

On a Characterization of Dirichlet Distribution

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Abstract

Let $Z = (X, Y)$ be a random variable taking values in a finite set $I \times J$. For any distribution P on $I \times J$, let $P(\cdot)$ be the marginal distribution of X and let $P(\cdot | i)$ be the conditional distribution of Y given $X = i$. Define $Q(\cdot), Q(\cdot | j)$ be the marginal and conditional distribution of Y and X given $Y = j$. Geiger and Heckerman [The Annals of Statistics, Vol. 25, No. 3. (Jun., 1997), pp. 1344-1369.] showed that if a prior Π for the distribution of Z , satisfies, under Π ,

- (1) $P(\cdot)$ is independent of $\{P(\cdot | i) : i \in I\}$ and
 - (2) $\{P(\cdot | j) \in j \in J\}$ are independent among themselves
 - (3) a similar statement holds for $Q(\cdot), Q(\cdot | j)$
- then Π is a Dirichlet distribution.

We provide a simpler proof of the above characterization. Then we indicate an extension of the result to general Dirichlet process and also examine the distribution of the conditional distribution $P|X$ for a general Dirichlet process. This is joint work with Laura Sangalli.