, a tendency for investors to end up with different beliefs about the values of firms even if exposed to the same reports, especially when the reports are complex.[6] Such disagreements follow in our setup because the pages of the books investors pay attention to are assumed to be independently drawn across investors. The disagreement between investors is what motivates trade in our setting.[7]

[3] In practice, firms have a lot of discretion in the way they report their performance to investors. Even relatively simple reports, like earnings announcements, are typically supplemented by a large set of information such as balance sheets, cash flows, and earnings disaggregated at various levels (say by products or geographic regions). The amount of additional information provided, as well as its format, is largely discretionary (Chen, DeFond and Park (2002); Francis, Schipper and Vincent (2002)).

[4] This probably represents an idealization of how much investor protection can be expected to impose on firms. Our findings that prices of stocks can be far away from the fundamentals would *a fortiori* hold in environments with weaker investor protection.

[5] Such a tendency is referred to as the law of small numbers by Tversky and Kahneman (1971). For evidence of extrapolation in surveys about stock market expectations see e.g. Shiller (2000); Dominitz and Manski (2011); Greenwood and Shleifer (2012); for evidence in actual financial choices, see e.g. Benartzi (2001); Baquero and Verbeek (2008); Greenwood and Nagel (2009).

[6] See e.g. Beaver (1968); Kandel and Pearson (1995); Hong and Stein (2007) on abnormal trade volumes around earning announcements and Bailey, Li, Mao and Zhong (2003); Sarkar and Schwartz (2009); Hope, Thomas and Winterbotham (2009) on the role of complex information. See also Morgan (2002); Flannery, Kwan and Nimalendran (2004) for studies in which firms' opacity and investors' disagreement appear so closely interrelated that the latter is used as a proxy for the former.

[7] As standard in models with heterogeneous beliefs, an (implicit) assumption here is that investors underestimate the informative role of market prices. That can be interpreted as a form of overconfidence or simply as a lack of understanding of the market functioning (see Hong and

In our baseline model, we also assume that investors are risk-neutral and can only trade one stock (either short sell or buy) and so they trade the stock for which they expect the highest gains from trade (that is, the highest difference between their perceived values of the firms and the corresponding prices). Competition among firms over investors' trades can be roughly described as follows. On the one hand, firms wish to attract investors with high valuations, and that may push towards reporting high profitability in some activities. On the other hand, since on aggregate firms cannot lie, the high profitability reported in some activities must be compensated by low profitability reported in other activities, which may increase the corresponding supply of stocks (through short selling). Equilibrium reporting strategies result from trading off these two effects. Our aim is to characterize how the reports of firms and market prices are affected by the level of sophistication of investors (how many pages of the book each investor considers) and/or by the degree of market competition (how many firms compete for investors' trades).

Because the average of what each firm reports is correct, and draws from the reports of firms are made independently across investors, observe that investors' estimates have to be on average correct. One might have thought that as a result no significant price distortions should arise. Yet, this intuition is incorrect. Consider the simplest setting in which a single firm faces investors who look at just one page in the financial report. In this case, the price would be the median of investors' beliefs, and so mispricing can be induced with a report in which the median profitability exceeds its average (typically using some skewed distribution of returns). We note that the mispricing observed in our simple unit demand/supply setting would continue to hold in the monopoly case, as long as the demand is not linear in the perceived gains from trade (see Section 6).

We then investigate the effect of increasing investors' sophistication. We show that, in a monopolistic market, mispricing can persist even if the sample size on which investors base their estimates grows very large. One might have thought that as investors consider a large number of pages their assessments should get very close to the true profitability of the firm, and so trade should occur at the right price or close to it. If investors could consider all pages, this would be true. But to the extent that the sample size is finite, no matter how large, a monopolist can guarantee a price bounded away from the fundamental by an appropriate choice of reporting strategy, as we show. Specifically, such a strategy endogenously depends

Stein (2007) for a discussion). We discuss in Section 6 how our analysis would be affected by introducing some (imperfect) inference from the price or from other investors' beliefs.

on investors' sophistication, and in particular it requires that the variance of the chosen distribution increases as investors become more sophisticated. In such a scenario, the law of large numbers does not apply. As a result, estimates are not necessarily close to the fundamental, and for well chosen distributions, prices are strictly above the fundamental.

Finally, we analyze the effect of competition. Our main result is that competition may magnify the mispricing of stocks. To show this, we mostly focus on the simple case in which firms have the same fundamental value, investors sample one page from the report of each firm, and we consider the symmetric equilibrium which induces the highest stock price (which we motivate based on tacit collusion considerations). We show that in this equilibrium the price of stocks is strictly above the fundamental, and that it increases in the number of competing firms.

While the complete logic of this result is somewhat involved, we suggest here why market clearing occurs at a higher price when more firms compete.[8] In this equilibrium, investors who sell are only those with a low assessment of *all* firms. That is, in equilibrium, if an investor has a high assessment of firm j and a low assessment of firm r, this investor buys a stock from firm j (as opposed to selling a stock from firm r). Fixing the reports of firms, the probability of ending up with a low assessment of all firms decreases with the number of firms. Low assessments are then less likely to be incorporated into prices when many firms compete in the market, thereby explaining the magnifying effect of competition on price distortion.

In Section 6, we show that such a magnifying effect of competition carries over if the fundamental values are heterogeneous among firms and/or if firms are privately informed of their fundamental value. We view such a finding (as well as the robustness checks) as strongly suggestive that it would be unwise to rely on the observation that there are many firms around in the stock market to dismiss the potential effect of strategic financial reporting onto the mispricing of stocks in a world with extrapolative investors.

Even though our paper emphasizes the extrapolative nature of investors' heuristics, the results reported here can be viewed as illustrative of a more general theme. It has long been understood, since Harrison and Kreps (1978), that speculative trade can arise if investors have heterogeneous (subjective) priors, say about the profitability of the various firms. However, the subjective beliefs held by investors (or what may determine them) are typically taken as exogenous in this story. What

[8] A more complete description of why no firm can unilaterally deviate and get a higher price, as well as a discussion of other possible equilibria, is left to Section 5.

our approach suggests is that firms may try to influence/manipulate the formation of subjective beliefs, here through their release of financial reports, and we have seen how, within our framework, such manipulation could lead to overpricing in the stock market. We believe such a theme of subjective prior manipulation should be the subject of active research in the future, as it seems relevant to explain a number of dysfunctionings in financial markets.

Related literature

The sampling heuristic we consider for investors was first proposed by Osborne and Rubinstein (1998) in a game-theoretic context. This heuristic has been applied in IO settings by Spiegler (2006*a*) and Spiegler (2006*b*), in which firms compete on prices and consumers choose their firms by a sampling procedure (the sampling bears on the quality in Spiegler (2006*b*) whereas it bears on the price draw as well in Spiegler (2006*a*)).[9] Our model follows the spirit of Spiegler in the modeling of investors' heuristics and in the questions that are being addressed (effect of sophistication, effect of competition). But our application is different, leading to different formulations of the game and different conclusions.[10]

Alternative models of investors' overextrapolation have been considered in the context of financial markets. De Long, Shleifer, Summers and Waldmann (1990*b*) study whether arbitrageurs have a stabilizing role in the presence of extrapolative investors, while Barberis, Shleifer and Vishny (1998), Rabin (2002) and Rabin and Vayanos (2010) focus on how extrapolative investors react to news. None of these papers studies the issue of strategic financial reporting, which is the main focus of our paper. Moreover, a growing literature studies financial markets in which investors hold heterogeneous beliefs (see Scheinkman and Xiong (2004) and Hong and Stein (2007) for reviews). As mentioned, our model seems to be the first to investigate the role of the financial reports of firms as a possible source of disagreement.

From a more general perspective, a few recent papers analyze stock prices in competitive equilibria with non-fully rational agents (in Gul, Pesendorfer and Strzalecki (2011), agents can only distinguish a limited number of contingencies;

[9] See also Rubinstein and Spiegler (2008), who consider a speculative market in which investors randomly sample one price in the history of posted prices and buy if the current price is below the sampled price.

[10] In particular, in our setting, prices are determined through market clearing, and what firms report have to be on average correct, a constraint that has no analog in Spiegler' settings. In terms of results, our findings that significant distortion persists even if investors consider arbitrarily large samples, and that more firms may induce higher prices, have no counterpart in Spiegler (there is also no idea of tacit collusion in Spiegler).

in Eyster and Piccione (2011) and Steiner and Stewart (2012), agents use a coarse reasoning to understand the dynamics of the market). An essential distinctive feature of our study is the focus on how investors' beliefs may be manipulated, which has no counterpart in these papers. Finally, the strategies of firms in financial reporting are analyzed in a large literature in accounting (see e.g. Verrecchia (2001) for a survey, and Hirshleifer and Teoh (2003) for a model in which investors have limited attention). This literature, however, generally abstracts from the role of improved investors' sophistication and of market competition on firms' reporting strategies.[11]

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we analyze the monopoly case in the simplest sophistication scenario. In Section 4 we study the effect of sophistication. In Section 5 we study the effect of competition. Section 6 offers a general discussion, allowing for heterogeneity and private information among firms, but also discussing the role of bounded rationality in the results as well as the introduction of alternative investment heuristics and more general demand specifications. Section 7 concludes.

2 Model

Consider a stock market consisting of F firms $j = 1, ...F$, each having fundamental value φ.[12] There is a unitary mass of investors trading on the stock market. Investors are unaware of the fundamental values of the firms. They assess the profitability of the various firms by taking at face value the financial reports they pay attention to (see details below). Each investor can only trade one stock (either buy or short sell), and he trades the one for which he perceives the highest gain from trade. The prices of the various stocks are determined through market clearing conditions.

Firms are assumed to know the procedure followed by investors, and they seek to maximize the price of their stocks. Each firm chooses a financial reporting strategy that consists in a distribution of signals meant to represent the returns of the various activities in the firm. The mean of this distribution is constrained to coin-

[11] Of course, there is also a large literature building on Crawford and Sobel (1982) that studies how much information can be transmitted from an informed sender to an uninformed decision-maker when the latter is assumed to be perfectly rational. We will discuss some of this literature in relation to the financial reporting application in Section 6.

[12] We chose a deterministic specification to stress the role of bounded rationality in our model. Later on, we discuss the more realistic scenario in which firms may have different fundamental values and/or the fundamental values may be stochastically drawn for each firm.

cide with the fundamental value φ, which holds because of regulatory constraints (all activities must appear somewhere). Moreover, signals in the distribution are assumed to be non-negative. This is either because investors know that activities cannot have too low returns (here normalized to 0) or because a too low report would somehow attract too much attention and so be too detrimental to the perceived profitability of the firm.[13]

There is complete information among firms, and we consider the Nash equilibria of the financial reporting game played by the firms. In particular, our analysis will focus on whether the prices of the stocks differ from the fundamental values, and how the sophistication of investors (see below for a measure of sophistication) and/or the degree of competitiveness (as measured by the number F of firms) affect the result.

Formally, let σ^j denote the distribution of signals chosen by firm j and X^j be the support of σ^j. We require that

$$X^j \subset \mathbb{R}_+, \tag{1}$$

and

$$E(X^j) = \varphi, \tag{2}$$

for each $j = 1, ..., F$. We denote by Σ the set of signal distributions satisfying conditions (1)-(2), and we allow firms to choose any distribution in Σ. In the sequel, we refer to (2) as the aggregation condition.

Investors do not know the fundamental values of the firms, and they employ a simple heuristic procedure in order to assess them. For each firm, they consider K independent random draws from the the signal distribution of the firm, and they interpret the average of these K signals as the fundamental value of the firm.[14] Hence, if investor i observes signals $x^j_{i,1}, x^j_{i,2}, ..., x^j_{i,K}$ from firm j, his assessment of the value of firm j is

$$\hat{x}^j_i = \frac{1}{K}\sum_{n=1}^{K} x^j_{i,n}.$$

[13] While in the main part of the paper we do not consider the dual possibility that there is an upper bound on signals, in the discussion section, we suggest that our most interesting findings are robust to such a consideration.

[14] In our analysis, the same signal can be drawn several times and that may appear at odds with our book story in which investors would look at a few pages of the book only (which suggests a sampling procedure without replacement). Yet, given that the firm can choose as many signals as it wishes, the models with or without replacement would deliver the same insights. We chose the formulation with replacement to make the mathematical analysis simpler.

We also assume that the draws are independent across investors and firms.[15] Such a heuristic can easily be interpreted along one of the general principles outlined in Kahneman (2011): "All there is (for investor i) to assess firm j's value is what investor i sees of firm j," and we implicitly assume here that investor i only sees $x^j_{i,n}$, $n = 1, ..., K$ of firm j, from which $\hat{x}^j_i$ is obviously a focal assessment.[16]

Based on his assessments of the values of firms, each investor trades one unit of stock and he can either buy or short sell it.[17] Hence, investor i is willing to trade stock r if stock r is perceived to offer the highest gains from trade. That is, if

$$r \in \arg\max_j \left| p^j - \hat{x}^j_i \right|, \tag{3}$$

where p^j denotes the price of stock j. Investor i buys stock r if $p^r < \hat{x}^r_i$ and he short sells stock r if $p^r > \hat{x}^r_i$. Note that $\arg\max_j \left| p^j - \hat{x}^j_i \right|$ may sometimes consist of several stocks r, in which case investor i is indifferent between several options. In case of indifference, a tie-breaking rule (to be determined endogenously) specifies the probability assigned to the various possible trades. We let Ω denote the set of tie-breaking rules and ω denote an element of Ω.

Based on investors' orders, and on the tie-breaking rule ω, demand and supply for firm j are denoted by $D^j(\sigma^j, \sigma^{-j}, p^j, p^{-j}, \omega)$ and $S^j(\sigma^j, \sigma^{-j}, p^j, p^{-j}, \omega)$, respectively where σ^{-j} and p^{-j} denote the distributions and prices for all firms except j. We denote the profile of demand and supply for all firms as $D(\sigma, p, \omega)$ and $S(\sigma, p, \omega)$, where $\sigma = \{\sigma^j\}$ and $p = \{p^j\}$, $j = 1, .., F$.

As far as firms are concerned, we assume that they are completely rational and that they know the procedure employed by investors (in particular, they know K).[18] Given that firm j seeks to maximize p^j, this leads to the following definition of equilibrium:

Definition 1 *(Equilibrium) The profile* (σ, p, ω) *is an equilibrium if: for each* j*,* $\sigma^j \in \Sigma$*, and*

[15] From a theoretical perspective, note that such an assumption is the most favorable to market efficiency. Introducing some systematic correlation in investors' draws, e.g. allowing that some signals are known to receive more attention than others, typically weakens the effect of condition (2) and is likely to increase the scope for distortions.

[16] That $\hat{x}^j_i$ is focal can possibly be related to a form of coarse reasoning. If one has to form a guess as to what the mean of a distribution is on the basis of K independent draws from the distribution, then without further information (meaning by averaging over all possible distributions) the empirical mean would be the right guess. Such a line of reasoning can be modeled using Jehiel (2005)'s analogy-based expectation equilibrium.

[17] We chose this specification to make the model as simple as possible. In Section 6, we discuss more general specifications in which demand and supply depend in a smoother way on the perceived gains of trade.

[18] In Section 6, we briefly consider the case in which investors have heterogeneous K.

a) $D(\sigma, p, \omega) = S(\sigma, p, \omega)$.
b) There is no distribution $\tilde{\sigma}^j \in \Sigma$, *prices* $\tilde{p}^j, \tilde{p}^{-j}$, *and tie-breaking rule* $\tilde{\omega} \in \Omega$ *such that* $D(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega}) = S(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega})$ *and* $\tilde{p}^j > p^j$.

Condition (a) requires that the markets clear. Condition (b) requires that there should be no profitable deviation for any firm j, where a profitable deviation $\tilde{\sigma}^j$ of firm j means that for the profile of distributions $(\tilde{\sigma}^j, \sigma^{-j})$, there exists a tie-breaking rule $\tilde{\omega}$ and prices $\tilde{p}^j, \tilde{p}^{-j}$ that clear the markets and such that firm j achieves a strictly higher price $\tilde{p}^j > p^j$.[19]

In the following analysis, we will prove the existence of an equilibrium (in a constructive manner). Discrete distributions with a finite number of signals will play an important role. We will denote by $\sigma = \{x_1, \mu_1; x_2, \mu_2; ..\}$ the distribution in which x_1 occurs with probability μ_1, x_2 occurs with probability μ_2, and so on.

3 Monopoly

We first focus on a monopolistic firm facing investors who just consider one dimension in the financial report. That is, we set $F = 1$ and $K = 1$.

As there is only one firm, the market clearing price corresponds to the median belief about the firm's value. At this price, half of the investors wants to buy and half of them wants to sell. Since each investor only trades one stock, the market clears. Moreover, given that investors only consider one signal, such a median belief corresponds to the median of the firm's distribution. Hence, the monopoly's problem is to choose a distribution with the maximal median that satisfies the constraints (1) and (2) that signals should be non-negative and that the mean of the distribution should coincide with the fundamental φ.

Such a maximization is achieved with a two-signal distribution that puts weight on 0 and h and such that the median is just h (requiring that the weight on h is just above that on 0). To see this, observe that any signal strictly above the median is a waste for the firm as reducing such a signal to the median while increasing all signals slightly so as to meet the aggregation condition (2) would be profitable. Similarly, any signal strictly in between 0 and the median is a waste, as lowering such signals to 0 while increasing all signals slightly so as to respect (2) would be profitable. Consider then $\sigma = \{0, 1 - \mu; h, \mu\}$ with $\mu \geq 1/2$. The

[19] There are alternative possible definitions of profitable deviations (based on other expectations about the ensuing market clearing prices). Note however that any equilibrium as defined here would *a fortiori* be an equilibrium under the alternative specifications of profitable deviations. None of our results depends on this specific choice.

aggregation condition (2) implies that $\mu h = \varphi$, and thus the maximum price that can be achieved by the monopolist is 2φ. The following Proposition collects these observations.[20]

Proposition 1 *Suppose $F = 1$ and $K = 1$. The firm chooses the distribution $\sigma_M = \{0, 1/2; 2\varphi, 1/2\}$. The price is $p_M = 2\varphi$.*

4 Monopoly and Sophistication

We now turn to a setting in which investors are more sophisticated in the sense of considering larger samples. More precisely, we consider a monopolist and we assume that investors sample several ($K > 1$) signals in order to evaluate the fundamental value of the firm. Our question of interest is whether the price gets close to the fundamental if we let K be sufficiently large.

Based on the law of large number, one might have expected that, for K large enough, investors would end up with (approximately) correct assessments of the fundamental value, and thus the market clearing price would have to be close to φ. Such an intuition would be true if the financial reporting strategy of the firm were set independently of K. But, this is not the relevant consideration here, given that the firm can adjust its financial reporting strategy to the number of draws made by investors (since we assume that firms know K). Thus, the distribution chosen by the firm will typically change with K, and the law of large number need not apply.

As we show now, the firm can always guarantee a price bounded away from the fundamental by a suitable choice of reporting strategy (that must depend on K by the previous argument). To see this, consider the following two-signal distribution:

$$\sigma_K = \left\{0, (1/2)^{1/K}; h(K), 1 - (1/2)^{1/K}\right\}, \tag{4}$$

and the price $p_K = h(K)/K$, with $h(K) = \varphi/[1 - (1/2)^{1/K}]$ so that the mean of the distribution is φ.

An investor who gets K draws from the distribution and samples z times the signal $h(K)$ is willing to buy if the price does not exceeds $zh(K)/K$. As the price equals $h(K)/K$, only those who sample K times signal 0 are willing to sell,

[20] The finding that the price coincides with the maximum signal in the support of σ_M would not carry over to smoother demand/supply specifications. Yet, the finding that $p_M > \varphi$ holds quite generally, as shown in Section 6.

which is a proportion $[(1/2)^{1/K}]^K = 1/2$ of investors. That is, at this price half of investors sell and half of the investors buy, so the market clears.

So given K, the monopolist can achieve a price of its stock no smaller than

$$p_K = \frac{\varphi}{K[1-(1/2)^{1/K}]}.$$

Simple algebra reveals that p_K is decreasing with K and that p_K converges to $\varphi/\ln 2$, which is strictly bigger than φ, as K grows arbitrarily large. Hence, we have established:

Proposition 2 *Suppose $F = 1$. Irrespective of K, the firm can attain a price no smaller than $\varphi/\ln 2$, which is strictly larger than φ.*

As described in (4), the distribution used to establish Proposition 2 requires that there is no upper bound on the signals that can be sent by the firm ($h(K) = \varphi/[1-(1/2)^{1/K}]$ goes to infinity as K goes to infinity). If there were an upper bound (as considered in Section 6), the variance of the distribution would have to be bounded, and the firm would not be able to obtain a price of its stock much away from the fundamental when K is large.

5 Competition

We now turn to investigate the effect of competition, i.e. having more than one firm $F > 1$.

Our main question of interest is whether more competition brings the prices of stocks closer to the fundamental values, and whether price distortions may persist as the number of competing firms grows large. It is not a priori clear in which way competition may drive mispricing. Inducing a higher market clearing price would require attracting more demand and so tilt the financial reporting distribution toward higher signals. Yet, since the mean of the distribution has to coincide with the fundamental, that would have to be counter-balanced by having more weight on low signals, which would trigger more supply. This makes it hard to identify how the most relevant deviations would look like and so what effect competition may have on stock prices. We divide our investigation into various subsections.

5.1 A non-transparency result

A first observation is that no matter how many firms are competing on the stock market, it cannot be an equilibrium that (all) firms choose a transparent financial reporting saying what their fundamental value is with probability 1. Indeed, if all firms choose $\sigma = \{\varphi, 1\}$, then obviously the market clearing price for all stocks is $p = \varphi$ and no firm would be perceived as offering any gain from trade. But, suppose that firm j chooses the distribution displayed in the monopoly case; that is, $\sigma^j = \{0, 1/2; 2\varphi, 1/2\}$ when $K = 1$. Then trading other stocks at price φ would be viewed as offering no gains from trade, and as a result one can assume that all trades take place on stock j. As shown in Section 3, firm j can obtain a price of its stock as high as 2φ, thereby showing that the deviation is profitable.[21] This observation carries over to any specification of K (by Proposition 2), thereby allowing us to derive:

Proposition 3 *Irrespective of F and K, there is no equilibrium in which firms report their fundamental value with probability* 1*.*

A second observation is that, irrespective of the strategy used by others, a firm can always guarantee that the price of its stock is at least the fundamental value. Indeed if firm j chooses $\sigma^j = \{\varphi, 1\}$ then $p^j = \varphi$ is necessarily a market clearing price for j (and there is no other possible market clearing price for j if some of the stocks j are to be traded).[22] This establishes the following Proposition:

Proposition 4 *In all equilibria, the price of stocks is no smaller than the fundamental value.*

5.2 The highest price equilibrium

Characterizing all equilibria is somewhat difficult because it requires getting into comparative statics properties of the Walrasian equilibria of the stock market as induced by the various possible choices of reporting strategies of the firms (which in turn affect in a complex way the demand and supply of the various stocks through the sampling heuristic).[23]

[21] Note that when $x_i^j = 2\varphi$, investor i is indifferent between trading stock j or any other stock. If we were concerned of inducing strict gains from trade, we could consider distributions of the form $\{0, 1/2 - \varepsilon; m, 2\varepsilon; h, 1/2 - \varepsilon\}$ with a market clearing price $p^j = m$. Clearly, m and h can be chosen as close as one wishes to 2φ, thereby showing that the conclusion does not hinge on the choice of tie-breaking rule.

[22] This insight establishes within our setup that prices are more likely to exceed than to fall short of fundamentals. Such an asymmetry results in our model from firms' incentives to distort signals whenever they can induce prices above fundamentals while turning to full transparency if prices were to fall below fundamentals.

[23] The theory of general equilibrium has essentially produced existence and efficiency results but very few instances in which Walrasian prices can be explicitly derived from the demand and supply structure. For our purpose, it is the latter that is required though.

To keep the analysis tractable, we consider the case in which investors only consider one dimension in the financial reporting, i.e. $K = 1$. Moreover, we restrict our attention to symmetric equilibria. That is, we require that in equilibrium firms choose the same distribution of signals, the prices of the various stocks are the same, and the tie-breaking rule is anonymous.[24] Among symmetric equilibria, we focus on the equilibrium that induces the highest prices of stocks. There are two ways to think of such a focus: 1) It highlights how much the prices can be far from the fundamental. 2) It is a natural benchmark equilibrium if we have in mind that the firms in the stock market can coordinate on the equilibrium they like best (a form of selection based on tacit collusion). We will also in the next subsection discuss other (symmetric) equilibria.

In order to characterize the highest price symmetric equilibrium, we proceed in several steps. First, we characterize among the symmetric distributions of signals (and anonymous tie-breaking rules) the one that induces the largest common clearing price of stocks. Then, we show that such a symmetric distribution of signals together with the corresponding profile of prices constitutes an equilibrium, thereby leading to a characterization of the highest price symmetric equilibrium.

Consider a strategy profile $\{\sigma, p, \omega\}$, such that each firm chooses the same distribution $\sigma = \{x_1, \mu_1; x_2, \mu_2; ...\}$, p denotes the common market clearing price and ω is an anonymous tie-breaking rule. We first note that σ may induce the highest price in this class only if it satisfies the following property: There must be no signal $x > 0$ which is in the support of σ and such that signal $\tilde{x} = 2p - x$ is not in the support of σ.[25] That is, all positive signals in the support of σ need to be paired around the price. To see this, suppose by contradiction that σ assigns mass $\mu_x > 0$ to an unpaired signal x (i.e., there is no mass on $2p - x$). Suppose also for the sake of the argument that $x > p$. Then one could obtain the same price by moving x to the lower adjacent signal $\hat{x}$ in the support of σ (or to p if there is no signal between x and p). The average of the distribution would be reduced by $\mu_x(x - \hat{x})$. This would then allow to increase all signals and so the price by the same amount, thereby showing that the distribution did not induce the highest price. In what follows, we say that $\widehat{\sigma} \in \hat{\Sigma}$ if $\widehat{\sigma} \in \Sigma$ (as defined by conditions (1) and (2)) and all positive signals in $\widehat{\sigma}$ are paired around some p interpreted as the price.

The second step in our argument is to observe that to achieve the largest price,

[24] That is, if a mass μ of investors ends up with the same assessment about a set of N firms, each of these firms receives a fraction μ/N of the trades.

[25] One can actually show the stronger result that any candidate symmetric equilibrium distribution (not only the best one) must meet this property.

the distribution $\hat{\sigma} \in \hat{\Sigma}$ should assign positive weight to at most three signals. To see this, suppose that $\hat{\sigma}$ assigns positive weight to n signals and $n > 3$. Then one can define another distribution $\sigma \in \hat{\Sigma}$ which involves at most $n-1$ signals and that induces a price $\tilde{p} \geq p$ (assuming again an anonymous tie-breaking rule and that $\sigma^j = \tilde{\sigma}$ for all j). The idea is to remove the two signals closest to the price and move their mass either to the price (if the weight of the higher of the two signals is no smaller than the weight of the smaller one) or to the adjacent signals further away from the price (if the weight of the smaller signal is bigger than the weight of the higher signal), and then increase all signals and the price upward so as to accommodate the aggregation condition.[26]

Iterating the argument, one gets a distribution with at most three signals, $0, p, 2p$. Then, one can move equal mass from p to 0 and $2p$ or vice-versa without changing the market clearing price. Thus, we end up with a two-signal distribution which takes one of the following forms: $\sigma_a = \{0, 1-\mu_a; 2p_a, \mu_a\}$ or $\sigma_b = \{0, 1-\mu_b; p_b, \mu_b\}$.[27] Consider σ_a. Investors are indifferent between trading stock j and stock r whenever they sample signal $2p_a$ from firm j and signal 0 from firm r. The highest aggregate demand is obtained by letting investors buy j whenever indifferent between buying j and selling another stock. In that case, the aggregate supply includes only those who sample signal 0 from all firms, which has probability $(1-\mu_a)^F$. Hence, market clearing requires $(1-\mu_a)^F \leq 1/2$. If $(1-\mu_a)^F < 1/2$, one can decrease slightly μ_a and increase all signals by ε and obtain a price which is ε higher. Hence, among distributions σ_a, the price is maximized by setting $\mu_a = \mu^*$ where

$$\mu^* = 1 - (1/2)^{1/F}. \tag{5}$$

The highest market clearing price from distributions σ_a is thus obtained with

$$\sigma^* = \{0, 1-\mu^*; \varphi/\mu^*, \mu^*\}, \tag{6}$$

and the resulting market clearing price is

$$p^* = \frac{\varphi}{2\mu^*}. \tag{7}$$

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[26]Intuitively, such a move can be done while respecting the market clearing conditions. The direction of the move is then dictated so that the aggregation condition can be satisfied by moving all signals (except possibly 0) as well as the price upwards.

[27]The distribution $\sigma_c = \{p_c, 1-\mu_c; 2p_c, \mu_c\}$ is easily ruled out given that by (2) it would imply that $p_c < \varphi$, which is clearly not the highest achievable price (see Proposition 4).

With simple algebra, one can show that no distribution in σ_b can achieve a price which is higher than p^*.[28] This in turn leads to the next Lemma, whose detailed proof appears in the Appendix.

Lemma 1 *Assume that for some* $\widehat{\sigma} \in \Sigma$, $\sigma^j = \widehat{\sigma}$ *for all* j, *and consider an anonymous tie-breaking rule. The resulting market clearing price* $\widehat{p}$ *is no larger than* p^*, *as defined in (7). Moreover,* p^* *is obtained with the distribution* σ^*, *as defined in (5) and (6).*

Our last step is to show that $\sigma^j = \sigma^*$ together with $p^j = p^*$ for all j and the anonymous tie-breaking rule that favors demand over supply in case of indifference defines an equilibrium.

Lemma 2 *There is a symmetric equilibrium in which firms choose the distribution* σ^* *and the price is* $p^*(F)$, *as defined respectively in (6) and (7).*

To get a sense of why Lemma 2 holds true, consider by contradiction a tentative deviation by one firm, say firm j that would lead to a market clearing price $p^j > p^*$ for firm j and a market clearing price p' for the non-deviating firms (in the Appendix, we allow for the case in which the non-deviating firms have different market clearing prices) and let $h = 2p^*(= \varphi/\mu^*)$.

We first note that $p' \leq h/2$ as $p' > h/2$ would imply excess supply for the non-deviating firm and thus make market clearing impossible (remember that $(1-\mu^*)^{F-1} > 1/2$ whenever $(1-\mu^*)^F = 1/2$).

We next observe that the total weight on signals of firm j above $p^j + p'$ should be no smaller than μ^* as otherwise there would be excess aggregate supply over the stocks of all firms (remember again that $(1-\mu^*)^F = 1/2$).

If $p' = h/2$ as before firm j's deviation, the aggregation condition (2) would not hold given the observation just made that the weight of firm j' signals above $p^* + p'$ should be no smaller than μ^*, and $p^j > h/2 = p'$.[29]

One may then wonder whether having a smaller $p' < h/2$ could help alleviating the constraint (2). To see that this cannot be the case, observe that the total weight of firm j signals strictly below $p^j - p'$ cannot exceed some threshold x as

[28] In fact, consider $\sigma_b = \{0, 1-\mu_b; p_b, \mu_b\}$. The aggregate demand equals at most those who sample signal p_b from all firms, so market clearing requires $(\mu_b)^F \geq 1/2$. Due to the aggregation condition (2), $p_b \leq \varphi(2)^{1/F}$ which is lower than $p^*(F)$.

[29] Observe that this property would not hold if we had considered a bimodal distribution on $(0, h)$ with a weight on h strictly larger than μ^*. For such distributions, a deviation of the form $(\varepsilon, 1-\mu(\varepsilon); h+\varepsilon, \mu(\varepsilon))$ with prices $p^j = h/2+\varepsilon$, $p' = h/2$ would constitute a profitable deviation.

otherwise there would be excess supply for firm j (investors sell stock j whenever such a signal is drawn with signals 0 from the non-deviating firms). Besides, such a threshold x gets arbitrarily small as F grows large.[30]

Given the above observations, the mean of the distribution of signals of firm j is no smaller than $\mu^*(p^j + p') + (1 - \mu^* - x)(p^j - p')$, but since x is small this expression decreases with p'. As for $p' = h/2$ it is strictly larger than φ, the aggregation condition (2) cannot be satisfied for any $p' \leq h/2$.

We conclude from the above that there can be no deviation of firm j that could possibly induce a market clearing price p^j strictly above $p^* = h/2$. Combining Lemmas 1 and 2, we get:

Proposition 5 *Suppose $K = 1$ and $F > 1$. The maximal price achieved in a symmetric equilibrium is $p^*(F) = \frac{\varphi}{2[1-(\frac{1}{2})^{1/F}]}$. This price increases in F.*

The reason why the price of stocks p^* increases with F is as follows. The price of a given firm j must reflect the median (here also the average) of the valuations *of those who trade firm j*. This however differs from the average valuation across *all* investors, as not all investors trade all firms. That is what opens the possibility of distorting prices when many firms compete in the market. More precisely, in the equilibrium of Proposition 5, investors sell stock j only when they sample F low signals. For a given probability of high signal, the more firms, the lower the chance that the signals drawn from all firms are low. In this way, bad evaluations are less likely to be reflected in market prices when the number of firms increases, and as a result the price p^* increases with F.

A question arises as to how the market clearing price in the competitive case compares with the monopoly price (see Proposition 1) for various F. Simple calculations reveal that the price in the duopoly case is smaller than in the monopoly case, but the price for any other market structure configuration ($F > 2$) is larger than in the monopoly case.[31]

Finally, it should be noted that in our setting the total number of signals that investors pay attention to increases with the number of firms (since investors

[30] Denote with ν the mass of signals of firm j strictly below $p^j - p'$ and with η the mass of those not smaller than $p^j + p'$. The supply of firm j cannot fall short of $\nu(1-\mu^*)^{F-1}$, while its demand cannot exceed η. Let F grow large. The price p^j grows large and so η must be small or the aggregation condition (2) would be violated. At the same time, $(1-\mu^*)^{F-1} \to 1/2$, so ν must be small or there would be excess supply for j. In the Appendix, we extend the argument to arbitrary F.

[31] These considerations imply that if a monopolistic firm could split its activity into several companies with different stocks, it would benefit from it given the heuristic of the investors.

consider one signal from each firm irrespective of F). In this way, we highlight the effect of competition among firms over investors' trades rather than over investors' attention.[32] One may ask how our construction would be affected if investors could process at most S signals say, and so they would sample one signal from at most S firms. Proposition 5 would hold by substituting F with $\min\{F, S\}$. When $F > S$, our result should then be interpreted as showing that mispricing increases when investors are allowed to pay attention to more firms.

5.3 Other Equilibria

As mentioned before, we think of the highest price symmetric equilibrium as the most meaningful one based on tacit collusion considerations. Yet, abstracting from such considerations, we wish here to highlight that there may be other (symmetric) equilibria. To illustrate this, we exhibit a symmetric equilibrium that induces a market clearing price as low as the fundamental (which combined with Propositions 4 and 5 allows us to show the range of market clearing prices that can be sustained in symmetric equilibria). More precisely, we have:

Proposition 6 *Suppose $K = 1$. For every $F > 1$, there is a symmetric equilibrium with market clearing prices $p = \varphi$. The common distribution of signals has support $(0, 2\varphi)$. It is centered around φ, and it is such that the probability of sampling $F - 1$ signals within distance z from φ is linear in z. When $F = 2$, it is the uniform distribution on $(0, 2\varphi)$.*

To get some intuition for Proposition 6, consider the duopoly case $F = 2$. If firm 1 chooses a uniform distribution of signals between 0 and 2φ, it is not hard to see that irrespective of the choice of distribution of firm 2, the market clearing price for firm 1 must be $p_1 = \varphi$. Indeed at this price, and given the symmetry of the distribution of firm 1 around φ, there is as much demand as there is supply for firm 1 (whatever the choice of distribution of firm 2). More important for our purpose though is the observation that when firm 1 chooses such a distribution, the market clearing price of firm 2 cannot be larger than φ. If the support of the distribution of firm 2 coincides with $(0, 2\varphi)$, one can show that the market clearing price of the two firms has to be φ. This is because 1) any signal $s_2 > p_2$ generates a demand for firm 2 proportional to $s_2 - p_2$ (that corresponds to the

[32] Hirshleifer, Lim and Teoh (2009) provide evidence consistent with the idea that information is less likely to be incorporated in market prices when many signals compete for investors' attention.

probability that s_1 satisfies $|s_1 - p_1| < s_2 - p_2$ conditional on s_2), 2) any signal $s_2 < p_2$ generates a supply for firm 2 proportional to $p_2 - s_2$, and 3)

$$\int_{\varphi}^{2\varphi} (s_2 - \varphi) f(s_2) ds_2 = \int_{0}^{\varphi} (\varphi - s_2) f(s_2) ds_2,$$

for all densities $f(\cdot)$ with support $(0, 2\varphi)$ satisfying the aggregation condition (2). Moreover, any positive measure of signal above 2φ would lead to a strictly lower price for firm 2. This in turn establishes Proposition 6 for the duopoly case and the argument can be generalized for an arbitrary number of firms (see the Appendix).

Two further comments about the equilibrium displayed in Proposition 6 are worth mentioning. First, as F increases, the corresponding distribution of signals becomes more concentrated around φ (so for this equilibrium, more competition eventually induces financial reports that get close to reporting the fundamental value with probability 1). Second, the equilibrium shown in Proposition 6 suffers from the following fragility. While the equilibrium requires that firms choose a distribution with continuous density, an obvious alternative (and simpler) best-response would be for the firms to choose a distribution putting mass 1 on the fundamental value. Yet, if firms were to choose such a financial reporting strategy, this would not be an equilibrium (see Proposition 3).

6 Discussion

We wish to address the following questions in this section: 1) Are our insights robust to modifications of the basic model? 2) Is bounded rationality needed after all, or more precisely what insights would have to go if investors were fully rational? 3) How are the insights affected if investors employ alternative heuristics and/or if firms have alternative objectives? We divide the investigations of these questions into several subsections.

6.1 Robustness

In this subsection, we consider more general specifications of demand and supply for investors, the case of heterogeneous firms (allowing for asymmetric and/or stochastic fundamental values), the case of heterogeneous investors (allowing investors to differ in their sampling size K), the case in which the financial report distribution must be bounded from above.

6.1.1 More general demand and supply

Assuming that investors can only trade one stock is, of course, special. Suppose more generally that trade orders are a smooth function of the perceived gains from trade.

To start with, consider the monopoly case $F = 1$ with a sample size $K = 1$. Letting $\hat{x}_i$ and p denote, respectively, the perceived value and price of the stock, we denote by $f(|\hat{x}_i - p|)$ the demand (resp. supply) for the stock if $\hat{x}_i - p > 0$ (resp. $\hat{x}_i - p < 0$), and we assume that $f(\cdot)$ is a smooth (in particular, continuous) function, thereby implying that $f(0) = 0$.

Given $f(0) = 0$, it is clear that it would not be possible to achieve a price equal to the highest signal as in Proposition 1 (since at that price there would be no demand). It is also clear that if $f(\cdot)$ is linear everywhere the price would correspond to the average of investors' beliefs and that would equal the fundamental value.[33] As we now show, however, the firm can achieve a price strictly above the fundamental whenever $f(\cdot)$ is non-linear (at least somewhere between 0 and φ). Suppose for example that $f(\cdot)$ is strictly concave and the firm chooses the distribution $\{\varphi - \varepsilon - \delta, 1/2 - \nu; \varphi + \varepsilon, 1/2 + \nu\}$, where ε, δ and ν are positive. At $p = \varphi$, the demand would be $(1/2 + \nu)f(\varepsilon)$ while the supply would be $(1/2 - \nu)f(\varepsilon + \delta)$. By concavity of $f(\cdot)$, the demand would exceed the supply and so market clearing would require $p > \varphi$. More generally, we have:

Proposition 7 *Suppose that $K = F = 1$ and $f(\cdot)$ is strictly concave or strictly convex in a neighborhood $W \subseteq [0, \varphi]$. The firm can achieve a price $p > \varphi$.*

While the above Proposition establishes that the kind of distortions exhibited in the monopoly case would continue to hold whenever the demand is not linear in the perceived gains from trade, we believe that there are many practical reasons why such non-linearities could arise, for example due to wealth and short selling constraints (in that investors are not always able to trade as much as they wish of a given firm), or due to trading costs (in that investors may not trade when perceived gains are too small).[34]

Consider next the oligopoly case with $F > 1$, and assume that $K = 1$ as in Section 5. An important observation is that in the equilibrium considered in

[33] In fact, for any distribution $\{x_1, \mu_1; x_2, \mu_2; ..\}$ chosen by the firm, market clearing requires $\sum_n \mu_n (x_n - p) = 0$, and since by (2) $\sum_n \mu_n x_n = \varphi$, we must have $p = \varphi$.

[34] In such more general formulations, the trade limit may depend on the stock price. Apart from making the algebra a bit more cumbersome, such a modification would not change much the logic developed above.

Proposition 5, both buyers and sellers perceive the same gain from trade (that is, p^*). Thus, it is unlikely that our main result that mispricing may increase with competition would be much affected if demand and supply were a smooth function of perceived gains from trade.[35]

6.1.2 Asymmetric and/or stochastic fundamentals

We have so far assumed that all firms have the same fundamental value φ, which is deterministic and commonly known among firms. Extending the definitions of the equilibrium to the cases of asymmetric and/or stochastic fundamentals raises no difficulties. We now show that the logic of our analysis extends to these cases.

Clearly, in the monopoly case, nothing would be changed by allowing the fundamental φ to be randomly determined. For each realization of φ, the obtained price would be the one derived above for this value of the fundamental. More challenging though is the analysis of competition when fundamental values may be asymmetric and/or stochastically determined.

Consider first the case of asymmetric (though deterministic) fundamentals. Firm j has fundamental value $\varphi^j = \varphi + \varepsilon^j$ and we assume that $0 \leq \varepsilon^1 \leq \varepsilon^2 \leq ...\varepsilon^F$ (without loss of generality). The following Proposition identifies an equilibrium in the same spirit as the one described in Proposition 5, in which $\mu^*(F)$ and $p^*(F)$ are defined as in (5) and (7) by $\mu^*(F) = 1 - (1/2)^{1/F}$ and $p^*(F) = \varphi/2\mu^*(F)$. This equilibrium requires that heterogeneity among firms is not too large; in particular, as detailed below, it requires

$$\varepsilon^F \leq \bar{\varepsilon}(F), \tag{8}$$

where

$$\bar{\varepsilon}(F) \equiv \begin{cases} \varphi\sqrt{2}/2 & \text{for} \quad F = 2, \\ 2(p^* - \varphi) & \text{for} \quad F > 2. \end{cases}$$

Proposition 8 *Suppose firm j has fundamental value $\varphi^j = \varphi + \varepsilon^j$, and assume that $K = 1$ and $\varepsilon^F \leq \bar{\varepsilon}(F)$. There is an equilibrium in which $\sigma^j = \{\varepsilon^j, 1 - \mu^*(F); 2p^*(F) + \varepsilon^j, \mu^*(F)\}$ and the prices are $p^j = p^*(F) + \varepsilon^j$ for all j. These prices increase in F.*

[35] In Proposition 5, the market clearing condition would not be affected by the specific form of the function $f(\cdot)$. While this is not enough to conclude that the equilibrium would be unchanged, we expect that the logic would be robust. Our baseline specification seems to make deviations easier, in that the deviating firm is allowed to offer little gains to buyers while making sellers very keen on selling, and that pushes the price closer to high signals. As we show that even in this case there is no profitable deviation, we conjecture that profitable deviations would not exist even in the more general setting.

Given that $\bar{\varepsilon}(F)$ grows arbitrarily large as F increases, if for all i, $\varepsilon^i < \tilde{\varepsilon}$ for some constant $\tilde{\varepsilon}$, then for F large enough, it must be that $\varepsilon^F < \bar{\varepsilon}(F)$ and thus Proposition 8 applies. Moreover, in this equilibrium, adding more firms has the effect of increasing $p^*(F)$ and thus the price of all firms.

Compared to Proposition 5, the main difference is that firm j may consider using signals lower than ε^j.[36] The reason for the extra condition (8) can be understood as follows. First, firm j with fundamental value $\varphi^j = \varphi + \varepsilon^j$ can deviate and choose the distribution $\{0, 1/2; 2\varphi + 2\varepsilon^j, 1/2\}$. At prices $p^j = 2\varphi + 2\varepsilon^j - p^*$ and $p^r = p^*$ for $r \neq j$, firm j would attract all investors and the market would clear. For this to be unprofitable, it should be that $p^* + \varepsilon^j \geq 2\varphi + 2\varepsilon^j - p^*$, which must hold in particular for the most profitable firm, thereby explaining that $\varepsilon^F \leq 2(p^*(F) - \varphi)$ is required. Second, when $F = 2$, a deviation of firm j to $\{0, \mu^*; p^j, 1 - \mu^*\}$ would induce prices $p^j = (\varphi + \varepsilon^j)/(1 - \mu^*)$ and $p^r = \varepsilon^r$ for $r \neq j$. For this to be unprofitable, it should be that $p^*(F) + \varepsilon^j \geq \sqrt{2}(\varphi + \varepsilon^j)$, which for the most profitable firm writes $\varepsilon^F \leq \varphi\sqrt{2}/2$. It turns out that all deviations are taken care of when $\varepsilon^F < \bar{\varepsilon}(F)$, as defined above.

It should be noted that in the equilibrium of Proposition 8, the strategy of firm j depends only on her own fundamental value φ^j. This has nice implications for the case in which the fundamental values would be stochastically drawn. Indeed, assume that the fundamental value φ^j of firm j, $j = 1...F$, is now stochastically drawn from a distribution with support $[\varphi, \varphi + \bar{\varepsilon}(F)]$, and that only firm j knows the realization of φ^j. Define for each firm j receiving the fundamental value φ^j the strategy $\sigma^j(\varphi^j) = \{\varepsilon^j, 1 - \mu^*(F); 2p^*(F) + \varepsilon^j, \mu^*(F)\}$ where $\varepsilon^j = \varphi^j - \varphi$, together with the price $p^j(\varphi^j) = p^*(F) + \varepsilon^j$. Because such strategies constitute an ex-post equilibrium (i.e. they remain in equilibrium after the realization of all fundamental values are known), we have:

Proposition 9 *The above strategies and prices are part of a Bayes-Nash equilibrium whatever the joint distribution of fundamentals.*

Together Propositions 8 and 9 show that our main results regarding the destabilizing effect of competition is robust to the introduction of asymmetries and randomness in firms' fundamental values.

[36] If such signals were not allowed, the proof of Proposition 8 would be identical to the proof of Lemma 2 (with no further requirement on how large ε^F is).

6.1.3 Heterogeneous sampling procedures

In the main analysis, all investors were assumed to have the same K, and in the competitive case ($F > 1$), we assumed that $K = 1$. What if investors can have different K? Dealing in general with the case of heterogeneous populations is quite involved. We consider now the special case in which investors are either fully rational ($K = \infty$) with probability α or they are assumed to follow the K-sampling procedure (with $K < \infty$) with probability $1-\alpha$. In line with our baseline model, we assume that investors whether fully rational or boundedly rational can trade only one stock, and that the fundamental values of all firms are the same.

Given that the fundamental value of each firm is deterministic, rational investors know it with certainty. Given that the price is typically above the fundamental value, rational investors would all go for short selling. Our equilibrium constructions of Sections 3, 4, 5 should then be modified by adding a fraction α to the aggregate supply. Yet, it is not difficult to show that, provided α is not too large, our previous insights carry through.[37]

Regarding the competitive case ($F > 1$), dealing with $K > 1$ is a bit cumbersome (even when all investors have the same K). Yet, we conjecture that similar insights obtain, and, in particular, more competition may still drive the price further away from the fundamental in the $K > 1$ case.[38]

6.1.4 Upper bound on firms' reports

In our baseline model, firms were free to report signals with arbitrarily large values. One may question how our results would be affected if we were to impose an upper bound on firms' distribution (as derived for example by a regulatory constraint whereby firms which appear too profitable in some dimension would be subject to investigation). As already mentioned, the logic developed in Proposition 2 would not hold in this case, and a monopolistic firm would not be able to obtain a price much larger than φ when the sample size K gets arbitrarily large. By contrast, and perhaps more interestingly, the result that competition need not eliminate and may even increase mispricing would be preserved.

[37] To see this, one can apply the same analysis as above and just modify the market clearing conditions. In Section 4, market clearing would require $(1-\mu)^{K^*}(1-\alpha)+\alpha = 1/2$. In Section 5, it would require $(1-\mu)^F(1-\alpha)+\alpha = 1/2$.

[38] Following the logic of our previous analysis, we conjecture that, for any F, there is a market clearing price which is bounded away from the fundamental no matter how large K is. Moreover, irrespective of K, we conjecture that the market clearing price can grow arbitrarily large as F grows large. (To get a sense of this, repeat the arguments of Sections 4 and 5 with the distribution $\sigma = \{0, 1-\mu; h, \mu\}$, where μ is defined by $(1-\mu)^{FK} = 1/2$.)

To see this, assume that the support of the distribution used by firm j must be in the range $[0, H]$.[39] If $H \geq \varphi/\mu^*(F)$, where $\mu^*(F)$ is defined in (5), then the upper bound does not bind and the previous analysis applies. Consider then $H < \varphi/\mu^*(F)$ and assume that $H \geq \tilde{H}$, where

$$\tilde{H} = \max\left\{\frac{\varphi}{1 - 2\mu^*(F)}, 2\varphi\right\}. \tag{9}$$

As we show in the next Proposition, the highest symmetric equilibrium price in this case is achieved with the distribution $\sigma_H = \{l, 1 - \mu^*(F); H, \mu^*(F)\}$ in which the tie-breaking rule is anonymous and most favorable to demand and μ^* is defined as in (5) by the market clearing condition $(1 - \mu^*)^F = 1/2$.[40] This price is

$$p_H(F) = \frac{H + l(F)}{2}, \tag{10}$$

where due to the aggregation condition (2)

$$l(F) = \frac{\varphi - \mu^*(F)H}{1 - \mu^*(F)}.$$

By the same logic as that explained in Section 5, μ^* decreases in F, which allows to increase l and so p_H. In the limit as F gets arbitrarily large, $l(F)$ gets close to φ and $p_H(F)$ converges to $(\varphi + H)/2$. That is, as in our previous analysis, the maximal equilibrium price increases in the number of competing firms, but now the maximum price never goes beyond $(\varphi + H)/2$. We collect these observations in the following Proposition.

Proposition 10 *Suppose that signals must be in the range $[0, H]$ and $H \geq \tilde{H}$, as defined in (9). The maximal price achieved in a symmetric equilibrium increases in F, and converges to $(\varphi + H)/2$ as F gets arbitrarily large.*

6.2 Is bounded rationality needed?

A key aspect of our model is the heuristic used by investors to assess the values of firms. What if investors are rational instead? More precisely, as in our main

[39] Removing the lower bound on X^j would make our problem trivial, as firms would be always able to reach a price equal to H irrespective of K and F.

[40] As detailed in the proof of Proposition 10, the condition $H \geq 2\varphi$ ensures that σ_H induces a higher price than a three signals distribution which puts positive mass on signals $0, s$ and H and induces a price $(s + H)/2$. The condition $H \geq \varphi/(1 - 2\mu^*)$ ensures that σ_H induces a higher price than a two signals distribution which puts positive mass on signals 0 and h and induces a price h.

model, assume that (1) Firms choose a report distribution σ in Σ; (2) Each investor makes K independent draws from the distribution of each firm; (3) Based on the draws and the prices, investors decide whether to buy or short sell one stock of their choice, and (4) Prices are determined so as to clear the markets. By contrast to our baseline model, assume now that investors are rational and thus make the correct inferences (about fundamental values) from the signals they receive and the levels of prices (as in the models of Grossman and Stiglitz (1980), say).

Clearly, in our basic setup in which the fundamental value is deterministically set at φ, the inference problem is somehow trivial, and no matter what strategy firms use and no matter what signals investors receive, any investor if rational should know the value of the firms is φ. But, consider the extension discussed in Section 6.1 in which the fundamental values of firms may be stochastically determined. Now the signals received by investors may be informative, and the inference problem is non-trivial. Note though that our setup is one in which one can apply the no-trade theorem (Milgrom and Stokey (1982)) given that trades occur purely for speculative reasons. Thus, if investors are fully rational and make their inferences based on a common prior about the distributions of fundamental values, there can be no trade in equilibrium irrespective of the reporting strategy chosen by the firms, or more precisely if there is trade on some stock it can only be at a price equal to the fundamental value of the corresponding firm.

Starting from this classic no trade result, one may consider various modifications of the basic setup that would restore the possibility of trade. One modification is to move away from the common prior assumption, allowing investors to have diverging priors (or opinions). As already mentioned, one can interpret our approach along such lines with the twist that in our model the subjective prior is directly shaped by the financial reports chosen by the firms. To the best of our knowledge, the literature on subjective prior has never considered such an endogenous approach to the prior formation.

Another modification consists in keeping the common prior assumption but allow the possibility of noise traders (as in Grossman and Stiglitz (1980)). It should be noted that the literature on noise traders has mostly considered the case in which the distribution from which investors get their signal is exogenously set.[41] Compared to that literature, our investigation would require endogenizing the distributions from which investors receive their signal, since these distributions

[41] This distribution generally takes the form of a normal distribution which, together with CARA preferences, allows to express demand and supply as linear functions of the signal received and the price.

are strategically chosen by the firms in our model. As far as we know, such extensions of the basic financial market setup have not been dealt with in the literature. Moreover, there could be various modelling of the disclosure choice of firms, and these are subject to active research on their own.[42] The complexity of this together with that of aggregation of information in financial markets makes it challenging to consider a model that would mix all this, as required for our purpose.

The above discussion even if incomplete should make it clear that there is no obvious and natural way to reproduce our main insights using an existing model with fully rational investors. We hope this helps appreciate why we think the heuristic we assume on investors' behaviors, which we find plausible from a behavioral viewpoint, is key in driving our results about mispricing in financial markets.

6.3 Further extensions

6.3.1 On alternative heuristic procedures

Our assumed heuristic reflects a general tendency agents have about extrapolating from small samples, and our aim was to investigate the impact this could have in financial markets. Yet, there are alternative ingredients that could be added to the considerations of investors. For example, investors could consider that the price itself is indicative of the fundamental value. Alternatively, investors could base their estimate of the fundamental value not only on the part of the financial report they pay attention to but also on the market sentiment (De Long, Shleifer, Summers and Waldmann (1990a)). Finally, investors could take into account that their estimate is noisy and adjust their investment decision accordingly.

There are several possible ways to incorporate such ideas into the heuristic of investors. We discuss now some of these.

In the main model, investor i made an estimate of the fundamental value of firm j based on the average sample signal $\hat{x}_i^j$ from j. Suppose instead that investor

[42] For example, one can assume that firms can commit ex ante (before they know their realized fundamental value) to whatever disclosure sounds best (as in Kamenica and Gentzkow (2011), Rayo and Segal (2010) or Jehiel (2011)) or alternatively that the disclosure strategy is chosen at the time the firms know the realization of their fundamental model (as in models of cheap talk, à la Crawford and Sobel (1982)), or else that firms can only lie by omission (as in Grossman (1981) or Milgrom (1981) or Shin (2003) for a finance application).

i assesses the fundamental value of firm j according to

$$v_i^j = \lambda_i p^j + (1 - \lambda_i)\hat{x}_i^j, \tag{11}$$

where $\lambda_i \in [0, 1)$ reflects the subjective weight attached by investor i to the informativeness of the price relative to the informativeness of the private signal $\hat{x}_i^j$. Trading j would be assessed to give gains of $\left|p^j - v_i^j\right| = (1-\lambda_i)\left|p^j - \hat{x}_i^j\right|$ and thus, our previous analysis would apply equally to this new specification. In a richer model, the weight λ_i could be derived endogenously (and it could a priori depend on j as well). The (specific) reduced-form approach in (11) is taken to illustrate that introducing some coarse inference from the price need not change the logic developed above.

The idea of "market sentiment" can be modelled as the average belief of the various investors about the profitability of the firm. Given that as already noted, the mean of the financial reporting distribution has to coincide with the fundamental, the average belief about firm j corresponds to its fundamental value. Thus, an investor i receiving an average sample $\hat{x}_i^j$ from j would assess firm j according to

$$v_i^j = \tau\varphi + (1 - \tau)\hat{x}_i^j$$

where $\tau \in [0, 1)$ represents the weight given to the market sentiment. The gains from trade attached to asset j would be perceived as $\left|p^j - \tau\varphi - (1 - \tau)\hat{x}_i^j\right|$, and the main messages of our previous analysis would remain qualitatively the same.[43]

Finally, we could incorporate the idea that investors would take into account that their estimate of the fundamental value is noisy. For example, when investors draw several signals $K > 1$, instead of simply considering the mean of the signal and reason as if it were the fundamental value, investors could also consider the empirical variance in the sample and reason as if the fundamental value was a random variable normally distributed with mean and variance coinciding with the corresponding empirical values in the sample. With risk neutral investors (as we assumed) this would have no consequence. With risk-averse investors, it is not clear a priori in which way our main analysis would change given that both buying and short selling would be perceived as equally risky. A more systematic investigation of such heuristics should be the subject of future work.

[43] As K grows large in the monopoly case, the price would be bounded by $\tau\varphi + (1-\tau)\varphi/\ln 2$, which exceeds φ. In the oligopoly case with $K = 1$, as F grows large, the maximal price sustainable in equilibrium would be $\tau\varphi + (1-\tau)p^*$, where p^* denotes the price characterized in Section 5.

6.3.2 Alternative objectives for firms

In our main model, firms were assumed to maximize the price of their stocks. We think this is a natural objective for managers whose compensation is to a non-negligible extent indexed on the value of the stock. Yet, there may be other motives driving the choice of financial reporting strategy. For example, reputation considerations may lead firms to look as transparent as possible, which would clearly alleviate the distortions highlighted in this paper. Alternatively, managers may try to look better than their competitors.

To investigate the effect of the relative performance criterion, assume there are two firms, $F = 2$, that the sample size of investors is $K = 1$, and that each firm j seeks to maximize $p^j - p^{-j}$.[44] This modified objective function leads to a different game between firms, that is a symmetric zero-sum game whose value is 0 (since picking the same strategy as the competitor gives in expectation 0).

Yet, knowing the value of the game does not tell us what the resulting price of stocks is. Exploiting the analysis of Section 5, we now observe that there is an equilibrium in which the price coincides with the fundamental $p = \varphi$, which is sustained by having the two firms using a uniform distribution between 0 and 2φ, that is, $\sigma_R \sim U(0, 2\varphi)$. This follows from the proof of Proposition 6, in which we show that if firm j chooses σ_R, then we have $p^j = \varphi$ and $p^{-j} \leq \varphi$ for any distribution chosen by firm $-j$. Hence, no firm can get a price higher than the competitor when the competitor chooses σ_R.[45]

Proposition 11 *Suppose $F = 2$ and firms seek to maximize relative price. There is a symmetric equilibrium in which firms choose the uniform distribution on $(0, 2\varphi)$ and the price of stocks is φ.*

To the extent that the equilibrium shown in Proposition 11 is the only one (which we conjecture to be the case), changing the objective of managers from absolute to relative stock price would eliminate the mispricing of firms in this symmetric duopoly setting.

[44]Condition (b) in the definition of equilibrium should be modified accordingly by requiring that there is no distribution $\tilde{\sigma}^j \in \Sigma$, prices $\tilde{p}^j, \tilde{p}^{-j}$, and tie-breaking rule $\tilde{\omega} \in \Omega$ such that $D(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega}) = S(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega})$ and $\tilde{p}^j - \tilde{p}^{-j} > p^j - p^{-j}$.

[45]Somehow unexpectedly, the same uniform distribution appears as an equilibrium in the Blotto game with a continuum of battle fields (see Gross and Wagner (1950) for an early result and Myerson (1993) for an application to political contests). While the aggregation condition is clearly the same in the two problems, the market clearing condition seems to have no analog in the Blotto game, making the connection unexpected.

7 Conclusion

This paper has considered a stylized financial market in which firms strategically frame their financial reports so as to influence investors' beliefs and induce higher stock prices. We have illustrated how the introduction of less sophisticated, extrapolative investors in such a setting could alter dramatically the analysis of market efficiency. We have shown that a form of investor protection requesting that overall there should be no lie in the financial reporting need not restore market efficiency. Moreover, capital market competition has been shown to be ineffective in ensuring that prices are close to fundamentals.

Our model is obviously stylized and open to several extensions. In particular, it would be interesting to explore more generally the incentives to manipulate beliefs as a function of which investors -along the distribution of beliefs- are key to determine the market price. Another interesting extension would be to add a time component to the belief formation given that some forms of accounting manipulation occur over time. Finally, future research may also explore the empirical implications of the model. Our results suggest that the complexity of information provided by firms should be positively correlated to investors' disagreement and to trading prices. According to Proposition 5, the effect is likely to be stronger in settings in which many firms compete for investors' trades. To our knowledge, this link remains to be explored empirically.

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8 Appendix

Proof of Proposition 1

As shown in the text, the price $p_M = 2\varphi$ clears the market when the firm sends the distribution $\sigma_M = \{0, 1/2; 2\varphi, 1/2\}$. We now show that no distribution induces a higher price. Suppose that the firm sends the distribution $\sigma = \{x_0, \mu_0; x_1, \mu_1; x_2, \mu_2; ..; x_N, \mu_N\}$ with $0 = x_0 < x_1 < x_2 < ... < x_N$ and $\mu_n \geq 0$ for $n = 0, ..., N$. (We consider a discrete distribution for simplicity of notation, the argument is the same if we consider continuous distributions.) Market clearing requires that the price is the median of the distribution. If there are several medians (because of the discreteness of the distribution), then considering the largest median is enough to characterize the largest market clearing price. Thus, we let $p = x_N$ if $\mu_N \geq 1/2$; $p = x_{N-1}$ if $\mu_N < 1/2$ and $\mu_N + \mu_{N-1} \geq 1/2$; and more generally for $n \in [1, N-1]$

$$p = x_n, \text{ if } \sum_{w=0}^{N-n-1} \mu_{N-w} < 1/2 \text{ and } \sum_{w=0}^{N-n} \mu_{N-w} \geq 1/2.$$

Maximizing x_n while satisfying the above constraints and the constraint in (2) requires setting $\mu_w = 0$ for all $w \in [1, n-1]$. Moreover, by setting $x_w = x_n$ for all $w \in [n+1, N]$, x_n can be increased, and so p can be increased, while still satisfying condition (2). Hence, we are left with a distribution $\sigma = \{0, 1-\mu; x_n, \mu\}$ with $\mu \geq 1/2$. Condition (2) requires $x_n \leq \varphi/\mu$. As we need $\mu \geq 1/2$ to have $p > 0$, it follows that $x_n \leq 2\varphi$. Thus, no alternative distribution can induce a price higher than p_M. **Q. E. D.**

Proof of Lemma 1

Part 1. We first show that the distribution σ cannot induce the highest price if there is a signal $x > 0$ which is in the support of σ (that is, to which the distribution σ assigns positive mass) and such that signal $\tilde{x} = 2p - x$ is not in the support of σ. Consider the equilibrium profile $\{\sigma, p, \omega\}$, where p is the market clearing price and ω is an anonymous tie-breaking rule, and suppose by contradiction that σ assigns mass $\mu_x > 0$ to signal x and no mass to signal $\tilde{x}$. Suppose first that $x > p$ and there is a signal $\hat{x} \in [p, x)$ such that σ assigns mass $\mu_{\hat{x}} > 0$ to $\hat{x}$ and no mass to any other signal between x and $\hat{x}$. Consider the alternative distribution $\tilde{\sigma}$ in which the mass μ_x is moved to signal $\hat{x}$. Under the original distribution σ, the demand for each firm can be written as

$$D = \mu_x \sum_{y=0}^{F-1} \binom{F-1}{y} \frac{1}{F-y} (\mu_x)^{F-1-y} (\mu_{\hat{x}} + \mu_z)^y + \\ \mu_{\hat{x}} \sum_{y=0}^{F-1} \binom{F-1}{y} \frac{1}{F-y} (\mu_{\hat{x}})^{F-1-y} (\mu_z)^y + W,$$

where μ_z denotes the total mass of signals that are at a distance to the price smaller than $\hat{x}$ is, $\mu_z = \sum_{\{n \text{ s.t. } 2p-\hat{x}<x_n<\hat{x}\}} \mu_n$, and W is unaffected by the proposed change in the distribution. The demand for each firm under the new distribution $\tilde{\sigma}$ can be written as

$$\tilde{D} = (\mu_x + \mu_{\hat{x}}) \sum_{y=0}^{F-1} \binom{F-1}{y} \frac{1}{F-y} (\mu_x + \mu_{\hat{x}})^{F-1-y} (\mu_z)^y + W.$$

Notice that the supply of each firm is unaffected by the proposed change in the distribution since signal $\tilde{x} = 2p - x$ is not part of σ. Hence, given that $D = \tilde{D}$, the same market clearing price p can be attained with the new distribution $\tilde{\sigma}$. At the same time, the distribution $\tilde{\sigma}$ has a lower mean than σ, the difference being $\mu_x(x - \hat{x})$. This allows to increase all signals in $\tilde{\sigma}$ and so the price by the same amount, thereby showing that the distribution σ did not induce the highest price.

If there is no signal between x and p the same argument applies by moving the mass μ_x to p. If $x < p$, the argument is symmetric and the proposed change is to move mass μ_x to the highest signal $\hat{x} < x$ in the support of σ or to zero if x is the lowest signal in the support of σ. Hence, in what follows, we can restrict our attention to distributions in the set $\hat{\Sigma}$ of distributions such that $\widehat{\sigma} \in \Sigma$ (as defined by conditions (1) and (2)) and all positive signals in $\widehat{\sigma}$ are paired around some p interpreted as the price.

Part 2. We now show that firms cannot attain a price larger than p^* with any distribution $\sigma \in \hat{\Sigma}$. Suppose all firms choose the same distribution $\sigma \in \hat{\Sigma}$ and consider a symmetric tie-breaking rule. Denote by p the market clearing price. Suppose that σ assigns positive mass to $2n+1$ signals, 0, $x_1^-, .., x_n^-, x_n^+, .., x_1^+$ with $0 \leq x_1^- < .. < x_n^- < p < x_n^+ < .. < x_1^+$ and $x_t^+ + x_t^- = 2p$ for all $t = 1, .., n$. Suppose there are also atomless parts of the distribution over the intervals $[a_1^-, b_1^-]$ and $[b_1^+, a_1^+]; ...; [a_v^-, b_v^-]$ and $[b_v^+, a_v^+]$, where $0 \leq a_1^- < b_1^- < .. < a_v^- < b_v^- < p < b_v^+ < a_v^+ < .. < b_1^+ < a_1^+$ and $a_t^+ + a_t^- = b_t^+ + b_t^- = 2p$ for all $t = 1, .., v$. In steps 1-4, we show that one can induce a price $\hat{p} \geq p$ by using a distribution with at most two signals. In step 5, we show that no distribution with at most two signals induces a price higher than p^*, as defined in (7). We conclude that p^* is the maximal market clearing price when firms choose a distribution $\sigma \in \hat{\Sigma}$.

Step 1. Consider signal x_n^-, x_n^+. Suppose $\mu_{x_n^+} \geq \mu_{x_n^-}$ and $b_n^- < x_n^-$; that is, there is no atomless part of the distribution at a lower distance from the price (we consider the atomless parts of the distribution in step 3 below). Define as $\hat{X}$ the set of signals x in the support of the distribution such that there exists a signal $2p - x$ in the support of the distribution, that is

$$\hat{X} = \left\{x \in \sigma : x \geq \min\left\{x_1^-, a_1^-\right\}\right\},$$

and denote by μ_0 the weight attached by σ to signal 0. Then one can induce a price $p + \Delta_1$, where $\Delta_1 \geq 0$ will be defined below, by first moving x_n^+ and x_n^- to p and then moving all signals $x \in \hat{X}$ up by Δ_1. To show this, we first show that by moving x_n^+ and x_n^- to p one can induce the same market clearing price p and employ a signal distribution whose mean is lower than φ. Then, we can move all signals $x \in \hat{X}$ up by Δ_1 to obtain a price $p + \Delta_1$ with a signal distribution in $\hat{\Sigma}$ whose average is φ.

Suppose firms assign weight $\mu_{x_n^+} + \mu_{x_n^-}$ to signal p instead of assigning weights $\mu_{x_n^+}$ and $\mu_{x_n^-}$ to signals x_n^+ and x_n^-, respectively. Those who sample signal p for all firms are indifferent between buying and selling. Denote by τ_1 the fraction of them who buy. Suppose first that, before the change in the distribution, whenever an investor sampled signal x_n^+ from firm j and signal x_n^- from firm $\hat{\jmath}$ he bought stock j. The old aggregate demand is

$$D_1 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_n^+})^y (\mu_{x_n^-})^{F-y} + Z_1,$$

where Z_1 depends on the signals further away from p and is unaffected with the proposed change. The new aggregate demand (after the change) is

$$\hat{D}_1 = \tau_1(\mu_{x_n^+} + \mu_{x_n^-})^F + Z_1.$$

Hence, one can define a $\tau_1 < 1$ such that $\hat{D}_1 = D_1$ and so the market clears at p. Similarly, if an investor sampling signal x_n^+ from firm j and signal x_n^- from firm $\hat{\jmath}$ sold stock $\hat{\jmath}$, the old demand is $\tilde{D}_1 = (\mu_{x_n^+})^F + Z_1$ and there exists a $\tau_1 < 1$ such that $\hat{D}_1 = D_1$. Notice that $\mu_{x_n^+} x_n^+ + \mu_{x_n^-} x_n^- \geq (\mu_{x_n^+} + \mu_{x_n^-})p$ since $\mu_{x_n^+} \geq \mu_{x_n^-}$ and by definition $x_n^- = 2p - x_n^+$. Hence, we can define

$$\Delta_1 = \frac{1}{1-\mu_0}[\mu_{x_n^+} x_n^+ + \mu_{x_n^-} x_n^- - (\mu_{x_n^+} + \mu_{x_n^-})p],$$

and move all signals $x \in \hat{X}$ up by Δ_1 so as to satisfy condition (2) and have a price $p + \Delta_1$. The resulting distribution still belongs to $\hat{\Sigma}$. The same logic will be applied in the next steps.

Step 2. The procedure in step 1 can be repeated until one considers signals x_m^-, x_m^+ where $m \equiv max_t \left\{ t : \mu_{x_t^+} < \mu_{x_t^-} \right\}$ (if $\mu_{x_n^+} < \mu_{x_n^-}$, then $m = n$), or until one encounters an atomless part of the distribution at a lower distance from the price. Suppose one ends up with weight μ_{p_2} on signal p_2 and market clearing requiring that a fraction τ_2 of those who sample signal p_2 for all firms buy. Consider first x_m^-, x_m^+. Following the same logic as in step 1, one can move x_m^- to x_{m-1}^- and x_m^+ to x_{m-1}^+ and then move all signals $x \in \hat{X}$ up by some $\Delta_2 \geq 0$ so as to induce a price $p_2 + \Delta_2$.

Consider the following weights for x_{m-1}^-, x_{m-1}^+ and p_2, respectively: $\hat{\mu}_{x_{m-1}^-} = \mu_{x_m^-} + \mu_{x_{m-1}^-} - k_2$, $\hat{\mu}_{x_{m-1}^+} = \mu_{x_m^+} + \mu_{x_{m-1}^+} - k_2$, and $\hat{\mu}_{p_2} = \mu_{p_2} + 2k_2$. Suppose a share τ_m of those who sample signal p_2 for all firms buy. We wish to define a $k_2 \in (0, \mu_{x_m^+})$ and a $\tau_m \in (0, 1)$ such that p_2 clears the market. Suppose first that whenever an investor samples signal x_m^+ from firm j and signal x_m^- from firm $\hat{\jmath}$ he buys stock j and similarly for signals x_{m-1}.[46] The pre-change aggregate demand is

$$D_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x^+_{m-1}})^y (\mu_{x^-_{m-1}} + \mu_{x^-_m} + \mu_{x^+_m} + \mu_{p2})^{F-y} + \sum_{y=1}^{F} \binom{F}{y} (\mu_{x^+_m})^y (\mu_{x^-_m} + \mu_{p2})^{F-y} + \tau_2(\mu_{p2})^F + Z_2.$$

The new aggregate demand (considering the same symmetric tie-breaking rule after the change of distribution) is

$$\hat{D}_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x^+_{m-1}} + \mu_{x^+_m} - k_2)^y (\mu_{x^-_{m-1}} + \mu_{x^-_m} + \mu_{p2} + k_2)^{F-y} + \tau_m(\mu_{p2} + 2k_2)^F + Z_2.$$

Using the binomial theorem and the convexity of $x \to x^F$ for $F \geq 2$, one can see that $\hat{D}_2 > D_2$ when $k_2 = 0$ and $\tau_m = 1$ and conversely $\hat{D}_2 < D_2$ when $k_2 = \mu_{x^+_m}$ and $\tau_m = 0$. Hence, there exists a $k_2 \in (0, \mu_{x^+_m})$ and a $\tau_m \in (0,1)$ such that $\hat{D}_2 = D_2$. A similar argument can be applied in the case that, before the change, whenever an investor sampled signal x^+_m from firm j and signal x^-_m from firm $\hat{j}$ he sold stock $\hat{j}$ and similarly for signals x_{m-1}. Now one can move all signals $x \in \hat{X}$ up by Δ_2, where

$$\Delta_2 = \frac{1}{1-\mu_0}[\mu_{x^+_m} x^+_m + \mu_{x^+_{m-1}} x^+_{m-1} + \mu_{x^-_m} x^-_m + \mu_{x^-_{m-1}} x^-_{m-1} - (\mu_{x^+_m} + \mu_{x^+_{m-1}} - k_2) x^+_{m-1} - (\mu_{x^-_m} + \mu_{x^-_{m-1}} - k_2) x^-_{m-1} - 2p_2 k_2],$$

and $\Delta_2 \geq 0$ since $\mu_{x^-_m} \geq \mu_{x^+_m}$, so as to satisfy condition (2) and have a price $p_2 + \Delta_2$.

Step 3. Suppose one encounters an atomless part of the distribution and there is no other signal at a lower distance from the price. Suppose the price is p_3 and consider the distribution with density $g(x)$ over the interval $[a^-_n, b^-_n]$ and density $h(x)$ over $[b^+_n, a^+_n]$. The logic of the previous steps can be applied by dividing the intervals $[a^-_n, b^-_n]$ and $[b^+_n, a^+_n]$ into sufficiently small subintervals.

[46] We can wlog assume the indifferences are broken in the same way when x^-_m vs x^+_m or x^-_{m-1} vs x^+_{m-1} are drawn by satiating demand in one or the other.

Consider first the intervals $[b_n^- - \varepsilon, b_n^-]$ and $[b_n^+, b_n^+ + \varepsilon]$, where ε is small. Define

$$\mu^+ = \int_{b_n^+}^{b_n^+ + \varepsilon} h(x)dx, \text{ and } x^+ = \frac{1}{\mu^+} \int_{b_n^+}^{b_n^+ + \varepsilon} xh(x)dx,$$

and similarly

$$\mu^- = \int_{b_n^- - \varepsilon}^{b_n^-} g(x)dx, \text{ and } x^- = \frac{1}{\mu^-} \int_{b_n^- - \varepsilon}^{b_n^-} xg(x)dx.$$

If $\mu^+ > \mu^-$ and $\varepsilon \to 0$, one can obtain a larger price by moving all signals $x \in [b_n^- - \varepsilon, b_n^-] \cup [b_n^+, b_n^+ + \varepsilon]$ to p_3 (following the logic of Step 1). If $\mu^+ < \mu^-$ and $\varepsilon \to 0$, it is profitable to move all signals $x \in [b_n^- - \varepsilon, b_n^-]$ to $b_n^- - \varepsilon$ and all $x \in [b_n^+, b_n^+ + \varepsilon]$ to $b_n^+ + \varepsilon$ (following the logic of Step 2). Finally, if $\mu^+ = \mu^-$ for all $\varepsilon \in [0, a_n^+ - b_n^+]$, the same price p_3 can be obtained by moving all the mass μ^+ into x^+ and all the mass μ^- into x^-.

Step 4. The argument in Steps 1-3 can be iterated until one obtains a distribution $0, x_1^-, p_4, x_1^+$, with $x_1^+ = 2p_4 - x_1^-$ and $x_1^- \geq 0$ with weights $\mu_0, \mu_{x_1^-}, \mu_{p_4}, \mu_{x_1^+}$. Suppose $\mu_{x_1^-} < \mu_{x_1^+}$. Then one can increase the price by repeating the argument in step 1 and moving x_1^- and x_1^+ to p_4. If $\mu_0 = 0$, we would end up with a one-signal distribution. If $\mu_0 > 0$, we would end up with a two-signals distribution with signals 0 and $\hat{p}_4$. Suppose instead $\mu_{x_1^-} \geq \mu_{x_1^+}$. Then one could increase the price by repeating the argument in step 2 and moving x_1^- to 0 and x_1^+ to $2p_4$. We would end up with a three-signals distribution with $0, \bar{p}, 2\bar{p}$. Now consider the distribution $0, \bar{p}, 2\bar{p}$, with weights respectively $\tilde{\mu}_0, \mu_p, \mu_{2p}$. The aggregate supply is at least

$$S_4 = \sum_{y=1}^{F} \binom{F}{y} (\mu_p)^{F-y} (\tilde{\mu}_0)^y = (\tilde{\mu}_0 + \mu_p)^F - (\mu_p)^F.$$

We show that there exists a two-signals distribution inducing a larger price. Suppose a mass k_4 is moved from $\bar{p}$ to 0 and a mass k_4 is moved from $\bar{p}$ to $2\bar{p}$. Condition (2) holds and there exists a tie breaking rule so that the new aggregate supply is

$$\tilde{S}_4 = \sum_{y=1}^{F} \binom{F}{y} (\tilde{\mu}_0 + k_4)^y (\mu_p - 2k_4)^{F-y} = (\tilde{\mu}_0 + \mu_p - k_4)^F - (\mu_p - 2k_4)^F.$$

That induces a higher price if $\tilde{S}_4 < S_4$, that is the case if $\tilde{S}_4$ decreases in k_4 at

$k_4 = 0$. Taking the derivative of $\tilde{S}_4$ with respect to k_4, we need that

$$\frac{d\tilde{S}_4}{dk_4} = (2)^{1/(F-1)}(\mu_p - 2k_4) - (\tilde{\mu}_0 + \mu_p - k_4) < 0.$$

Notice that $d\tilde{S}_4/dk_4$ is decreasing in k_4 ($1 - 2^{\frac{F}{F-1}} < 0$ for all $F \geq 2$), hence if $d\tilde{S}_4/dk_4 < 0$ at $k_4 = 0$ then it is negative everywhere. Hence, we need that

$$(2)^{1/(F-1)}(\mu_p) \leq (\tilde{\mu}_0 + \mu_p). \tag{12}$$

If condition (12) holds, setting $k_4 = \mu_p/2$ we obtain a two-signals distribution which induces a higher price. A similar argument can be applied if condition (12) is violated by moving a mass $\tilde{k}_4 = \min\{\tilde{\mu}_0, \mu_{2p}\}$ from 0 to $\bar{p}$ and from $2\bar{p}$ to $\bar{p}$. Hence, the highest market clearing price is obtained with a two-signals distribution.

Step 5. We are then left with two-signals distributions which take one of the following forms: $\sigma_a = \{0, 1-\mu_a; 2p_a, \mu_a\}$ or $\sigma_b = \{0, 1-\mu_b; p_b, \mu_b\}$ or $\sigma_c = \{p_c, 1-\mu_c; 2p_c, \mu_c\}$. For the argument developed in the main text, among those distributions, the highest price is p^*, as defined in (7), and it is achieved by σ^*, as defined in (6). **Q. E. D.**

Proof of Lemma 2

We show that σ^* and p^*, as defined respectively in (6) and (7), are part of an equilibrium. To simplify the notation, denote with h the positive signal which is in the support of σ^*, that is $h = \varphi/\mu^*$, where μ^* is defined in (5). First, we show that there exists a tie-breaking rule such that (σ^*, p^*) clears the market. Suppose that whenever indifferent between buying firm r and selling another firm j the investor buys r. Then since $p^* = h/2$ only those who sample a signal 0 for all firms sell. The aggregate supply is $(1-\mu^*)^F$ that equals 1/2. As the equilibrium is symmetric, that implies that the market clears for each firm.

Consider the possibility of deviations and suppose firm j deviates and achieves a price $p^j > h/2$. Let p^r denote the price of non-deviating firm r, with $r \neq j$.

Step 1. We show that $p^r \leq h/2$ for all firms $r \neq j$.

The demand for non-deviating firm r is at most

$$D^r \leq \mu^* \prod_{w \neq r} \Pr(|p^w - x^w| \leq h - p^r),$$

while the supply for r is at least

$$S^r \geq (1-\mu^*)\prod_{w\neq r}\Pr(|p^w - x^w| < p^r).$$

Suppose by contradiction that $p^r > h/2$. Since $\mu^* < 1-\mu^*$ for all $F \geq 2$ and $\Pr(|p^w - x^w| \leq h - p^r) \leq \Pr(|p^w - x^w| < p^r)$, there is always excess supply for r. Hence, we must have $p^r \leq h/2$ for all $r \neq j$.

We first assume that non-deviating firms are traded at the same price, and we denote this price by p' (see Step 5 for the case in which non-deviating firms are traded at different prices). In what follows, let μ_1 denote the mass assigned by σ^j to signals strictly below $p^j - p'$, that is, $\mu_1 = \Pr(x^j < p^j - p')$. Similarly, let $\mu_2 = \Pr(p^j + p' \leq x^j < p^j + h - p')$ and $\mu_3 = \Pr(x^j \geq p^j + h - p')$.

Denote by D^j and S^j respectively the demand and supply for the deviating firm j, and similarly by D^{-j} and S^{-j} the demand and supply for non-deviating firms. Investors sell firm j when they sample a signal $x^j < p^j - p'$ together with signals 0 from $-j$, which occurs with probability $(1-\mu^*)^{F-1}$, and they demand $-j$ whenever a signal h from $-j$ is sampled with a signal $x^j < p^j + h - p'$ from firm j. Hence, we have

$$S^j + D^{-j} \geq \mu_1(1-\mu^*)^{F-1} + (1-(1-\mu^*)^{F-1})(1-\mu_3).$$

Investors demand j at most when a signal $x^j \in [p^j, p^j + h)$ is sampled together with signals 0 from $-j$ or whenever a signal $x^j \geq p^j + h$ is sampled with any signal from $-j$, and they sell $-j$ whenever they sample signals 0 from firms $-j$ and a signal $x^j \in (p^j - p', p^j + p')$ from firm j. That is,

$$D^j + S^{-j} \leq \mu_3 + (1-\mu^*)^{F-1}(1-\mu_1-\mu_3).$$

Since market clearing requires $S^j + D^{-j} = D^j + S^{-j}$ and $(1-\mu^*)^F = 1/2$, we must have

$$\mu_1 \leq \mu^* + \mu_3(1-2\mu^*). \tag{13}$$

Step 2. We show that $p' > 0$. Suppose by contradiction $p' = 0$, then $\mu_1 + \mu_2 + \mu_3 = 1$ and condition (13) writes as

$$\mu_3 \geq \frac{1-\mu_2-\mu^*}{2(1-\mu^*)}. \tag{14}$$

The average of firm j's distribution is at least $\mu_2 p^j + \mu_3(h + p^j)$, which exceeds

$\mu_2 h/2 + 3\mu_3 h/2$ since we are assuming $p^j > h/2$. Hence, given that $\mu^* h = \varphi$, condition (2) and (14) require

$$\frac{\mu_2}{2} + \frac{1-\mu^*-\mu_2}{2(1-\mu^*)}\frac{3}{2} < \mu^*.$$

Since the left hand side of the abobe inequality decreases in μ_2, the condition must be satisfied when μ_2 is the largest, i.e. $\mu_2 = 1-\mu^*$ (as derived by letting $\mu_3 = 0$). The condition writes as $1-\mu^* < 2\mu^*$, which is violated for all $F \geq 2$. We conclude that we cannot have $p' = 0$.

Step 3. The aggregate supply of all firms is at least

$$S^j + S^{-j} \geq (1-\mu_2-\mu_3)(1-\mu^*)^{F-1},$$

as obtained when a signal 0 from all non-deviating firms is sampled with a signal $x^j < p^j + p'$. Since market clearing requires $S^j + S^{-j} = 1/2$, the previous condition requires

$$\mu_2 + \mu_3 \geq \mu^*. \tag{15}$$

Step 4. The average of firm j's distribution is minimized when all signals $x^j < p^j - p'$ are concentrated at $x^j = 0$, all signals $x^j \in [p^j + p', p^j + h - p')$ are concentrated at $x^j = p^j + p'$, all signals $x^j \geq p^j + h - p'$ are concentrated at $x^j = p^j + h - p'$ and all other signals $x^j \in [p^j - p', p^j + p')$ are concentrated at $x^j = p^j - p'$. That is, for condition (2) to hold, we need

$$\mu_2(p^j + p') + \mu_3(p^j + h - p') + (1-\mu_1-\mu_2-\mu_3)(p^j - p') \leq \varphi. \tag{16}$$

Given (13) and (15), the left hand side is minimized when $\mu_1 = \mu^* + \mu_3(1-2\mu^*)$ and $\mu_2 + \mu_3 = \mu^*$. Substituting into (16), we have

$$(\mu^* - \mu_3)(p^j + p') + \mu_3(p^j + h - p') + (1-\mu_3)(1-2\mu^*)(p^j - p') \leq \varphi.$$

The left hand side decreases in p' and so it is minimized when $p' = h/2$. Hence, $p^j > h/2$ requires $\mu^* h < \varphi$, which is violated since by construction $\mu^* h = \varphi$. We conclude that there is no profitable deviation for firm j when non-deviating firms are traded at the same price p'.

Step 5. Suppose now $F > 2$ and non-deviating firms are traded at a different price. Suppose two non-deviating firms, say firm 1 and firm 2, are traded respectively at prices p_1 and p_2. Assume wlog that $p_1 < p_2$. When signal 0 from firm

1 is drawn together with signal 0 from firm 2 an investor prefers trading stock 2 (since $p_1 < p_2$) and when signal 0 from firm 1 is drawn with signal h from firm 2 an investor prefers trading stock 2 (since from Step 1 $p_1 < h - p_2$). Hence, $S_1 = 0$ and firm 1 is not traded. We are left with a market in which $F-1$ firms are traded. Suppose that, among them, the non-deviating firms are traded at the same price p. We can repeat the above argument and conclude that there is no profitable deviation. Suppose instead that among the $F-1$ traded firms there exist two non-deviating firms which are traded at a different price. We can repeat the above argument and end up with $F-2$ traded firms. Iterating, we end up with 2 traded firms, in which case we have already shown that there is no profitable deviation. We conclude that the profile (σ^*, p^*) defines an equilibrium. **Q. E. D.**

Proof of Proposition 6

Suppose firms choose a distribution with support on $[0, 2\varphi]$, density g symmetric around φ and cdf G such that $[1-2G(x)]^{F-1} = 1 - x/\varphi$ for $x < \varphi$ and $[2G(x)-1]^{F-1} = x/\varphi - 1$ for $x \geq \varphi$.

Step 1. Market clearing requires $p = \varphi$. In fact, at $p = \varphi$, aggregate demand is

$$D = F\int_{\varphi}^{2\varphi} g(x)[2G(x)-1]^{F-1}dx = F\frac{1}{\varphi}\int_{\varphi}^{2\varphi} g(x)(x-\varphi)dx,$$

while aggregate supply is

$$S = F\int_{0}^{\varphi} g(x)[1-2G(x)]^{F-1}dx = F\frac{1}{\varphi}\int_{0}^{\varphi} g(x)(\varphi - x)dx.$$

Since $\int_0^{2\varphi} g(x)dx = 1$ and $\int_0^{2\varphi} xg(x)dx = \varphi$ due to condition (2), we have $D = S$. That is, the market clears.

Step 2. There is no profitable deviation. To see this, suppose firm j deviates to a distribution H with density h. Denote with p^j the price of j and with p the price of non-deviating firms. Notice first that market clearing requires $p = \varphi$. In fact, if $p = \varphi$ aggregate demand for non-deviating firms is

$$D^{-j} = (F-1)\int_{\varphi}^{2\varphi} g(x)[2G(x)-1]^{F-2}[H(p^j + x - \varphi) - H(\varphi + p^j - x)]dx,$$

while aggregate supply is

$$S^{-j} = (F-1)\int_0^{\varphi} g(x)[1-2G(x)]^{F-2}[H(p^j+\varphi-x)-H(p^j+x-\varphi)]dx.$$

By symmetry of g, for any $x \le \varphi$ there exists a signal $v = 2\varphi - x$ such that $g(x) = g(v)$. Hence, $G(x) = 1 - G(v)$, $H(x-\varphi) = H(\varphi - v)$ and $H(\varphi - x) = H(v - \varphi)$, which imply $D^{-j} = S^{-j}$ for $p = \varphi$. To see that $p = \varphi$ is the only market clearing price for non-deviating firms, suppose $p > \varphi$. Then the new aggregate demand is

$$\hat{D}^{-j} = (F-1)\int_p^{2\varphi} g(x)[G(x)-G(2p-x)]^{F-2}[H(p^j+x-p)-H(p+p^j-x)]dx.$$

Notice that $p > \varphi$ implies $G(x) - G(2p-x) < G(x) - G(2\varphi - x)$ and $H(p^j + x - p) - H(p+p^j-x) > H(p^j+x-\varphi) - H(\varphi+p^j-x)$, so it must be that $\hat{D}^{-j} < D^{-j}$. Similarly, the new aggregate supply is $\hat{S}^{-j} > S^{-j}$. Hence, there is excess supply and so $p > \varphi$ does not clear the market. The argument which rules out $p < \varphi$ is symmetric. Suppose then $p^j = \varphi$. The demand for j is

$$D^j = \int_{\varphi}^{\infty} h(x)[2G(x)-1]^{F-1}dx = \frac{1}{\varphi}\int_{\varphi}^{2\varphi} h(x)(x-\varphi)dx + \int_{2\varphi}^{\infty} h(x)dx,$$

while the supply of j is

$$S^j = \int_0^{\varphi} h(x)[1-2G(x)]^{F-1}dx = \frac{1}{\varphi}\int_0^{\varphi} h(x)(\varphi - x)dx.$$

Since $D^j \le S^j$ at $p^j = \varphi$, it must be that $p^j \le \varphi$. Hence, there is no profitable deviation. **Q. E. D.**

Proof of Proposition 7

Suppose first that $f(\cdot)$ is strictly concave in gains from trade. Suppose the firm chooses the distribution $\{\varphi - \varepsilon - \delta, 1/2 - \nu; \varphi + \varepsilon, 1/2 + \nu\}$, where ε and δ are positive and $\nu = \delta/(4\varepsilon + 2\delta)$ due to condition (2). At $p = \varphi$, the demand is $(1/2+\nu)f(\varepsilon)$ while the supply is $(1/2-\nu)f(\varepsilon+\delta)$. Since $(1/2+\nu)/(1/2-\nu) = (\delta+\varepsilon)/\varepsilon$ and by concavity of $f(\cdot)$ that exceeds $f(\varepsilon+\delta)/f(\varepsilon)$, we have excess demand at $p = \varphi$. That is, market clearing requires $p > \varphi$. If $f(\cdot)$ is strictly convex in gains from trade, the same argument applies considering the distribution $\{\varphi - \varepsilon + \delta, 1/2 + w; \varphi + \varepsilon, 1/2 - w\}$, where ε and δ are positive and $w = \delta/(4\varepsilon - 2\delta)$. **Q. E. D.**

Proof of Proposition 8

Suppose firm j has fundamental $\varphi^j = \varphi + \varepsilon^j$, $\sigma^j = \{\varepsilon^j, 1 - \mu^*; 2p^* + \varepsilon^j, \mu^*\}$ and prices are $p^j = p^* + \varepsilon^j$. The logic to show that the market clears is the same as in Lemma 2. Let $h = \varphi/\mu^*$, where $\mu^* = 1 - (1/2)^{1/F}$, and suppose firm j deviates and achieves a price $p^j > h/2 + \varepsilon^j$. Let p^r denote the price of non-deviating firm r, with $r \neq j$.

Step 1. Following the argument of Step 1 in the proof of Lemma 2, we establish that $p^r \leq h/2 + \varepsilon^r$ for all $r \neq j$ as otherwise there would be excess supply for firm r.

Assume first that the price of the non-deviating firms takes the following form:

$$p^r = \varepsilon^r + \lambda h \text{ for all } r \neq j,$$

where, due to Step 1, $\lambda \leq 1/2$. In Step 5, we consider the general case in which $p^r = \varepsilon^r + \lambda^r h$. Let $\mu_0 = \Pr(x^j < p^j - (1-\lambda)h)$; $\mu_1 = \Pr(p^j - (1-\lambda)h \leq x^j < p^j - \lambda h)$; $\mu_2 = \Pr(p^j + \lambda h \leq x^j < p^j + (1-\lambda)h)$; and $\mu_3 = \Pr(x^j \geq p^j + (1-\lambda)h)$.

Following the logic of Lemma 2, we have that market clearing requires

$$\mu_0 + \mu_1 \leq \mu^* + \mu_3(1 - 2\mu^*). \tag{17}$$

Step 2. We show that $\lambda > 0$ when $F > 2$.

Suppose by contradiction that $\lambda = 0$ and so $\mu_0 + \mu_1 + \mu_2 + \mu_3 = 1$. Hence, (17) writes as

$$\mu_3 \geq \frac{1 - \mu_2 - \mu^*}{2(1 - \mu^*)}. \tag{18}$$

The average of firm j's distribution is at least $\mu_1(p^j - h) + \mu_2 p^j + \mu_3(h + p^j)$, which exceeds $\mu_1(\varepsilon^j - h/2) + \mu_2(h/2 + \varepsilon^j) + \mu_3(3h/2 + \varepsilon^j)$ since we are assuming $p^j > h/2 + \varepsilon^j$. Suppose first $\varepsilon^j < h/2$. Conditions (2) and (18) require

$$\mu_2(\frac{h}{2} + \varepsilon^j) + (\frac{1 - \mu_2 - \mu^*}{2(1 - \mu^*)})(\frac{3h}{2} + \varepsilon^j) < \varphi + \varepsilon^j. \tag{19}$$

Since the left hand side of the above inequality decreases in μ_2, it must be satisfied when μ_2 is the largest. Substituting $\mu_2 = 1 - \mu^*$ and $\varphi = \mu^* h$ into (19), we have $h(1 - 3\mu^*) < 2\mu^* \varepsilon^j$, which is violated for all $F > 2$. A similar argument applies when $\varepsilon^j \geq h/2$ by noticing that the average of firm j's distribution is minimized when $\mu_2 = 1 - \mu^*$ and so in particular we must have $(1 - \mu^*)h/2 < \mu^*(h + \varepsilon^j)$. Given (8), that requires $(1 - \mu^*)h/2 < \mu^*(h + (1 - 2\mu^*)h)$, which is violated for any $F > 2$. We conclude that we cannot have $\lambda = 0$ when $F > 2$.

Step 3. Suppose $\lambda > 0$. The aggregate supply of all firms is at least

$$S^j + S^{-j} \geq \mu_0 + (1 - \mu_2 - \mu_3 - \mu_0)(1 - \mu^*)^{F-1},$$

as obtained when a signal $x^j < p^j - (1 - \lambda)h$ from firm j is sampled with any other signal from firms $-j$ and when a signal ε^r from all firms $r \neq j$ is sampled with a signal $x^j < p^j + \lambda h$ from firm j. Hence, market clearing requires

$$\mu_2 + \mu_3 \geq \mu^* + \mu_0(1 - 2\mu^*). \tag{20}$$

Step 4. Condition (2) requires

$$\mu_1(p^j - (1 - \lambda)h) + \mu_2(p^j + \lambda h) + \mu_3(p^j + (1 - \lambda)h) + \\ (1 - \mu_0 - \mu_1 - \mu_2 - \mu_3)(p^j - \lambda h) \leq \varphi + \varepsilon^j. \tag{21}$$

Suppose first $F > 2$ and so $\lambda > 0$. Given (17) and (20), the left hand side is minimized when $\mu_1 = \mu^* + \mu_3(1 - 2\mu^*) - \mu_0$ and $\mu_2 = \mu^* + \mu_0(1 - 2\mu^*) - \mu_3$. Substituting into (21), we see that the left hand side of (21) decreases in λ and so it must hold when $\lambda = 1/2$. This in turn requires $\varepsilon^j > (1 - 2\mu^*)h$, which violates condition (8) in the text.

Suppose finally that $F = 2$. The left hand side of inequality (21) is linear in λ, so it must hold either for $\lambda = 1/2$ or for $\lambda = 0$. If $\lambda = 1/2$, we can repeat the argument above and conclude that $\varepsilon^j > (1 - 2\mu^*)h$ violates condition (8) in the text (notice that $2(p^* - \varphi) = \varphi\sqrt{2}$ when $F = 2$ and so $\varepsilon^j \leq \varphi\sqrt{2}/2$ implies $\varepsilon^j \leq 2(p^* - \varphi)$). Suppose then $\lambda = 0$. Condition (21) requires in particular $\mu_2 p^j + \mu_3(p^j + h) \leq \varphi + \varepsilon^j$, which given (18) requires

$$\mu_2 p^j + \frac{1 - \mu_2 - \mu^*}{2(1 - \mu^*)}(p^j + h) \leq \varphi + \varepsilon^j.$$

The left hand side is linear in μ_2 and so it must hold when either $\mu_2 = 0$ or $\mu_2 = 1 - \mu^*$. Suppose $\mu_2 = 0$, we have $p^j \leq 2\varepsilon^j - (1 - 2\mu^*)h$, that implies $p^j < h/2 + \varepsilon^j$ since by condition (8) $\varepsilon^j \leq \varphi\sqrt{2}/2$. Suppose then $\mu_2 = 1 - \mu^*$, we have $p^j \leq (\varphi + \varepsilon^j)/(1 - \mu^*)$. Condition (8) is equivalent to $(\varphi + \varepsilon^j)/(1 - \mu^*) \leq h/2 + \varepsilon^j$. We conclude that there is no profitable deviation for firm j when non-deviating firms are traded at prices $p^r = \varepsilon^r + \lambda h$.

Step 5. Suppose each non-deviating firm is traded at a price $p^r = \varepsilon^r + \lambda^r h$. The argument to show that there are no profitable deviations follows Step 5 of the proof of Lemma 2. **Q. E. D.**

Proof of Proposition 10

Suppose that $X^j \in [0, H]$ and let $\mu^* = 1 - (1/2)^{1/F}$. If $\varphi/\mu^* \leq H$, the upper bound does not bind and the analysis of Proposition 2 applies. Suppose instead $\varphi/(1-\mu^*) \geq H$. In this case, one can obtain a price $p = H$ with the distribution $\sigma = \{0, 1-\eta; H, \eta\}$ in which $(\eta)^F \geq 1/2$, that is $\eta \geq 1-\mu^*$. Since $\varphi \geq (1-\mu^*)H$, condition (2) is satisfied. This is obviously the highest price irrespective of F. Hence, in what follows, we focus on

$$H \in (\frac{\varphi}{1-\mu^*}, \frac{\varphi}{\mu^*}). \tag{22}$$

We first show that if $H \geq \tilde{H}$, as defined by condition (9), the highest market clearing price is achieved with $\sigma_H = \{(\varphi - \mu^* H)/(1-\mu^*), 1-\mu^*; H, \mu^*\}$ and it is defined as in (10) by

$$p_H = \frac{1}{2}(H + \frac{\varphi - \mu^* H}{1-\mu^*}). \tag{23}$$

By the same argument a the one developed in Lemma 1, the highest market clearing price is obtained when the distribution takes one of the following forms. Either, $\sigma_a = \{0, 1-\mu_a; p_a, \mu_a\}$ with $(\mu_a)^F \geq 1/2$ and so the highest price is obtained when $\mu_a = 1-\mu^*$ and it writes as

$$p_a = \frac{\varphi}{1-\mu^*}. \tag{24}$$

The price defined in (23) exceeds p_a in (24) if

$$H \geq \frac{\varphi}{1-2\mu^*}. \tag{25}$$

Or $\sigma_b = \{0, \tilde{\mu}, l, 1-\mu_b-\tilde{\mu}; H, \mu_b\}$ with $p_b = (H+l)/2$ and because of market clearing

$$1 - (1-\tilde{\mu})^F + (1-\mu_b-\tilde{\mu})^F = 1/2. \tag{26}$$

Differently from Lemma 1, it may be optimal to have $\tilde{\mu} > 0$ since shifting signals l and H further away from the price is not feasible. Since σ_H is obtained as a special case of σ_b when $\tilde{\mu} = 0$, we investigate under which condition p_b is maximized by $\tilde{\mu} = 0$. Given the market clearing condition (26), l is defined by condition (2) and so we have

$$p_b = \frac{1}{2}(H + \frac{\varphi - (1 - \tilde{\mu} - ((1-\tilde{\mu})^F - \frac{1}{2})^{\frac{1}{F}})H}{((1-\tilde{\mu})^F - \frac{1}{2})^{\frac{1}{F}}}). \tag{27}$$

Differentiating p_b with respect to $\tilde{\mu}$, we see that p_b decreases in $\tilde{\mu}$ if

$$2\varphi - H \leq 0. \tag{28}$$

That is, under condition (28), p_b is maximized by $\tilde{\mu} = 0$. Hence, if conditions (25) and (28) are satisfied, as required by condition (9) in the text, the highest market clearing price is defined by (23). This price increases in F since μ^* decreases in F.

We now show that p_H can be sustained in equilibrium. The logic follows closely the proof of Lemma 2. Suppose firm j deviates and achieves a price $p^j > (H+l)/2$ and let p^r denote the price of non-deviating firm r, with $r \neq j$.

Step 1. Following the same argument as the one in Step 1 of the proof of Lemma 2, we must have $p^r \leq (H+l)/2$ for all $r \neq j$ or there would be excess supply for firm r.

We first assume that non-deviating firms are traded at the same price, and we denote it with p' (see Step 5 for the case in which non-deviating firms are traded at different prices). Let $\mu_0 = \Pr(x^j < p^j + p^{'} - H)$; $\mu_1 = \Pr(p^j + p^{'} - H \leq x^j < p^j - p' + l)$, and $\mu_2 = \Pr(p^j + p' - l \leq x^j \leq H)$.

Following the logic of Lemma 2, we have that market clearing requires

$$\mu_0 + \mu_1 \leq \mu^*. \tag{29}$$

Step 2. We show that $p' > l$. Suppose by contradiction that $p' = l$, then $\mu_0 + \mu_1 + \mu_2 = 1$ and (29) writes as

$$\mu_2 \geq 1 - \mu^*. \tag{30}$$

The average of firm j's distribution is at least $\mu_2 p^j$, and so given (30) and $p^j > (H+l)/2$, condition (2) requires

$$(1-\mu^*)(H+l) < 2\varphi,$$

that is, $H + \varphi - 2H\mu^* < 2\varphi$, and that violates (25). We conclude that we cannot have $p = l$.

Step 3. Following the logic of Step 3 in the proof of Lemma 2, we need

$$\mu_2 \geq \mu^* + \mu_0(1 - 2\mu^*), \tag{31}$$

or we would have excess aggregate supply.

Step 4. Condition (2) requires

$$\mu_1(p^j + p^{'} - H) + \mu_2(p^j + p' - l) + (1 - \mu_0 - \mu_1 - \mu_2)(p^j - p' + l) \leq \varphi. \tag{32}$$

The previous expression is linear in $p^{'}$, so it must hold either for $p^{'} = (H + l)/2$ or for $p^{'} \to l$. Suppose $p^{'} = (H + l)/2$, we must have $\mu_2 H + (1 - \mu_0 - \mu_2)l < \varphi$, which given (29) and (31) must hold when $\mu_0 = \mu^* - \mu_1$ and $\mu_2 = \mu^* + (\mu^* - \mu_1)(1 - 2\mu^*)$. That is, we need $(\mu^* - \mu_1)(H - 2l - 2H\mu^* + 2l\mu^*) < 0$, which is equivalent to $H < 2\varphi$, and that violates condition (28). Suppose instead $p^{'} \to l$, (32) requires $(1 - \mu_0 - \mu_1)(H + l) < 2\varphi$, which given (29) requires $(1 - \mu^*)(H + l) < 2\varphi$. As shown in Step 2, this violates (25).

Step 5. The argument to show that there are no profitable deviations when some non-deviating firms are traded at a different price follows Step 5 of the proof of Lemma 2. **Q. E. D.**