Mechanism Design in Two-Sided Markets: Auctioning Users

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Abstract

Many two-sided platforms (such as search engines and business directories) make profits from auctioning their user base to advertisers. Yet, auctioning users is different from selling standard commodities, since the participation decision by users (and, therefore, the size of the platform's user base) depends on the benefit users expect to receive from joining the platform. In this setting, what is the profit-maximizing auction? And how should a platform structure its user fees? First, I show that if bidders profit from the match more than users, it is optimal for the platform to offer subsidies to users, and recoup losses on the user side of the market by inducing aggressive bidding on the bidder side (*loss leader strategy*). Second, I show that if the bidders' willingness to pay for the match is positively affiliated with the value users derive from bidders, the revenue-maximizing mechanism favors bidders with low values to users (*search diversion*). In turn, when charging or subsidizing users is not feasible, the platform favors bidders with high (low) user values as a substitute for the subsidies (fees) it would otherwise implement. In this setting, I also show that competition between two-sided platforms can decrease total welfare when the supply of users is sufficiently inelastic. This result implies that applying standard antitrust economics to two-sided markets may be misleading.

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1 Introduction

The most valuable asset of many two-sided platforms is their user base. In a celebrated example, Internet search engines (such as *Google*) derive most of their revenue from auctioning off the eyeballs of millions of users to advertisers, who pay to be displayed alongside the search results associated with particular queries.[1]

Yet, auctioning users is fundamentally different from selling standard commodities (e.g., timber). As is typical in many two-sided markets, more users will participate in the platform the higher the benefit they expect to receive. Much of the prior work on two-sided markets assumes this benefit is simply a function of the number of participants from the other side of the market (cross-network externalities). However, in many important examples, users care about the quality (as opposed to quantity) of the participants (advertisers) on the other side of the market. In the case of search engines, fewer users click on sponsored links if the allocation mechanism often selects advertisers who poorly match the users' queries.

Other two-sided platforms that face similar market design problems:

- Business directories, such as *YellowPages.com*, sell to firms space in online listings offered to users who conduct specific searches (e.g., a restaurant in "Chicago's Gold Coast Neighborhood").[2] Clearly, fewer users employ the platform's services if recommendations consistently favor unsatisfying/expensive restaurants.

- Job-matching agencies, such as *Monster.com*, sell to potential employers access to millions of resumes posted by online job seekers.[3] Fewer job seekers set up professional profiles if the platform often connects job seekers to low-quality employers.

In this paper, I follow a mechanism-design approach to answer the following question: What is the optimal auction for selling to bidders the right to be matched with users, when the total supply of users for sale depends on the surplus that the users expect to obtain from the mechanism?

In my baseline model, a monopolistic platform (Google) has to select one of many bidders (advertisers) to match with users (in the search-engine example, this simplification means

[1] In 2007, the US search traffic totalled 86.4 billion queries and paid search revenue amounted to more eight billion dollars (see Evans (2008)). Dai, Nie, Wang, Zhao, Wen and Li (2006) estimate that 40% of the queries have commercial potential.

[2] *YellowPages.com* processes 140 million searches per month. Reportedly, 74% of its users contact a listed merchant and 55% conclude transactions with these merchants. Listings with "priority placement" for a given search may cost more than $1,000 per month (October 2009 values).

[3] *Monster.com* claims to have a catalog of 150 million resumes. Employers can access 400 resumes in a two-week period for $975. Improved matching capabilities and access to 20,000 resumes within a year cost $9,995 (August 2009 values).

that Google has a single link to sell). Users are heterogeneous in their outside options relative to joining the platform, but are homogeneous on the values they derive from being matched with each bidder (this is the *user value* associated with a bidder). In turn, bidders differ in the value they attach to being matched with users (this is the *bidder value* associated with a bidder). User and bidder values summarize the payoffs obtained by users and bidders in their subsequent interaction (e.g., if the advertiser is an online retailer, user and bidder values represent the consumer and producer surplus from a sale).

The platform observes an informative signal about the user and bidder values associated with each bidder. In the search-engine example, this signal is shorthand for the web site's content and its clicking history from previous searches. The platform's objective is to design a mechanism for selling users to bidders, taking into account that the supply of users for sale increases with the surplus that users expect to obtain from the mechanism.

First, I analyze this problem in a setting where, besides running an auction among bidders, the platform can charge or subsidize users (I call this a *two-sided mechanism*). In this case, the surplus that users expect to obtain from the mechanism is the user value of the winning bidder net of the fees/subsidies set by the platform. I show that:

- By setting the appropriate fees/subsidies for users, the platform can fix the supply of users at any desired level (since it can transfer money across sides). As a consequence, the matching rule associated with the revenue-maximizing auction selects the bidder who produces the match with highest total virtual value in expectation (the sum of user and bidder values, adjusted for informational rents).

- If bidders profit more from the match than users in expectation, the platform should follow a loss leader strategy; subsidize users (using proceeds collected from bidders) to boost supply and further extract rents from bidders. If users profit more from the match than bidders, the optimal mechanism charges both sides of the market.

In my leading example, search engines provide users with a vast wealth of nonmonetary subsidies (basic searches, scholarly searches and reader capabilities, for example, are all given for free). Moreover, many business directories (such as *Coupon.com*) offer discount coupons from advertisers in order to expand their user base. In contrast, some job-matching agencies specialized in high-paying jobs (such as *TheLadders.com*) charge access fees to users who want to create professional profiles. My model shows that the platform's decision to adopt different business models depends on the relative values that bidders and users expect to obtain from a match.

Incidentally, the optimal mechanism derived in this paper offers an explanation for why many two-sided platforms often produce matches with surprisingly low user values (Hagiu and Jullien (2009) call it *search diversion*). Indeed, the sponsored links offered by search engines are seldom similar to their organic counterparts and usually display links of much lower relevance to users. Likewise, restaurant recommendations from *YellowPages.com* are usually at odds with Zagat's suggestions. Why do platforms under-provide value to users? My model provides an informational rationale for this distortion. The same signal used by the platform to infer the user values associated with bidders (advertisers) reveals information about the bidders' willingness to pay for a match. As a consequence, the revenue-maximizing mechanism distorts the matching rule away from efficiency in order to provide stronger incentives for high bidding. Specifically:

- When bidder and user values are positively affiliated, the platform adopts a matching rule that favors bidders with *low users values*. Intuitively, affiliation implies that advertisers with high user values are *more* likely to have high bidder values. As such, with the purpose of maximizing profits, the platform must distort the matching rule towards bidders with low user values, in order to encourage those with high bidder values to pay more for a match. This distortion tradeoffs a larger user base to increase the rent extraction from bidders (advertisers).

The results discussed above assume that the platform can charge users or subsidize them (either monetarily or by providing free services). However, many two-sided platforms are not able (or decide not) to engage in direct transfers to users. Job-matching agencies, for example, cannot reward users for posting their resumes online, as it would attract a large number of fictional job seekers, who create fake resumes with the sole objective of collecting subsidies. Other examples of two-sided platforms that choose not to make transfers to users include business directories (such as *YellowPages.com* and *Ariba.com*) and online rental agencies (such as *Rent.com*).[4]

With these examples in mind, I study the revenue-maximizing matching mechanism when charging or subsidizing users is ruled out (I call it a *one-sided mechanism*). In this setting,

[4]The literature on electronic commerce emphasizes three reasons why some platforms choose not to charge/subsidize users. The first is based on *decision making and transaction costs*. If a platform provides services that are of low value to users ("micro services"), then charging users could have a negative effect on the business. For example, how much should Google Maps charge users for a search? Many authors argue that setting up "micro payments" (e.g., a penny per search) would sharply decrease the number of users due to the decision-making costs (see Szabo (1996)) and transaction costs involved (see Párhonyi, Lambert and Pras (2005)). The second reason is outlined by Peha and Khamitov (2004), who argue that high rates of *identity theft* and *financial fraud* have induced many internet platforms to adopt business models that do not require users to make payments. According to the third reason, accepting payments only by credit card could have drastic effects on the number of users who patronize a platform, especially in markets where users have limited access to financial (and payment) instruments. These frictions could render "zero charges" policies optimal.

the matching rule induced by the optimal auction is the only instrument the platform can deploy to attract users while providing the right incentives for bidders to pay high prices for the match. I then show that:

- When the optimal two-sided mechanism subsidizes users, the optimal one-sided mechanism favors bidders with high user values as a substitute for the subsidies it would otherwise implement. In turn, when the optimal two-sided mechanism charges users, the optimal one-sided mechanism favors bidders with high bidder values.[5]

Without fees/subsidies to users, the expected user value induced by the optimal auction is the sole determinant of the size of the platform's user base. As a consequence, the user-supply elasticity plays an important role in the analysis:

- The more elastic is the supply of users, the more the platform distorts its matching rule to select bidders with high user values. This effect hinders the platform's ability to extract rents from bidders. Moreover, if the supply of users is sufficiently elastic, I show that banning transfers may lead to a more efficient matching rule. This result has interesting implications for the regulation of online platforms.

Two-sided platforms often participate in oligopolistic markets: *Google* competes for searches with *Yahoo!* and *Bing*, *YellowPages.com* competes for business searchers with *SwitchBoard.com*, and *Monster.com* competes for job seekers with *CareerBuilder.com*. It is then natural to ask what impact competition has on equilibrium matching rules. Is competition welfare increasing? Do users and bidders benefit from a more competitive matching market?

To answer these questions, I embed the optimal mechanism problem of each auctioneer into a Hotelling model where users are heterogeneous in their preferences for platforms. As in Evans (2008) and in line with casual empiricism, I assume that users join at most one platform (single-homing), while bidders join multiple platforms (multi-homing). In my model of competition, platforms simultaneously announce their mechanisms. The users choose the one platform, and the bidders, the possibly multiple platforms, they wish to join. I show that:

- When platforms are allowed to use two-sided mechanisms, competition benefits users due to lower payments (or higher subsidies). Nevertheless, since bidders can advertise

[5] Interestingly, this result predicts that improvements in the micro payments technology that capacitate platforms to charge users (when doing so was unfeasible beforehand) should lead to more efficient matching rules.

in multiple platforms, a "competitive bottleneck" (see Armstrong (2006)) emerges. As such, the platforms do not need to compete for bidders, and follow in equilibrium the same auction protocol derived in the monopolistic case. Since competition helps increase the market's user base, a duopolistic matching market produces more welfare than a monopoly.

- The situation is different when platforms compete using one-sided mechanisms. In this case, competition for users pushes platforms to adopt matching rules that favor bidders with high user values. This effect has two important consequences. First, it weakens bidders' incentives to pay high prices for the match. As a result, competition in one-sided mechanisms is beneficial for users while reducing rent extraction from bidders. Second, competition in one-sided mechanisms could *reduce* total welfare when the supply of users is inelastic (that is, the number of users who join *some* platform is fixed). Indeed, as the matching rule is distorted to favor bidders with high user values, the platform foregoes selecting bidders with high bidder values. If bidders appropriate a higher share of the total value from the match, this may result in a reduction of total welfare relative to monopoly.

This last result calls for caution when applying standard antitrust economics to two-sided markets. In particular, concentrated markets could be welfare improving when platforms compete with one-sided mechanisms.

1.1 Related Literature

This paper extends the theory of mechanism design to a two-sided market setting in order to answer the following question: how to sell users to bidders, when the total supply of users for sale depends on the expected benefit that users obtain from the mechanism. As such, this paper belongs to the large body of literature originating from the fundamental works of Myerson (1981) and Riley and Samuelson (1981).

My analysis hinges on the platform's conflict between inducing participation and extracting rents from both sides of the market. A similar tradeoff lies at the heart of the two-sided-market literature (see Rochet and Tirole (2003, 2006), Armstrong (2006), Caillaud and Jullien (2001, 2003), Evans (2003), Hagiu (2006) and Weyl (2009)). This literature emphasizes cross-network externalities: the payoff from agents on one side of the market depends on the participation (total "quantity") of agents on the other side.[6] Instead, in

[6] Rochet and Tirole (2003) derives the monopoly's optimal pricing formulas when network externalities take the form of usage benefits. In contrast, Armstrong (2006) solves the monopolist's problem by assuming users have heterogeneous membership values. Rochet and Tirole (2006) proposes a general model of two-sided

my model, while the bidders' payoffs increase with the total number of users (network or quantity externality), the participation decision by users depends on the *expected quality* associated with bidders from the other side.

The second key difference between this paper and the two-sided-market literature concerns the pricing instruments used by the platform. While the two-sided-market literature assumes that platforms choose linear prices or two-part tariffs, I allow the platform to design a mechanism to sell users to bidders.[7]

This paper is also related to the burgeoning literature on online advertising. Athey and Ellison (2007) study a model in which advertisers bid for sponsored slots in a generalized second price (GSP) auction, and rational users sequentially search through a list of sponsored links.[8] Assuming bidders' types are one-dimensional (bidder and user values perfectly coincide), the authors derive the optimal reserve prices of the GSP. In contrast, I allow bidders to have bi-dimensional types (bidder and user values differ) and consider the platform's problem from a mechanism-design point of view. By following this approach, I'm able to tackle novel questions, such as the optimal fees/subsidies to users and the effects of competition between two-sided platforms. In turn, their analysis illuminates many issues that I do not discuss here. Most notably, they consider multiple advertising slots and discuss how "learning-by-searching" by users affects the platform's design choices.[9]

In Rayo and Segal (2008), users are randomly assigned to an advertiser and then receive a signal sent by the platform about the quality of the advertiser's product. In choosing an optimal disclosure policy, the platform trades off providing credible information to users and inducing them to make purchases from advertisers. Although concerned with similar issues, our models differ considerably. While I allow the platform to choose an allocation

markets that unifies these two approaches. Weyl (2009) generalizes Rochet and Tirole (2006) to contrast the implications of different forms of users' heterogeneity for pricing and welfare.

[7] In recent work, Hagiu (2009) develops a model in which a two-sided platform charges access prices to agents on both sides and can exclude agents from one side of the market if their quality (or user value) is too low (threshold exclusion policy). As the analysis of my paper shows, restricting attention to threshold policies is suboptimal, as the the optimal mechanism excludes bidders on the basis of their total virtual value.

[8] In the simplest version of the generalized second-price auction, each advertiser submits one bid that represents his willingness to pay for a click on his link. Slots are then assigned in decreasing order of bids and each advertiser pays per click the bid submitted by the advertiser immediately below him. See Aggarwal, Goel and Motwani (2006), Edelman, Ostrovsky and Schwarz (2007), Varian (2007) and Gomes and Sweeney (2009) for an equilibrium analysis of this auction with different solution concepts.

[9] The advertising literature studies how competition in the product market is affected by investments in advertising (cf., Butters (1977) and Grossman and Shapiro (1984)). More recently, the advent of online media and targeted advertising motivated the work of Akçura and Srinivasan (2005), Iyer, Soberman and Villas-Boas (2005), Gal-Or, Gal-Or, May, and Spangler (2006), Esteban and Hernandez (2007) and Galeotti and Moraga-González (2008). Focusing specifically in sponsored search advertising, Chen and He (2006) and Chen, Liu and Whinston (2009) study models that integrate the advertisers' pricing decisions with their bidding behavior in the position auction that determines their advertising exposure.

(matching) mechanism, Rayo and Segal consider information-disclosure rules.[10]

Hagiu and Jullien (2009) derive the optimal level of search diversion by a platform that trades off consumer traffic with the extra rents that come from "accidental" shopping. They assume that the platform has complete information about the advertisers' types, therefore ignoring the mechanism-design issues that are at the core of this work.

The present work is also related to the literature on auctions with elastic supply (see Hansen (1988), Lengwiler (1999), Ausubel and Cramton (2004) and LiCalzi and Pavan (2005) and McAdams (2007)). In these papers, the auctioneer is allowed to condition the number of units to be sold on the profile of bids submitted at the auction. In contrast, the supply is endogenous in my model due to the users' participation constraints.

Other works have studied competition between two-sided platforms. McAfee (1993), Peters and Severinov (1997), Caillaud and Jullien (2001, 2003), Ellison, Fudenberg and Möbius (2004) and Damiano and Li (2007) assume that buyers and sellers have to make exclusive decisions on which platform to join (universal single-homing). More in line with the present paper, Rochet and Tirole (2003) and Armstrong (2006) embed a two-sided market model into a Hotelling duopoly game in which one side single-homes and the other multi-homes.

This paper is organized as follows. In section 2, I present the primitives of my model. As a benchmark, I derive the efficient allocation in section 3. In section 4, I study the platform's revenue-maximization problem. In section 5, I extend the analysis to consider competition between two-sided platforms, and conclude in section 6. All proofs omitted in the text appear in the Appendix.

2 The Model

Two-sided platforms match users with firms, employers or advertisers that share reciprocal needs. For a group of users with a certain objective (such as acquiring an airline ticket to Orlando or finding a job as an accountant), the platform encounters N bidders with corresponding interests. The platform then selects one bidder to be matched with users (this amounts to saying that Google has a single link to sell or business directories issue a single recommendation per query).[11]

If bidder $j \in \mathbf{N} \equiv \{1, ..., N\}$ is selected, all users derive from a match with j value u_j,

[10] Calzolari and Pavan (2006) study the optimal disclosure policies for principals who contract sequentially with an agent. They show that privacy is the optimal disclosure policy when principals impose no externalities on each other.

[11] This assumption greatly simplifies the analysis but does not affect the main insights of this work. I discuss in the conclusion how to extend this model to allow the platform to offer multiple bidders to users.

which I refer to as the *user value* of bidder j.[12] In turn, bidder j earns a value v_j per user he is matched with. I refer to v_j as the *bidder value* associated with bidder j.[13]

The values associated with each bidder, v_j and u_j, should be seen as a reduced-form description of how users and bidders interact. Consider the following examples:

- Users are searchers and bidders are advertisers on a search engine. The user value of advertiser j, u_j, is the payoff that searchers derive from visiting the advertiser's website. In turn, the bidder value, v_j, is the advertiser's willingness to pay for additional searchers.

- Users are Rabelaisian gourmands looking for a restaurant in a business directory. In this case, the bidder value associated with j, v_j, is the extra profit earned by restaurant j from being listed online. In turn, the user value, u_j, is the consumer surplus derived from a meal at the restaurant.

- Users are job seekers and bidders are employers who list vacancies with a job-matching agency. In this case, the user value associated with employer j, u_j, is the net surplus derived by a job seeker who decides to work with j, while the bidder value, v_j, is the net profit from employing an extra worker.

2.1 Platform Information

The platform does not observe the bidder and user values, v_j and u_j. Instead, it observes for each bidder a signal, θ_j (also observed by the bidder), which provides information about v_j and u_j. In turn, bidders privately know their bidder values, v_j, but do not know their user values, u_j (only observed by users). I refer to the pair $t_j \equiv (\theta_j, v_j)$ as bidder j's *type*. In light of the examples discussed above:

- The search engine does not observe the payoff that searchers derive from advertiser j, u_j, nor the advertiser's willingness to pay for additional searchers, v_j. Instead, the search engine has information on the content of the advertiser's site and knows its click history from previous searches, which is captured by θ_j.

- The business directory does not observe the extra profits from being listed online, v_j, nor the consumer surplus derived from a meal at the restaurant, u_j. Instead, it has

[12] Alternatively, I can assume that users are heterogeneous in their values from interacting with bidders, but that the platform does not observe (nor can it contract on) this heterogeneity. In this case, let user i derive value u_j^i from being matched to bidder j, and let $u_j \equiv E[u_j^i]$ be the expected user value associated with this bidder.

[13] The bidder value, v_j, can also be interpreted as the average profit accross users: $v_j \equiv E[v_j^i]$.

access to a description provided by the restaurant and to reviews posted by users. This information is summarized by signal θ_j.

- The job-matching agency does not observe the net surplus from either job seekers or employers, u_j and v_j, respectively. Nevertheless, it can infer (v_j, u_j) from the employer's job description (and wage), which I represent by θ_j.

For each bidder j, the triple (θ_j, v_j, u_j) is an independent draw from a trivariate distribution F with discrete support on

$$T \equiv \Theta \times V \times U \equiv \{\theta^1, \theta^2, ..., \theta^{T_1}\} \times \{v^1, v^2, ..., v^{T_2}\} \times \{u^1, u^2, ..., u^{T_3}\}$$

and pdf $f(\theta, v, u) > 0$ for all $(\theta, v, u) \in T \subset \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$. For convenience, I assume that the support of F is a grid, i.e., $\theta^k = k \cdot \varepsilon$ for all k, $v^l = l \cdot \epsilon$ for all l and $u^m = m \cdot \xi$ for all m.[14] Finally, I impose the usual monotone hazard rate condition for each realization of θ: $\frac{f(v|\theta)}{1-F(v|\theta)}$ is weakly increasing in $v \in V$ for all $\theta \in \Theta$.

Most importantly, I consider environments in which, first, (θ, v, u) are positively affiliated:

Assumption 1 (θ, v, u) *are positively affiliated random variables.*

This assumption implies that the bidders with a higher willingness to pay to be matched with users are more likely to provide users with greater benefits. Considering once again the examples provided in the previous subsection, this means that:

- Advertisers who are willing to pay more for new searchers, v_j, are more likely to offer products that are more relevant to user queries, i.e., have higher u_j.

- Restaurants that derive greater expected profits per user, v_j, are more likely to deliver greater expected surplus for users, u_j.

- Employers who pay higher wages to job seekers, u_j, are more likely to derive greater profits from extra hirings, v_j.

One can think of many economic situations in which user and bidder values are not positively affiliated. As an example, consider the case of online retailers that sell some homogenous product (e.g., a camera). Clearly, the retailer who charges higher prices (and therefore produces lower user values) should have higher bidder values! In subsection 4.2.1, I relax Assumption 1 to analyze environments in which (v, u) are negatively affiliated.

[14] This purely technical assumption appears in other papers on mechanism design with discrete support (see, e.g., Pai and Vohra (2008)).

I denote by $\mathbf{t} \equiv (t_1, ..., t_N)$ a profile of bidders' types and by $f(\mathbf{t})$ the joint probability of profile $\mathbf{t}$. I will now describe the platform's problem of designing a matching mechanism in order to maximize profits.

2.2 Mechanisms

A matching mechanism has three components. The first component is the *matching rule* Z, which picks from the set of available bidders the one to be matched with users (in a possibly non-deterministic manner). According to the Revelation Principle, I can restrict attention to direct-revelation mechanisms. Therefore, I will consider matching rules that map the profile of bidders' types, $\mathbf{t} \equiv (t_1, ..., t_N)$, into probability distributions over assignments. I represent this rule by a function $Z : T^N \to \triangle(\mathbf{N} \cup \{0\})$, where $\triangle(\mathbf{N} \cup \{0\})$ is the set of probability distributions with support on the set of bidders $\mathbf{N} \cup \{0\}$ ($\{0\}$ means that the platform assigns no match with users, in which case $v_0 = \theta_0 = 0$). Accordingly, Z maps each profile of types $\mathbf{t}$ to a distribution, $Z(\mathbf{t})$, where $Z(\mathbf{t})$ is represented by a vector that lists the probability that each bidder j obtains a match:

$$Z(\mathbf{t}) \equiv (Z_j(\mathbf{t}) : j \in \mathbf{N} \cup \{0\}) \quad \text{such that} \quad Z_j(\mathbf{t}) \geq 0 \;\; \forall j \quad \text{and} \quad \sum_{j \in \mathbf{N} \cup \{0\}} Z_j(\mathbf{t}) = 1. \qquad (1)$$

The second component of a matching mechanism is the bidders' payment rule, $P : T^N \to \mathbb{R}^N$, which assigns to each bidder j a payment, $P_j(\mathbf{t})$, that depends on the whole profile of bidders' types $\mathbf{t}$.

The final component of the platform's matching mechanism is the (possibly negative) user fee, Q. One could allow Q to depend on the whole profile of bidders' types $\mathbf{t}$. Mainly in order to economize on notation, I refrain from doing so. As we will see shortly, users preferences are quasi-linear, due to which this dependence is redundant and restricting the platform to constant fees is without loss of generality.

For future use, I will refer to the triple (Z, P, Q) as a *two-sided mechanism*, since it specifies transfers to both sides of the market. I will also analyze the platform's problem when charging users is not feasible. In this case, the platform chooses a pair (Z, P), which I call a *one-sided mechanism*.

An important class of matching rules can be described by *scoring rules*. They work as follows. Each bidder j is assigned a score, $s(t_j)$, as a function of his type (θ, v) and the bidder with the highest score obtains the match (ties are broken arbitrarily). Formally:

Definition 1 *A matching rule Z is said to be described by the scoring rule $s : t \mapsto s(t)$ if for any two bidders j_1 and j_2, $s(t_{j_1}) > s(t_{j_2})$ implies $Z_{j_2}(\mathbf{t}) = 0$.*

The following example introduces an important matching rule.

Example 1 (Efficient Matching Rule) *Denote by $\mu(\theta, v)$ the expectation of the user value u conditional on signal θ:*

$$\mu(\theta, v) \equiv E[u|\theta, v].$$

Let Z^E be the matching rule that selects the bidder with highest expected total surplus, $\mu(\theta, v) + v$. Clearly, Z^E is described by the scoring rule $s^E(t) = \mu(\theta, v) + v$. I refer to Z^E as the efficient matching rule (for reasons that will be clear shortly).

A truthful matching rule Z naturally induces a probability distribution, $f(\cdot|Z)$, over the type of the bidder who is matched with users. In order to derive it, I first define the set of profiles $T((\theta, v))$ for which some bidder has type (θ, v):

$$T((\theta, v)) \equiv \{\mathbf{t} : \exists j \text{ s.t. } t_j = (\theta, v)\}.$$

Moreover, for each profile $\mathbf{t} \in T((\theta, v))$, let us denote by $N(\theta, v)$ the set of bidders' indexes for which $t_j = (\theta, v)$. It then follows that

$$f((\theta, v)|Z) = \sum_{\mathbf{t} \in T((\theta,v))} \sum_{j \in N(\theta,v)} Z_j(\mathbf{t}) \cdot f(\mathbf{t}).$$

From the measure $f(\cdot|Z)$, I can readily derive the marginal distributions $f_\theta(\cdot|Z)$ and $f_v(\cdot|Z)$ over the signal and bidder values from the selected bidder. Having derived distributions $f(\cdot|Z)$, $f_\theta(\cdot|Z)$ and $f_v(\cdot|Z)$, I can compute $E[\psi(t)|Z]$ for any function $\psi : T \to \mathbb{R}$.

2.3 Payoffs

As in Armstrong (2006), users are heterogeneous in their outside options relative to using the platform's matching services. To capture this heterogeneity, I normalize the population of users to one and index users by their reservation values c^i, distributed according to the cdf G_c with compact support $S_c \equiv [0, C]$. The distribution $G_c(\cdot)$ is twice differentiable and log concave.[15]

Users make their participation decisions before learning the realized match induced by mechanism (Z, P, Q).[16] It then follows that a user with reservation value c^i joins the platform

[15] This assumption is satisfied by many popular distributions of compact support, e.g., the uniform and the beta with parameters $\alpha \leq 1$ and $\beta \geq 1$.

[16] More generally, one can consider mechanisms in which the matching rule Z and the user fee Q for each user i are allowed to depend on the user's outside option c^i. Because of the timing assumption, this depence

if and only if

$$E[u|Z] - Q - c^i \geq 0, \tag{2}$$

where $E[u|Z]$ is the expected user value from the winning bidder.[17]

In light of our leading example, web searchers click on sponsored links (or use the search engine in the first place) only when their alternative search resources, captured by c^i, offer a lower payoff. This implies that the total number of users available to the platform equals

$$S(Z,Q) = G_c(E[u|Z] - Q).$$

I refer to $S(Z,Q)$ as the platform's *user base* (or *supply of users*). Note that the supply of users increases in the surplus that users expect to obtain from the platform, $E[u|Z] - Q$.

Denote by $z(\theta, \hat{v})$ the probability that a bidder reporting bidder value $\hat{v}$ obtains the match when his signal is θ and all other bidders truthfully report their bidder values,

$$z(\theta, \hat{v}) \equiv E_{\mathbf{t}_{-j}}\left[Z_j((\theta, \hat{v}); \mathbf{t}_{-j})\right],$$

and by $p_j(\theta, \hat{v})$ the expected payment from doing so,

$$p_j(\theta, \hat{v}) \equiv E_{\mathbf{t}_{-j}}\left[P_j((\theta, \hat{v}); \mathbf{t}_{-j})\right].$$

Equipped with this notation, I can now write the individual rationality (IR) constraints for each bidder j who joins the platform:

$$S(Z,Q) \cdot z_j(\theta, v) \cdot v - p_j(\theta, v) \geq 0. \tag{3}$$

The first term in (3) is the expected value appropriated by a bidder with type (θ, v) from joining the platform. Importantly, this term crucially depends on the total number of users, $S(Z,Q)$. In an analogous manner, the incentive compatibility (truth-telling) constraint for each bidder j and for each type (θ, v) takes the following form:

$$\begin{aligned} & S(Z,Q) \cdot z_j(\theta, v) \cdot v - p_j(\theta, v) \\ \geq\ & S(Z,Q) \cdot z_j(\theta, \hat{v}) \cdot v - p_j(\theta, \hat{v}) \quad \text{for all } v, \hat{v}. \end{aligned} \tag{4}$$

turns out to be redundant, since in the optimal implementable mechanism Z would be invariant on c^i and ask the platform to post uniform prices for users. Therefore, in order to simplify the exposition, I restrict attention to the relevant class of mechanisms in which Z and Q are anonymous.

[17] The participation decision from users can be interpreted as a decision to avoid exposure to advertising. See Van Zandt (2004), Anderson and de Palma (2007), Anderson and Gans (2006), and Johnson (2009) for detailed models of advertising avoidance.

As is standard in the mechanism-design literature, I say a matching rule Z is *implementable* if there is a pair of bidder payments and user fee (P, Q) such that (Z, P, Q) satisfies the IR and IC constraints (3) and (4).

The platform's expected profits equal its revenue from charging users plus the expected payments from bidders:

$$\Pi(Z, P, Q) = S(Z, Q) \cdot Q + \sum_{j=1}^{N} E\left[P_j(\mathbf{t})\right]. \tag{5}$$

The expression for profits in (5) makes clear the two-sidedness of the platform's problem. On the one hand, the choice of the matching rule Z affects the user base, $S(Z, Q)$. On the other hand, it affects the rents that can be extracted from bidders, as the payment rule $P(\mathbf{t})$ has to satisfy constraints (3) and (4). Thus, the profit-maximizing matching rule Z, together with the payment rule P and the user fee Q, has to weigh the effects on extracting rents from both sides of the market.

The timing of the model is as follows:

1. The platform announces its matching mechanism (Z, P, Q),
2. Users and bidders simultaneously decide to join the platform's matching services,
3. Participating bidders report $\hat{v}$ and the platform observes signals θ,
4. The platform selects the match according to Z, and
5. Users and bidders make payments according to Q and P.

I start the analysis by deriving the efficient allocation.

3 Efficiency

To set a benchmark, I will now characterize the allocation of users and bidders that maximizes the ex-ante efficiency produced by the platform.

Denote by $\bar{c}$ the threshold such that all users with reservation values $c^i \leq \bar{c}$ join the platform and all users with $c^i > \bar{c}$ do not join the platform. The efficient allocation of users and bidders is described by a matching rule Z and a threshold $\bar{c}$ that solve:

$$\max_{Z, \bar{c}} G_c(\bar{c}) \cdot \left[E\left[u + v | Z\right] - E[c | c \leq \bar{c}]\right].$$

The expression above is intuitive. The term inside the brackets is the expected value of a match, $E[u+v|Z]$, net of the average (opportunity) cost of all users that join the platform, $E[c|c \leq \bar{c}]$. The term outside the brackets, $G_c(\bar{c})$, is the total number of matches produced by the platform.

Now note that by the law of iterated expectations:

$$E[u|Z] = E[E[u|\theta, v]|Z] = E[\mu(\theta, v)|Z],$$

As a consequence, the platform's problem becomes:

$$\max_{Z,\bar{c}} G_c(\bar{c}) \cdot [E[\mu(\theta, v) + v|Z] - E[c|c \leq \bar{c}]]. \tag{6}$$

Recall that the matching rule Z^E (see Example 1) picks for every profile of bidders' types the one that produces the match with the highest score $s^E(t) = \mu(\theta, v) + v$. Since the objective function in (6) is strictly increasing in $E[\mu(\theta, v) + v|Z]$, this obviously implies that no matching rule can produce matches that lead to higher welfare than Z^E. Moreover, it is immediate from (6) that, at the optimum, a user with reservation value c^i should join the platform if and only if $c^i \leq \bar{c}^* = E\left[\mu(\theta, v) + v|Z^E\right]$.

The next proposition describes a variant of the Vickrey-Clark-Groves (VCG) mechanism that implements the efficient allocation in our two-sided-market setting. In order to do so, let's denote by $t^{(l)} = (\theta^{(l)}, v^{(l)})$ the type of the bidder with the l-th highest score $s^E(t)$ (ties are broken arbitrarily). Define by $\mathring{v}$ the solution (if one exists) to:

$$\mathring{v} \equiv \min\{v : \mu(\theta^{(2)}, v^{(2)}) + v^{(2)} \leq \mu(\theta^{(1)}, v) + v\}.^{18} \tag{7}$$

We can then state:

Proposition 1 (Efficiency) *The ex-ante efficient allocation is such that:*

1. *the winning bidder is selected according to the matching rule Z^E, and*
2. *all users with reservation values $c^i \leq \bar{c}^* = E\left[u+v|Z^E\right]$ join the platform.*

   Consider a mechanism (Z^E, P^E, Q^E) such that $Q^E = -E\left[v|Z^E\right]$ and

$$P_j^E(\mathbf{t}) = G_c(\bar{c}^*) \cdot \mathbf{1}\{t_j = t^{(1)}\} \cdot (\mu(\theta^{(2)}, v^{(2)}) + v^{(2)} - \mu(\theta^{(1)}, \mathring{v})),$$

where $\mathbf{1}\{t_j = t^{(1)}\}$ is an indicator function that equals one if and only if $t_j = t^{(1)}$. Then

[18] If $\mu(\theta^{(2)}, v^{(2)}) + v^{(2)} > \mu(\theta^{(1)}, v) + v$ for all $v \in V$, then I set $\mathring{v} \equiv +\infty$.

the efficient mechanism (Z^E, P^E, Q^E) *satisfies the IR and IC constraints (3) and (4), that is,* Z^E *is implementable.*[19]

Though simple, the ex-ante efficient solution has two important features: first and foremost, users and bidders are matched to maximize the total expected surplus from the match. Second, the deadweight loss from the supply of users is zero, since all users whose reservation values, c^i, are smaller than the expected total surplus from the match, $E\left[u + v | Z^E\right]$, join the platform.

4 The Monopolistic Platform

By the Revelation Principle, all matching rules that can be supported in a Bayes-Nash equilibrium of an indirect mechanism can also be supported as the outcome of a truthful direct revelation mechanism. As such, the platform's problem is to select a mechanism (Z, P, Q) to maximize the profit function (5) subject to the bidders' participation constraints (3) and incentive compatibility constraints (4). We first characterize the set of implementable matching rules.

4.1 Implementability

The next lemma uses standard mechanism design techniques to pin down the IC and IR constraints that bind at the optimum:

Lemma 1 *Fix the user fee* Q *and let* Z *and* P *be matching and payment rules that maximize profits (5) subject to the IR constraints (3) and the IC constraints (4). Then the only constraints that bind at the optimum are the IC constraints*

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) = S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^l - p_j(\theta^k, v^{l-1})$$
$$\textit{for all} \quad k \in \{1, ..., T_1\},\ l \in \{1, ..., T_2\} \tag{8}$$

and the IR constraints for bidders with bidder values v^1*:*

$$p_j(\theta^k, v^1) = S(Z,Q) \cdot z_j(\theta^k, v^1) \cdot v^1$$
$$\textit{for all} \quad k \in \{1, ..., T_1\}. \tag{9}$$

[19] The mechanism (Z^E, P^E, Q^E) extends to a two-sided-market setting the construction of Dasgupta and Maskin (2000).

Moreover, (8) and (9) imply conditions (3) and (4), provided $z_j(\theta, v)$ weakly increases in v for all θ and j.

By solving the recursion (8) with initial condition (9), we obtain that the expected payment of a bidder with type (θ, v) induced by the allocation Z and the fee Q is given by

$$p_j(\theta, v) = G_c(E[u|Z] - Q) \cdot \left\{ z_j(\theta, v) \cdot v - \epsilon \cdot \sum_{v'<v} z_j(\theta, v') \right\}. \tag{10}$$

Expression (10) extends the celebrated payoff equivalence formula to a two-sided setting. Unlike the one-sided problem, though, bidders' payments depend not only on the matching rule Z, but also on the user fee Q (as it determines participation from users).

Plugging this expression into the objective function (5), we can characterize the platform's revenue in terms of Z and Q. This is the subject of our next lemma:

Lemma 2 *Fix the user fee Q, and let Z and P be the matching rule and the payment rule that maximize profits (5) subject to the IR constraints (3) and the IC constraints (4). Then the revenue generated by this mechanism can be expressed as a function of Z and Q:*

$$G_c(E[u|Z] - Q) \cdot (Q + E[\omega(\theta, v)|Z]),$$

where

$$\omega(\theta, v) = v - \epsilon \cdot \frac{1 - F(v|\theta)}{f(v|\theta)} \qquad \text{for all} \qquad (\theta, v) \in \Theta \times V.$$

Assumption 1 implies that virtual bidder value, $\omega(\theta, v)$, is weakly decreasing in θ for all $v \in V$. In turn, the monotone hazard rate condition implies that $\omega(\theta, v)$ is strictly increasing in v for all $\theta \in \Theta$.

The lemma above is of fundamental importance for solving the platform's problem. Indeed, it implies that one can eliminate bidders' payments and write the platform's problem as one of choosing a matching rule Z and fees Q to

$$\max_{Z,Q} \; G_c(E[u|Z] - Q) \cdot (Q + E[\omega(\theta, v)|Z]). \tag{11}$$

I start with two-sided mechanisms.

4.2 The Revenue-Maximizing Two-Sided Mechanism

I will now derive the matching rule Z and the user fee Q that solve problem (11). To this end, consider the following matching rule:

Definition 2 (Virtual Efficient Matching Rule) *The matching rule Z^{II} selects the bidder with the highest nonnegative score:*

$$s^{II}(t) = \mu(\theta, v) + \omega(\theta, v). \tag{12}$$

I refer to Z^{II} as the virtual efficient matching rule.

The next lemma shows that, for any matching rule Z, the platform can always increase profits by moving to rule Z^{II} and adjusting fees appropriately:

Lemma 3 *Let a two-sided mechanism (Z, P, Q) satisfy the IR constraints (3) and the IC constraints (4). Now consider the supply-preserving fee $\bar{Q}$ satisfying*

$$E\left[u|Z\right] - Q = E\left[u|Z^{II}\right] - \bar{Q},$$

where Z^{II} is the virtual efficient matching rule of Definition 2 and P^{II} are payments satisfying equation (10). The platform's profits are weakly greater under $(Z^{II}, P^{II}, \bar{Q})$ than under (Z, P, Q).

Proof. First, note that scoring rule $s^{II}(t)$ is strictly increasing in v, since $\mu(\theta, v)$ weakly increases in v by Assumption 1 and $\omega(\theta, v)$ strictly increases in v from the monotone hazard rate condition. Therefore, the interim probability $z^{II}(t)$ implied by Z^{II} is strictly increasing in v, which implies that Z^{II} is implementable by Lemma 1.

Second, note that, by the law of iterated expectations,

$$E\left[u + \omega(\theta, v)|Z\right] = E\left[\mu(\theta, v) + \omega(\theta, v)|Z\right],$$

which implies that the matching rule Z^{II} from Definition 2 maximizes $E\left[u + \omega(\theta, v)|Z\right]$ among all rules Z that are implementable.

Moreover, for any fixed P, the platform's payoff from adopting the mechanism (Z, P, Q) is

$$\begin{aligned}
\Pi(Z, P, Q) &= G_c\left(E\left[u|Z\right] - Q\right) \cdot \left(Q + E\left[\omega(\theta, v)|Z\right]\right) \\
&= G_c\left(E\left[u|Z^{II}\right] - \bar{Q}\right) \cdot \left(Q + E\left[\omega(\theta, v)|Z\right]\right) \\
&= G_c\left(E\left[u|Z^{II}\right] - \bar{Q}\right) \cdot \left(E\left[u + \omega(\theta, v)|Z\right] - E\left[u|Z^{II}\right] + \bar{Q}\right) \\
&\leq G_c\left(E\left[u|Z^{II}\right] - \bar{Q}\right) \cdot \left(E\left[u + \omega(\theta, v)|Z^{II}\right] - E\left[u|Z^{II}\right] + \bar{Q}\right) \\
&= G_c\left(E\left[u|Z^{II}\right] - \bar{Q}\right) \cdot \left(\bar{Q} + E\left[\omega(\theta, v)|Z^{II}\right]\right) \\
&= \Pi((Z^{II}, P, \bar{Q}),
\end{aligned}$$

where the second equality follows from the construction of $\bar{Q}$ and the inequality (in the fourth line) uses the fact that Z^{II} maximizes $E[u + \omega(\theta, v)|Z]$ among all rules Z that are implementable. This proves the lemma.■

To build intuition for Lemma 3, consider the extreme case in which all users have reservation value $c^i = 0$ (that is, G_c is degenerate at 0). In this case, the platform can fully extract rents from users by setting $Q = E[u|Z]$. Since the platform extracts $E[\omega(\theta, v)|Z]$ from bidders for every user who joins the platform, maximizing profits amounts to selecting the matching rule Z that produces matches with the highest total virtual value in expectation: $E[u + \omega(\theta, v)|Z]$. As shown above, this is accomplished by rule Z^{II}.

More generally, the platform has to leave some rents to users (as the support of G_c has positive measure). Nevertheless, Lemma 3 shows that Z^{II} is still optimal. As the proof above makes clear, the ability to charge the user fee Q is key to this result: by setting the appropriate Q, the platform can fix the supply of users at any given level. Once it has done so, the rents enjoyed by the platform depend only on the total virtual value, $E[u + \omega(\theta, v)|Z]$, which is maximized by Z^{II}.

We can now solve for the optimal user fee, Q^{II}. It follows from equation (11) and Lemma 3 that we can rewrite the platform's problem (11) only in terms of Q:

$$\max_{Q} G_c\left(E\left[\mu(\theta, v)|Z^{II}\right] - Q\right) \cdot \left(Q + E\left[\omega(\theta, v)|Z^{II}\right]\right). \tag{13}$$

Taking the first-order condition leads to

$$Q^{II} + E\left[\omega(\theta, v)|Z^{II}\right] = \frac{E\left[\mu(\theta, v)|Z^{II}\right] - Q^{II}}{\eta\left(E\left[\mu(\theta, v)|Z^{II}\right] - Q^{II}\right)}, \tag{14}$$

where $\eta(\cdot)$ denotes the elasticity of supply with respect to user surplus:

$$\eta(x) \equiv x \cdot \frac{g_c(x)}{G_c(x)}.$$

The formula (14) is reminiscent of the classical Lerner formula: its left-hand side captures the marginal gain from adding one extra user to the platform ($E\left[\omega(\theta, v)|Z^{II}\right]$ accounts for the rents collected from the bidder side of the market and reads like a negative marginal cost), while its right-hand side captures the inframarginal losses from decreasing user fees.

To guarantee an interior solution to the problem, I make the technical assumption that the support of G_c, $[0, C]$, is such that $E\left[u + \omega(\theta, v)|Z^{II}\right] < C$. I can then state:

Proposition 2 *The optimal two-sided mechanism employs the virtual efficient matching rule*

Z^{II}, sets user fees Q^{II} according to the Lerner formula (14) and sets payments for bidders P^{II} according to the payoff equivalence formula (10) evaluated at Z^{II} and Q^{II}. Moreover, users are subsidized ($Q^{II} < 0$) if and only if

$$\frac{E\left[\mu(\theta,v)|Z^{II}\right]}{\eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right)} \leq E\left[\omega(\theta,v)|Z^{II}\right]. \tag{15}$$

As Proposition 2 makes clear, the division of the expected total value $(\mu(\theta,v)+\omega(\theta,v))$ from the match determines whether users are charged or subsidized at the optimum. Indeed, after controlling for the supply conditions captured in $\eta(\cdot)$ and replacing bidder values by virtual values (which adjusts for informational rents), condition (15) simply compares the expected bidder and user values from the match. If the platform can extract more rents from bidders than from users, then the optimal pricing policy is to subsidize users (using proceeds collected from bidders) to boost supply and further extract rents from bidders. This is the familiar loss leader strategy so common in online platforms. The next example illustrates Proposition 2:

Example 2 (Linear Supply) *Let the users' reservation values be uniformly distributed over $[0,1]$, such that $S(Z^{II},Q) = E\left[\theta|Z^{II}\right] - Q^{II}$. Then the following fee is optimal:*

$$Q^{II} = \frac{E\left[\mu(\theta,v)-\omega(\theta,v)|Z^{II}\right]}{2}.$$

In this case, $\eta\left(E\left[\theta|Z^{II}\right]\right) = 1$ and users are subsidized if and only if

$$E\left[\mu(\theta,v)|Z^{II}\right] < E\left[\omega(\theta,v)|Z^{II}\right].$$

In practice, some platforms do offer subsidies per transaction to users. In keyword advertising, for example, LiveSeacrh (now Bing) introduced its cash-back program in May 2008. Through this program, Microsoft mails back to users a fraction of the posted price of a purchase from qualifying sponsored links.[20] Perhaps the most notorious example of subsidies in online matching platforms is that of *Coupon.com*. This platform matches users to retailers offering discount coupons on selected products.

Aside from monetary subsidies awarded on a per-transaction basis, two-sided platforms offer a variety of services intended to attract users with whom the platform matches its bidders. In the case of search engines, users are attracted by free search services, which are directly financed by the revenue collected from advertisers. In my model, Q^{II} can be

[20] See Chen, Ghosh, McAfee and Pennock (2008) for an analysis of the impact of different cash-back rules on advertisers' bidding behavior and on the auction revenue.

interpreted as the optimal level of free services for users.

In contrast, platforms that generate more surplus for users than for bidders tend to charge the former group: job-matching agencies specializing in high-paying jobs (such as *TheLadders.com*) charge users for posting professional profiles online.

The direct revelation mechanism (Z^{II}, P^{II}, Q^{II}) has many features in common with the actual auctions used by search engines. Google uses a modified Generalized Second Price auction (see footnote 8) that assigns a score to each advertiser based on his bid and on his quality to users.

We will now compare the revenue-maximizing matching rule Z^{II} with the efficient rule Z^E regarding the vale they bring to users. This is subject of our next definition:

Definition 3 *Call matching rule Z more user-friendly than rule $\hat{Z}$ if $E[u|\hat{Z}] \leq E[u|Z]$.*

The next proposition compares the revenue-maximizing two-sided mechanism with the efficient mechanism derived in Proposition 1. Before presenting this result, let us define the *bias function* $B(\theta, v)$, which computes the expected difference between scoring rules $s^{II}(t)$ and $s^E(t)$ as a function of (θ, v):

$$B(\theta, v) \equiv E\left[s^E(t) - s^{II}(t)|\theta, v\right].$$

The bias function captures how, relative to efficiency, the virtual efficient rule Z^{II} distorts the assignment of bidders to users. Assume for a moment that the bias function $B(\theta, v)$ increases in θ and decreases in v. This means that the efficient scoring rule, $s^E(t)$, increases more quickly in θ and less quickly in v than scoring rule $s^{II}(t)$. By the positive affiliation between θ and u, this means that bidders with higher user values and lower bidder values are more likely to be selected by the efficient rule Z^E than by the virtual efficient rule Z^{II}. We can then state:

Proposition 3 *Relative to the efficient mechanism (Z^E, P^E, Q^E), the optimal two-sided mechanism (Z^{II}, P^{II}, Q^{II}):*

1. *attracts a smaller user base:*

$$S(Z^{II}, Q^{II}) < S(Z^E, Q^E),$$

2. *reduces total welfare by precluding socially efficient matches for which bidder values v are smaller than the "reserve prices" defined by:*

$$r(\theta) \equiv \inf\{v \in V : \ \mu(\theta, v) + \omega(\theta, v) \geq 0\}, \tag{16}$$

3. is less user-friendly, as $E[u|Z^{II}] \leq E[u|Z^E]$. *Moreover, its bias function, given by*

$$B(\theta, v) = \epsilon \cdot \frac{1 - F(v|\theta)}{f(v|\theta)},$$

increases in θ *and decreases in* v.

The first distortion from Proposition 3 indicates that the revenue-maximizing two-sided mechanism serves fewer users than efficiency dictates, as we can see from equation (14). This follows from the standard trade-off in monopoly pricing between attracting a larger user base and increasing the revenue per transaction.

The second distortion introduces reserve prices. Reserve prices handicap bidders with low bidder values in order to encourage those with high bidder values to pay more for the match. Interestingly, reserve prices vary across θ. In the search-engine example, this translates into Google setting up different reserve prices for bidders with different click-through rates.

The third distortion is a key insight of my model. At the heart of the platform's problem lies a conflict between extracting rents from bidders and attracting users. On the one hand, bidders with high signals θ are more likely to yield high user values, as captured by the fact that $\mu(\theta, v)$ weakly increases in θ (it follows from Assumption 1). As such, in order to attract users to the platform, the optimal matching rule has to assign higher scores to bidders with higher signals θ (this is the *user-base effect*). On the other hand, because of positive affiliation, bidders with high signals θ are more likely to have high bidder values v. In order to encourage them to pay more for the match, the platform has to distort the matching rule away from efficiency to favor bidders with low signals. This is captured by the fact that $\omega(\theta, v)$ weakly decreases in θ (I call it the *rent-extraction effect*). Proposition 3 shows that the rent-extraction effect biases the revenue-maximizing matching rule Z^{II} towards bidders with low user values.

The analysis above offers an intuitive explanation to the search diversion puzzle (see, for example, Hagiu and Jullien (2009)). In the revenue-maximizing mechanism, the platform uses the information provided by signals θ to incentivize bidders to pay high prices for the match. The optimal way to do so is to handicap bidders with high signals, which lowers the expected user value from the match relative to the efficient matching.

The next example narrows our discussion to the case in which θ is perfectly informative about the user value, u.

Example 3 (Fully Informative Signal) *Assume that* $\theta = u$, *that is, the platform can directly observe user values,* u. *In this case, the profit-maximizing matching rule is described*

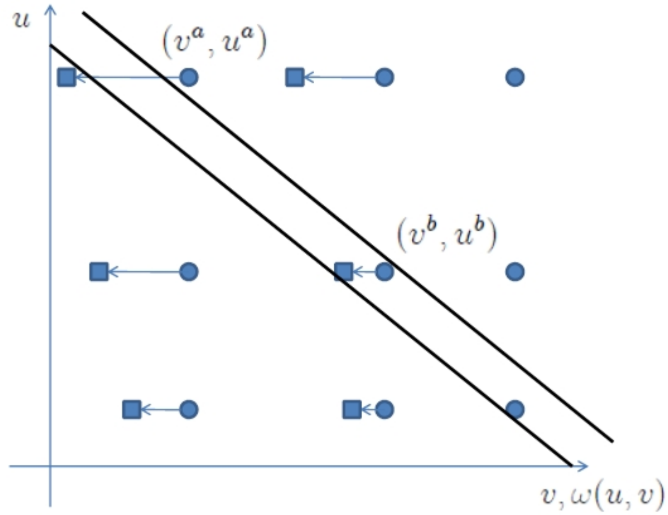

Figure 1: The profit-maximizing matching rule is less user-friendly than the efficient rule. Original types are represented by circles and virtual types by squares.

by the scoring rule:

$$s^{II}(t) = u + \omega(u, v).$$

Figure 1 depicts the indifference curves associated with scoring rules $s^{II}(t)$ *and* $s^E(t)$ *in the* (v, u) *axis. Circles represent original types* (v, u)*, while squares represent "virtual types"* $(\omega(u, v), u)$*. Clearly, type* (v^a, u^a) *is more efficient than* (v^b, u^b)*. Nevertheless, in order to maximize profits,* Z^{II} *favors bidders with low user values:* $s^{II}(t)$ *gives a higher score to* (v^b, u^b) *than to* (v^a, u^a)*. Therefore,* $E[u|Z^{II}] < E[u|Z^E]$*.*

The next example shows that if the signal θ provides no information regarding user and bidder values, then the optimal mechanism can be implemented by a standard second-price auction. More importantly, the expected user values from rules Z^{II} and Z^E are the same:

Example 4 (Uninformative Signal) *Assume that the signal* θ *is independent of user and bidder values* u *and* v*. In this case, the matching rule* Z^{II} *can be implemented by a second-price (Vickrey) auction with reserve price* r^* *given by*

$$\mu(r^*) + \omega(r^*) = 0,$$

where $\mu(r^*) \equiv \mu(\theta, r^*)$ *and* $\omega(r^*) \equiv \omega(\theta, r^*)$ *(which are invariant in* θ*). If* $\mu(v^1) + \omega(v^1) \geq 0$*, in which case the optimal reserve price is zero, the expected user value of the winning bidder is the same under the revenue-maximizing rule,* Z^{II}*, or the efficient rule* Z^E*:* $E[u|Z^{II}] = E[u|Z^E]$*.*

Before proceeding to one-sided mechanisms, I will analyze in the next subsection how the extend the characterization of Propositions 2 to environments in which the assumption of positive affiliation is relaxed.

4.2.1 The Case of Negative Affiliation

Consider the example of N online retailers selling a homogeneous product. In order to be matched with potential customers, retailers join a two-sided platform. Since products are homogeneous across stores, the retailers who charge higher prices (and have lower user values) are the ones who exhibit higher bidder values (profits per sale). This example violates Assumption 1, which posits that user and bidder values are positively affiliated.

In order to analyze economic situations like the one described above, I replace Assumption 1 with the following:

Assumption 2 (v, u) *are negatively affiliated.*

Importantly, from the negative affiliation between u and v, it follows that $\mu(\theta, v)$ is now weakly decreasing in v. As before, the monotone hazard rate condition implies that $\omega(\theta, v)$ is strictly increasing in v.

The next proposition shows that the mechanism (Z^{II}, P^{II}, Q^{II}) remains optimal under Assumption 2 if and only if the effect of bidder values v is higher on virtual values $\omega(\theta, v)$ than on expected user values $\mu(\theta, v)$.[21]

Proposition 4 *Under the alternative Assumption 2, the optimal two-sided mechanism employs the virtual efficient matching rule Z^{II} if and only if:*

$$\left|\mu(\theta, v^{l+1}) - \mu(\theta, v^{l})\right| \leq \left|\omega(\theta, v^{l+1}) - \omega(\theta, v^{l})\right| \qquad \text{for all } l \text{ and } \theta \in \Theta.$$

If this condition holds, the optimal user fee Q^{II} is given by equation (14) and bidder payments P^{II} satisfy equation (10) evaluated at Z^{II} and Q^{II}.

The condition for the matching rule Z^{II} to be optimal under Assumption 2 requires that the scoring rule s^{II} is strictly increasing in v. This condition may be violated if the negative affiliation between u and v is so strong that $\mu(\theta, v)$ decreases faster in v than $\omega(\theta, v)$ increases in v. In this case, the optimal matching rule is an "ironed" version of Z^{II}.[22].

Proposition 4 shows that the optimality of (Z^{II}, P^{II}, Q^{II}) is robust to environments in which the positive affiliation between user and bidder values is relaxed. I will now analyze

[21] This condition is analogous to that of Dasgupta and Maskin (2000).

[22] See Skreta (2007) for a general treatment of ironing procedures, for discrete, mixed and continuous distributions.

the platform's problem with one-sided mechanisms. From now on, I assume that Assumption 1 holds.

4.3 The Revenue-Maximizing One-Sided Mechanism

In many two-sided markets, charging or subsidizing users is not practical. Online job-matching agencies cannot pay users for each uploaded resume, as one can easily create fake profiles with the sole intent of collecting subsidies. More generally, transaction and decision-making costs may prevent matching platforms from directly charging users for their services.

In such cases, the platform has to choose among one-sided mechanisms (Z, P). This amounts to solving problem (11) subject to the additional constraint that $Q = 0$. We can then rewrite this program as

$$\max_Z \; G_c(E[u|Z]) \cdot E[\omega(\theta, v)|Z] = \max_Z \; G_c(E[\mu(\theta, v)|Z]) \cdot E[\omega(\theta, v)|Z], \tag{17}$$

where the equality follows from the law of iterated expectations.

In order to solve this problem, take a matching rule Z and an arbitrary profile of types $\mathbf{t}$. Let Z assign a positive probability to the event that bidder j with type (θ, v) obtains the match, and consider a bidder $\hat{\jmath}$ with type $(\hat{\theta}, \hat{v})$. Now let us perturb Z by picking bidder $\hat{\jmath}$ instead of j with probability q, whenever bidder j is selected for the match. The marginal gain from doing so is given by

$$\begin{aligned} \frac{\partial \Pi}{\partial q}(Z) \quad \propto \quad & \underbrace{g_c(E[\theta|Z]) \cdot (\mu(\hat{\theta}, \hat{v}) - \mu(\theta, v)) \cdot E[\omega(\theta, v)|Z]}_{\text{user-base effect}} \\ & + \underbrace{G_c(E[\theta|Z]) \cdot (\omega(\hat{\theta}, \hat{v}) - \omega(\theta, v))}_{\text{rent-extraction effect}}. \end{aligned} \tag{18}$$

The first term in expression (18), $g_c(E[\theta|Z]) \cdot (\mu(\hat{\theta}, \hat{v}) - \mu(\theta, v)) \cdot E[v|Z]$, accounts for the impact of swapping (θ, v) and $(\hat{\theta}, \hat{v})$ on the supply of users. If, for example, $\mu(\hat{\theta}, \hat{v}) > \mu(\theta, v)$, then more users come to the platform once its matching rule picks bidders with type $(\hat{\theta}, \hat{v})$ rather than (θ, v). This is the familiar *user-base effect*, in the context of one-sided mechanisms.

The second term, $G_c(E[\theta|Z]) \cdot (\omega(\hat{\theta}, \hat{v}) - \omega(\theta, v))$, captures the impact on the total rents collected from bidders. If, for example, $\omega(\hat{\theta}, \hat{v}) < \omega(\theta, v)$, then the platform extracts less rents from bidders as its matching rule favors $(\hat{\theta}, \hat{v})$ rather than (θ, v). This is the *rent-extraction effect.*

Denote by Z^I the solution to problem (17). Clearly, expression (18) evaluated at Z^I has to satisfy $\frac{\partial \Pi}{\partial q}(Z^I) \leq 0$ for any bidder $\hat{\jmath}$ in profile $\mathbf{t}$, as otherwise the platform could strictly increase profits by matching users with $\hat{\jmath}$ (instead of j). Therefore, at the optimum Z^I, a bidder with type (θ, v) obtains the match when a bidder $\hat{\jmath}$ with type $(\hat{\theta}, \hat{v})$ is available if and only if

$$\begin{aligned} &\omega(\hat{\theta}, \hat{v}) + \eta\left(E\left[\mu(\theta, v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta, v)|Z^I\right]}{E\left[\mu(\theta, v)|Z^I\right]} \cdot \mu(\hat{\theta}, \hat{v}) \\ &\leq \omega(\theta, v) + \eta\left(E\left[\mu(\theta, v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta, v)|Z^I\right]}{E\left[\mu(\theta, v)|Z^I\right]} \cdot \mu(\theta, v). \end{aligned} \tag{19}$$

Interestingly, condition (19) reveals that the gain from swapping any two bidders with signals and bidder values (θ, v) and $(\hat{\theta}, \hat{v})$ is the same regardless of the profile $\mathbf{t}$. This implies that the optimal matching rule Z^I can be described by a scoring rule, as stated by the next proposition:

Proposition 5 *The profit-maximizing one-sided mechanism employs the matching rule Z^I implicitly described by the scoring rule:*

$$s^I(t) = \omega(\theta, v) + \eta\left(E\left[\mu(\theta, v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta, v)|Z^I\right]}{E\left[\mu(\theta, v)|Z^I\right]} \cdot \mu(\theta, v). \tag{20}$$

Moreover, the bidders' payment rule P^I satisfies equation (10) evaluated at the matching rule Z^I and $Q \equiv 0$.

The fixed-point formula (20) shows that the platform places more weight on the expected user values, $\mu(\theta, v)$, as it can extract more rents from the bidder side of the market, $\frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]}$. This is the one-sided analogue of the loss leader strategy described in the context of two-sided mechanisms: the platform wishes to provide a higher surplus to users (and increase supply) as it extracts more rents from bidders.

Assume for a moment that $E\left[\mu(\theta, v)|Z^{II}\right] = \eta\left(E\left[\mu(\theta, v)|Z^{II}\right]\right) \cdot E\left[\omega(\theta, v)|Z^{II}\right]$, which implies by Proposition 2 that $Q^{II} = 0$. Under this condition, as direct inspection can confirm, the profit-maximizing two-sided matching rule Z^{II} solves the fixed-point equation (20) and, therefore, $Z^I = Z^{II}$. In this case, Proposition 3, which lists the distortions relative to efficiency produced by Z^{II}, also applies to Z^I. Namely, Z^I produces three types of inefficiencies: it serves a smaller user base, it introduces inefficient reserve prices and it is biased towards bidders with low user values.

But there is more: whenever the two-sided mechanism charges or subsidizes users, the one-sided mechanism has to distort its matching rule further to account for the fees/subsidies it can no longer implement.

To formalize this idea, let us now define the class of scoring rules, Υ, that linearly combines the virtual bidder value, $\omega(\theta, v)$, and the expected user value, $\mu(\theta, v)$:

$$\Upsilon = \{s(t) : s(t) = \omega(\theta, v) + b \cdot \mu(\theta, v) \text{ for some } b \in \mathbb{R}\}.$$

Clearly, the scoring rules $s^I(t)$ and $s^{II}(t)$ that represent rules Z^I and Z^{II}, respectively, belong to class Υ. Now, consider two matching rules, Z and $\hat{Z}$, represented by scoring rules $s(t), \hat{s}(t) \in \Upsilon$ with coefficients b and $\hat{b}$, respectively. It is easy to show that Z is more user-friendly than $\hat{Z}$ (that is, $E[u|Z] \geq E[u|\hat{Z}]$) if and only if $b \geq \hat{b}$.

Our next proposition uses this observation to establish that matching rule Z^I is more user-friendly than the virtual efficient rule Z^{II} if the profit-maximizing two-sided mechanism subsidizes users, and is more bidder-friendly if the profit-maximizing two-sided mechanism charges users.

Proposition 6 *The matching rule Z^I is more user-friendly than the virtual efficient rule Z^{II} if and only if users are subsidized under the profit-maximizing two-sided mechanism, that is,*

$$b^I = \eta\left(E\left[\mu(\theta, v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta, v)|Z^I\right]}{E\left[\mu(\theta, v)|Z^I\right]} \geq 1 \ = b^{II} \qquad \Leftrightarrow \qquad Q^{II} \leq 0.$$

The next example illustrates Proposition 6 when the supply of users is linear:

Example 5 (Linear Supply with One-Sided Mechanisms) *Consider the primitives of Example 2, where the supply of users assumes the linear form $S(Z, Q) = E[\theta|Z] - Q$. The one-sided matching rule Z^I is more user-friendly than the virtual efficient rule Z^{II} if and only if $E\left[\mu(\theta, v)|Z^{II}\right] < E\left[\omega(\theta, v)|Z^{II}\right]$.*

The scoring rule in Proposition 5 optimally trades off attracting users to the platform and extracting rents from bidders. Absent the fees/subsidies for users, the matching rule Z^I is the sole instrument the platform can deploy to manage both sides of the market. As a consequence, it must directly incorporate the user-supply elasticity, $\eta(\cdot)$.

The supply of users $S(Z, Q) = G_c(E[u|Z] - Q)$ is said to be *more elastic* than $\hat{S}(Z, Q) = \hat{G}_c(E[u|Z] - Q)$ if their respective elasticity mappings, $\eta(\cdot)$ and $\hat{\eta}(\cdot)$, satisfy $\eta(\cdot) \geq \hat{\eta}(\cdot)$ at every point. The next corollary shows that the profit-maximizing one-sided matching rule is more user-friendly the more elastic is the supply curve $S(Z, Q)$. Intuitively, a more elastic supply curve tips the balance towards the user-base effect, inducing the matching rule Z^I to give more weight to the expected user value from bidders.

Naturally, as the user-base effect becomes more prominent, the platform foregoes extracting rents from bidders. This effect is more pronounced for bidders with the highest possible signal θ^{T_1}. To understand why, let's compute from (10) the informational rents per user earned by a bidder with type (θ, v):

$$\epsilon \cdot \sum_{v'<v} z_j(\theta, v')$$

As the matching rule Z^I becomes more user-friendly (giving more weight to $\mu(\theta, v)$), the probability that bidders with signals θ^{T_1} win the auction increases. From the expression above it then follows that the informational rents per user earned by bidders with types (θ^{T_1}, v) must increase. The opposite happens to bidders with the lowest possible signal θ^1, as their interim probabilities of obtaining the match decreases for all possible bidder values $v \in V$. For intermediate signals $\theta^1 < \theta < \theta^{T_1}$, the direction of change on informational rents depends nontrivially on the distribution F. Yet, the overall effect in the total rents collected from bidders is clearly negative, that is, $E\left[\omega(\theta, v)|Z^I\right]$ decreases as Z^I becomes more user-friendly.

Corollary 1 *The optimal matching rule in a one-sided mechanism Z^I is more user-friendly the more elastic is the supply of users. Moreover, bidders with types $\left\{(\theta^{T_1}, v) : v \in V\right\}$ enjoy higher informational rents per user, while bidders with types $\left\{(\theta^1, v) : v \in V\right\}$ enjoy lower informational rents per user. Overall, as the supply of users becomes more elastic, the platform extracts lower rents from bidders in expectation, $E\left[\omega(\theta, v)|Z^I\right]$.*

Regulatory interventions in two-sided markets may lead to unexpected outcomes. What is the effect of banning fees/subsidies for users of online recommendation systems (such as those of Amazon, Borders and Barnes and Noble)? The next example shows that, when $Q^{II} < 0$, the new distortion brought by Z^I (intended to compensate for the lack of subsidies) countervails the bias towards bidders with low user values exhibited by the virtual efficient rule Z^{II}. As it turns out, these conflicting forces may actually render matching rule Z^I more efficient.

Example 6 (Banning Subsidies for Users May Lead to More Efficient Matching) *Assume that $\theta = u$, in which case the profit-maximizing two-sided mechanism employs the virtual efficient rule Z^{II}. Further, assume that u, v are drawn from a distribution with binary support $U \times V = \{u^L, u^H\} \times \{v^L, v^H\}$.*

As depicted in Figure 2, type (v^L, u^H) is more efficient than type (v^H, u^L). Nevertheless, the rule Z^{II} scores (v^H, u^L) higher than (v^L, u^H), introducing a bias towards bidders with low

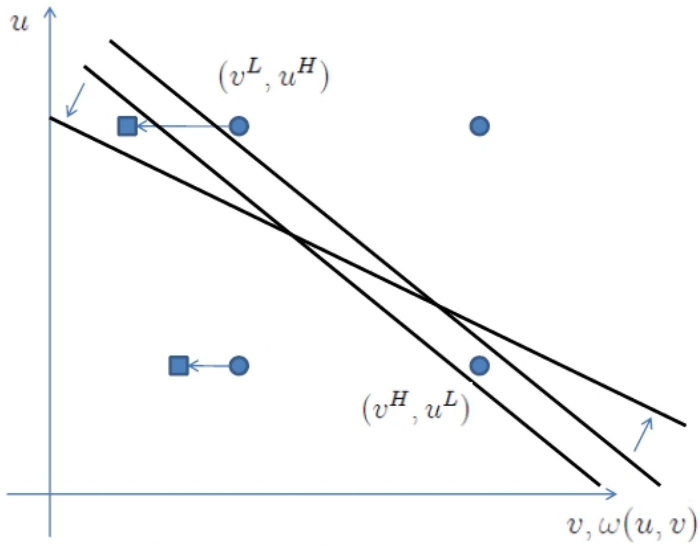

Figure 2: Banning subsidies to users may lead to more efficient matching rules.

user values.[23] *When* $Q^{II} < 0$*, the one-sided matching rule* Z^I *introduces a countervailing bias, represented by the counter-clockwise rotation in the indifference curve. If the supply of users* $S(Z, Q)$ *is sufficiently elastic, this new bias is such that* $s^I(\theta^L, v^H) < s^I(\theta^H, v^L)$*. Therefore, banning subsidies for users restores efficiency in matching!*

I now examine competition between matching platforms.

5 Competing Platforms

Markets in which two-sided platforms operate are usually dominated by a handful of firms, which compete with one another through their matching rules as well as by setting payments to each side of the market. Examples of such oligopolies can be found in disparate businesses, such as Internet-based travel agencies (*Expedia*, *Orbitz*, *Kayak,* etc.), rental agencies (*Rent.com*, *ForRent.com,* etc.) and job agencies (*Monster.com*, *CareerBuilder.com,* etc.).

In this section, I extend my baseline model to study the competition between matching platforms. Following Evans (2008) and in line with casual empiricism, I assume that users patronize a single platform (*single-homing*), while bidders can join more than one platform (*multi-homing*). I first analyze the case in which platforms can freely charge or subsidize users. I then proceed to the case in which platforms are not able to charge user fees.

[23] Z^{II} scores (v^H, u^L) higher than (v^L, u^H), overturning efficiency, if and only if $\frac{v^H - v^L}{\epsilon} \leq 1 + \frac{f(v^H, u^H)}{f(v^L, u^H)}$.

5.1 Competition with Two-Sided Mechanisms

For simplicity, only two platforms, A and B, compete in the user market. They simultaneously choose two-sided matching mechanisms $M_A = (Z_A, P_A, Q_A)$ and $M_B = (Z_B, P_B, Q_B)$, after which bidders and users decide which platform they wish to join.

Users have innate preferences regarding platforms A and B. Some users may be more inclined to consult platform A, for example, because that was the one recommended by their Internet browser. To capture the users' heterogeneity regarding the choice of platform, I introduce the idiosyncractic preference parameter d^i, which (by convention) measures user i's inclination for platform B. As before, instead of joining one of the two platforms, each user i may choose an outside option that gives him a payoff c^i. As a consequence, user i joins platform A if and only if

$$E\left[u|Z_A\right] - Q_A - c^i \geq \max\{E\left[u|Z_B\right] - Q_B - c^i + d^i, 0\}$$

and joins platform B if and only if

$$E\left[u|Z_B\right] - Q_B - c^i \geq \max\{E\left[u|Z_A\right] - Q_A - c^i - d^i, 0\}.$$

Note that preference parameter d^i does not affect the users' decision on whether to stick to the outside option or join *some* platform. Instead, it only affects the comparison across platforms, and, accordingly, should be thought of as a brand effect.

As before, let c^i be distributed according to the cdf $G_c(\cdot)$ with support on $[0, C]$. Further, d^i is distributed according to the unimodal symmetric cdf $G_d(\cdot)$ with support on $[-D, +D]$. The distributions $G_c(\cdot)$ and $G_d(\cdot)$ are assumed to be log concave and twice differentiable. To simplify the analysis, c^i and d^i are independent for each user i. As a consequence, platforms A and B face the following supply system:

$$S_A(M_A, M_B) = G_d\left(E\left[u|Z_A\right] - Q_A - E[u|Z_B] + Q_B\right) \cdot G_c\left(E\left[u|Z_A\right] - Q_A\right), \quad (21)$$

$$S_B(M_A, M_B) = \left(1 - G_d\left(E\left[u|Z_A\right] - Q_A - E[u|Z_B] + Q_B\right)\right) \cdot G_c\left(E[u|Z_B] - Q_B\right). \quad (22)$$

I look for a profile of matching mechanisms (M_A^{II}, M_B^{II}) that constitute a Nash equilibrium of this game. As such, for each platform J, the selected mechanism $(Z_J^{II}, P_J^{II}, Q_J^{II})$ should constitute a best response to M_{-J}^{II}:

$$M_J^{II} = \text{argmax}_{M_J}\ S_J(M_J, M_{-J}^{II}) \cdot Q_J + \sum_{j=1}^{N} E\left[P_{J,j}(\mathbf{t})\right], \quad (23)$$

subject to the bidders' IR, IC and feasibility constraints (3), (4) and (1), respectively.

Since bidders are allowed to multi-home, their decisions to join each platform are independent, that is, the decision whether to join platform A depends on M_A but is not affected by the mechanism M_B adopted by platform B. As a consequence, we can apply the same techniques used before to assess which IR and IC constraints bind at each platform's maximization problem. In particular, Lemmas 1 and 2 can be immediately replicated, and we can rewrite (23) as

$$M_J^{II} = \text{argmax}_{M_J}\ S_J(M_J, M_{-J}^{II}) \cdot \left(Q_J + E\left[\omega(\theta,v)|Z_J\right]\right), \tag{24}$$

subject to the now familiar monotonicity constraint that $z_{J,j}(\theta,v,u)$ is weakly increasing in v for all θ, u and j.

The next lemma extends the results about the optimal matching rule derived in Lemma 3 to a strategic setting:

Lemma 4 *In any Nash equilibrium of duopolistic competition with two-sided mechanisms, platforms A and B choose the virtual efficient matching rule Z^{II} from Definition 2.*

As in the monopoly case, the ability to charge users is key to Lemma 4: by setting the appropriate fee Q, the platform can adjust the size of its user base (taking M_B as given). Once it has done so, the rents enjoyed by platform A only depend on the total virtual value $E\left[u + \omega(\theta,v)|Z\right]$, which is maximized by Z^{II}.

We can now derive the equilibrium user fees, Q_A^{II} and Q_B^{II}. Lemma 4 transforms each platform's maximization problem into a one-dimensional program with control variable Q_J. As such, by taking the first-order conditions of problem (24), we obtain that the equilibrium fees Q_A^{II} and Q_B^{II} must satisfy the following system of best replies:

$$\frac{1}{Q_A^{II} + E\left[\omega|Z^{II}\right]} = \frac{\eta_c\left(E\left[\mu(\theta,v)|Z^{II}\right] - Q_A^{II}\right)}{E\left[\mu(\theta,v)|Z^{II}\right] - Q_A^{II}} + \frac{\eta_d\left(Q_B^{II} - Q_A^{II}\right)}{Q_B^{II} - Q_A^{II}} \tag{25}$$

$$\frac{1}{Q_B^{II} + E\left[\omega|Z^{II}\right]} = \frac{\eta_c\left(E\left[\mu(\theta,v)|Z^{II}\right] - Q_B^{II}\right)}{E\left[\mu(\theta,v)|Z^{II}\right] - Q_B^{II}} + \frac{h_d\left(Q_B^{II} - Q_A^{II}\right)}{Q_B^{II} - Q_A^{II}} \tag{26}$$

where $\eta_d(x) \equiv \frac{x g_d(x)}{G_d(x)}$ is the elasticity associated with the "supply" $G_d(\cdot)$ (from the perspective of firm A), and $h_d(x) \equiv \frac{x \cdot g_d(x)}{1-G_d(x)}$ is the elasticity associated with the supply $1 - G_d(\cdot)$ (from the perspective of firm B). As before, $\eta_c(x) \equiv \frac{x g_c(x)}{G_c(x)}$. We are now ready to characterize the unique equilibrium of this duopoly game:

Proposition 7 *The unique Nash equilibrium of duopolistic competition with two-sided mechanisms is symmetric. In this case, both platforms choose the matching rule Z^{II}, and the* *equilibrium user fees Q_A^{II} and Q_B^{II} are given by*

$$\frac{1}{Q_J^{II} + E\left[\omega|Z^{II}\right]} = \frac{\eta_c\left(E\left[\mu(\theta, v)|Z^{II}\right] - Q_J^{II}\right)}{E\left[\mu(\theta, v)|Z^{II}\right] - Q_J^{II}} + 2g_d(0). \tag{27}$$

Finally, the equilibrium payment rules for bidders P_A^{II} and P_B^{II} satisfy equation (10) when evaluated at the matching rule Z^{II} and Q_J^{II} as above.

Consider equation (27), which pins down the duopoly equilibrium fees, Q_A^{II} and Q_B^{II}. The only difference between equation (27) and the monopolistic Lerner formula (14) lies in the extra term $2g_d(0)$. This term accounts for the effect of user competition in the platforms' pricing decisions. The higher is $g_d(0)$, the higher is the number of users who are indifferent between platforms A and B.

A market described by the cdf $G_d(\cdot)$ is said to have a *higher degree of substitutability* than a market with cdf $\hat{G}_d(\cdot)$ if $g_d(0) > \hat{g}_d(0)$. Since by assumption $G_d(\cdot)$ is a unimodal symmetric distribution, the above definition implies that in a market described by $G_d(\cdot)$ users are more responsive to changes in the relative surplus between both platforms. The next corollary shows that users pay lower fees when platforms are closer substitutes:

Corollary 2 *The higher is the degree of substitutability between platforms, the lower are the expected user payments Q_A^{II} and Q_B^{II} in equilibrium.*

The analysis above shows that competition between two-sided platforms leads to lower payments (or higher subsidies) for users relative to monopoly. As such, competition reduces the deadweight loss from the supply of users. Nevertheless, since bidders can join multiple platforms, a competitive bottleneck emerges, and in equilibrium both platforms choose the same auction protocol as in a monopoly. Therefore, competition has the effect of reducing the distortions on the user side of the market, but not on the bidder side.

The supply of users is said to be *perfectly inelastic* when the distribution of users' outside options G_c is degenerate at zero. It follows from Corollary 2 that the total number of users who join *some* platform, $S_A(M_A, M_B) + S_B(M_A, M_B)$, weakly increases as platforms become closer substitutes (but stays constant when the supply of users is perfectly inelastic). The next corollary builds on this observation to assess the effect of competition on welfare:

Corollary 3 *Competition in two-sided mechanisms weakly increases welfare relative to monopoly. If the supply of users is perfectly inelastic, welfare is the same under duopoly or monopoly.*

This result is no longer true when platforms compete using one-sided mechanisms. This is the subject of the next subsection.

5.2 Competition with One-Sided Mechanisms

Many platforms are unable to (or decide not to) charge or subsidize users. In this case, they must compete in one-sided mechanisms. How does competition in one-sided mechanisms differ from competition in two-sided mechanisms? Is it beneficial for users and bidders? What is its impact on welfare? To investigate these issues, I extend the duopoly model developed above to a setting in which platforms cannot charge or subsidize users.

By definition, one-sided mechanisms are such that $Q_A = Q_B = 0$. Therefore, the platforms face the following supply system:

$$\begin{aligned} S_A(Z_A, Z_B) &= G_d\left(E\left[u|Z_A\right] - E[u|Z_B]\right) \cdot G_c\left(E\left[u|Z_A\right]\right), \\ S_B(Z_A, Z_B) &= \left(1 - G_d\left(E\left[u|Z_A\right] - E[u|Z_B]\right)\right) \cdot G_c\left(E\left[u|Z_B\right]\right). \end{aligned}$$

By the same arguments developed in the previous section, a Nash equilibrium of this duopoly game boils down to a profile of matching rules (Z_A^I, Z_B^I) that best respond each other:

$$Z_J^I = \text{argmax}_{Z_J}\ S_J(Z_J, Z_{-J}^I) \cdot E\left[\omega(\theta, v)|Z_J\right]. \tag{28}$$

I proceed as in the monopoly case (subsection 4.3) to find the equilibria of (28). I first derive the best-reply function of platform A. In order to do so, I fix Z_B and take an arbitrary matching rule Z_A and a profile $\mathbf{t}$. Let Z_A assign a positive probability to the event that bidder j with type (θ, v) obtains the match, and consider a bidder $\hat{\jmath}$ with type $(\hat{\theta}, \hat{v})$. Now let rule Z pick bidder $\hat{\jmath}$ instead of j with probability q whenever bidder j is selected for the match. The marginal gain from doing so is proportional to

$$\begin{aligned} \frac{\partial \Pi_A}{\partial q}(Z_A, Z_B) \;\propto\; & \left(\frac{\eta_c\left(E\left[\mu(\theta, v)|Z_A\right]\right)}{E\left[\mu(\theta, v)|Z_A\right]} + \frac{\eta_d\left(E\left[\mu(\theta, v)|Z_A\right] - E[\mu(\theta, v)|Z_B]\right)}{E\left[\mu(\theta, v)|Z_A\right] - E[\mu(\theta, v)|Z_B]}\right) \\ & \cdot(\mu(\hat{\theta}, \hat{v}) - \mu(\theta, v)) \cdot E\left[\omega(\theta, v)|Z_A\right] + (\omega(\hat{\theta}, \hat{v}) - \omega(\theta, v)). \end{aligned} \tag{29}$$

Denote by $Z_A(Z_B)$ the best response of platform A to Z_B. Clearly, the expression above evaluated at $Z_A(Z_B)$ must satisfy $\frac{\partial \Pi_A}{\partial q}\left(Z_A(Z_B)\right) \leq 0$ for any bidder $\hat{\jmath}$ in profile $\mathbf{t}$, as otherwise the platform could strictly increase profits by matching the bidder with $\hat{\jmath}$. By the same reasoning, one can compute the best reply $Z_B(Z_A)$ for platform B (which is similar to the expression above with indexes appropriately changed).

As in the monopoly case, the platforms' best-reply matching rules do not depend on the profile $\mathbf{t}$ (as one can see from (29)). Therefore, $Z_A(Z_B)$ and $Z_B(Z_A)$ can be described by scoring rules. The next proposition characterizes the unique symmetric equilibrium of this game:

Proposition 8 *In the unique symmetric Nash equilibrium of duopolistic competition with one-sided mechanisms, platforms A and B choose matching rules implicitly described by the scoring rule:*

$$s_J^I(\theta, v) = \omega(\theta, v) + \left(\frac{\eta_c \left(E\left[\mu(\theta, v)|Z_J^I\right]\right)}{E\left[\mu(\theta, v)|Z_J^I\right]} + 2g_d(0) \right) \cdot E\left[\omega(\theta, v)|Z_J^I\right] \cdot \mu(\theta, v),$$

The equilibrium payment rules P_A^I and P_B^I follow from the payoff equivalence formula (10) evaluated at Z_J^I and $Q_A = Q_B \equiv 0$.

The only difference between the duopoly matching rule from Proposition 8 and its monopoly counterpart (20) is the extra term $2g_d(0)$. This term captures the impact of competition on the weight the platform gives to the expected user value $\mu(\theta, v)$ from bidders.

As it turns out, as platforms become closer substitutes in the eyes of users, equilibrium matching rules become more user-friendly. As before, this extra distortion hinders the platforms' ability to extract rents from bidders.

Corollary 4 *The higher is the degree of substitutability between platforms, the more user-friendly are the equilibrium matching rules Z_A and Z_B. Moreover, the platform extracts lower rents from bidders in expectation, $E\left[\omega(\theta, v)|Z^I\right]$.*

Interestingly, competition between platforms could lead to a reduction in total welfare when the supply of users is perfectly inelastic (that is, number of users who join some platform is fixed). To see why, let us first consider a monopolistic market. It is clear from (17) that with a perfectly inelastic supply of users, the profit-maximizing matching rule selects the bidder with the highest virtual bidder value. This matching rule, denoted by Z^W, can be described by the scoring rule $s^W(t) = \omega(\theta, v)$ and leads to total welfare:

$$E\left[\theta + v|Z^W\right].$$

Now consider a duopolistic market in which users regard both platforms as perfect substitutes, that is, G_d is degenerate at zero. In this case, by a Bertrand-type argument, the unique symmetric equilibrium involves both platforms selecting the bidder with the highest expected user value. This is accomplished by the matching rule Z^O described by the scoring rule $s^O(t) = \mu(\theta, v)$, which leads to total welfare:

$$E\left[\theta + v|Z^O\right].$$

It is easy to construct examples in which $E\left[\theta+v|Z^O\right] < E\left[\theta+v|Z^W\right]$ (in the case of a binary state space, all we need is for v^H to be sufficiently larger than θ^H). We just established:

Proposition 9 *Let* $E\left[\theta+v|Z^O\right] < E\left[\theta+v|Z^W\right]$. *If the platforms are perfect substitutes and the supply of users is perfectly inelastic, then welfare decreases as the market moves from monopoly to duopoly.*

Intuitively, competition between platforms has two effects: first, it increases the user base relative to a monopolistic market and, second, it pushes both platforms to adopt more user-friendly matching rules. This first effect has a clearly positive impact on total welfare. The second effect may be negative, though, if the bidder side of the market contributes more to welfare than the user side. As a consequence, competition could reduce total welfare when the supply of users is sufficiently inelastic (in which case the first effect is negligible).

6 Conclusion

This paper derives the optimal mechanism for a platform willing to sell to bidders access to its user base. My analysis offers positive as well as normative implications.

On the positive side, I first show that the platform's decision whether to charge or subsidize users depends on the shares that users and bidders expect to derive from the match. If bidders appropriate most of the surplus from matches, the platform should follow a loss leader strategy by subsidizing users and recouping losses on the bidder side of the market. In contrast, platforms in which user profits are higher than those of bidders tend to adopt business models that charge users access or subscription fees.

Second, my model develops an informational rationale for why many two-sided platforms generate matches with inefficiently low user values (search diversion). Since the signal observed by the platform regarding the user value of a bidder contains information on his bidder value, the revenue-maximizing auction distorts the matching rule as a way to induce high bidding. When user and bidder values are positively affiliated, this distortion favors bidders with low user values, resulting in search diversion.

Third, my analysis predicts that, when platforms compete with one-sided mechanisms, more competitive markets lead to matching rules that favor bidders with high user values. This prediction can be tested by assessing how online recommendation systems (e.g., from Amazon, Barnes and Noble, and Borders) react to different market conditions.

On the normative side, this work suggests that the conventional wisdom from one-sided markets may be misleading when it comes to evaluating the welfare effects of mergers in two-sided markets. Indeed, I show that when two-sided platforms compete with one-sided mechanisms, monopoly may produce more welfare than duopoly.

This work can be extended to accommodate the case in which the platform selects multiple bidders (a list) to match with users. In such a model, users sequentially search through the list of selected bidders and complete a transaction when (given the mechanism) the expected gains from further searches are no grater than the payoff from purchasing from previous bidders. This extension might bring new and interesting insights regarding the design of revenue-maximizing position auctions.

This work is a first step towards incorporating price discrimination (here, in the form of an auction mechanism) in a two-sided market context. Still, much work remains to be done on the subject. In particular, the special form of bidder heterogeneity (only in interaction values) and user heterogeneity (only in membership values) calls for a more general analysis.

First, I plan to analyze the general nonlinear pricing problem in two-sided (or N-sided) markets when agents on both sides are heterogeneous in interaction *and* membership values (extending the one-sided model of Rochet and Stole (2002)). Such a model conveniently describes business to business electronic commerce platforms (see Lucking-Reiley and Spulber (2001)) and can be applied to study the optimal regulation of N-sided platforms.

Second, I plan to study a model in which the platform designs one-to-one matching rules (unlike the one-to-many matching studied here) to associate agents (heterogeneous in interaction values) from both sides of the market. Such a model captures important features of labor and marriage markets.

Appendix: Proofs

Proof of Proposition 1: Note that, given the mechanism (Z^E, P^E, Q^E), the user with the highest reservation value, $\bar{c}$, that joins the platform is such that:

$$\bar{c} = E[u|Z^E] - Q^E = E[u+v|Z^E] = \bar{c}^*.$$

I will now show that the mechanism (Z^E, P^E, Q^E) satisfies the IC constraints (4). It is immediate from 18 that $s^E(\theta^{(1)}, v^{(1)}) \geq s^E(\theta^{(2)}, v^{(2)})$ if and only if $v^{(1)} \geq \mathring{v}$. Consider a bidder with type (θ, v) who reports $b > v$. If there is no type $(\tilde{\theta}, \tilde{v})$ such that $s^E(\theta, b) > s^E(\tilde{\theta}, \tilde{v}) \geq s^E(\theta, v)$, the bidder is no better by reporting $b > v$ than by reporting v. If instead there is a type $(\tilde{\theta}, \tilde{v})$ such that $s^E(\theta, b) > s^E(\tilde{\theta}, \tilde{v}) \geq s^E(\theta, v)$, then the bidder's payoff is:

$$\begin{aligned} & v - (\mu(\tilde{\theta}, \tilde{v}) + \tilde{v} - \mu(\tilde{\theta}, \mathring{v})) \\ = \quad & v + \mu(\tilde{\theta}, \mathring{v}) - \mu(\tilde{\theta}, \tilde{v}) - \tilde{v} < \mathring{v} + \mu(\tilde{\theta}, \mathring{v}) - \mu(\tilde{\theta}, \tilde{v}) - \tilde{v} = 0, \end{aligned}$$

where the inequality follows from the fact $v < \mathring{v}$ and the last equality follows from the definition of $\mathring{v}$. Now consider a bidder with type (θ, v) who reports $b < v$. If there is no type $(\tilde{\theta}, \tilde{v})$ such that $s^E(\theta, v) \geq s^E(\tilde{\theta}, \tilde{v}) \geq s^E(\theta, b)$, the bidder is no better by reporting $b < v$ than by reporting v. If instead there is a type $(\tilde{\theta}, \tilde{v})$ such that $s^E(\theta, v) \geq s^E(\tilde{\theta}, \tilde{v}) \geq s^E(\theta, b)$, than by deviating to $b < v$ the bidder is worse off, since he obtains a zero payoff while by reporting v he would have enjoyed a payoff:

$$v - (\mu(\tilde{\theta}, \tilde{v}) + \tilde{v} - \mu(\tilde{\theta}, \mathring{v}) > \mathring{v} + \mu(\tilde{\theta}, \mathring{v}) - \mu(\tilde{\theta}, \tilde{v}) - \tilde{v} = 0,$$

where the inequality follows from the fact $v > \mathring{v}$. This shows that the mechanism (Z^E, P^E, Q^E) induces truth-telling in dominant strategies. It is immediate from the discussion above that this mechanism induces nonnegative payoffs to bidders, therefore satisfying the IR constraints (3).■

Proof of Lemma 1: The arguments here are standard in the literature (see for example Bolton and Dewatripont (2004) for a textbook treatment of the baseline mechanism design problem with discrete types). It is easy to see that the IR constraint has to bind for v^1 for all j and θ^k. Otherwise, the platform could strictly increase profits by increasing the bidder payments $P_j(\theta^k, v^1)$ by $\varepsilon > 0$ small enough, in which case the IR would still hold and no IC would be affected. Hence, using the IC constraints for types (θ^k, v^l) with $l > 1$, we see that:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) \geq S(Z,Q) \cdot z_j(\theta^k, v^1) \cdot v^l - p_j(\theta^k, v^1)$$
$$> S(Z,Q) \cdot z_j(\theta^k, v^1) \cdot v^1 - p_j(\theta^k, v^1) = 0,$$

what shows that IR constraints are slack for all signals and bidder values (θ^k, v^l) with $l > 1$.

Fixing θ^k, consider the reciprocal IC constraints for types (θ^k, v^l) and $(\theta^k, v^{l'})$:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) \geq S(Z,Q) \cdot z_j(\theta^k, v^{l'}) \cdot v^l - p_j(\theta^k, v^{l'}),$$
$$S(Z,Q) \cdot z_j(\theta^k, v^{l'}) \cdot v^{l'} - p_j(\theta^k, v^{l'}) \geq S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^{l'} - p_j(\theta^k, v^l).$$

Summing them up leads to:

$$(v^l - v^{l'}) \cdot (z_j(\theta^k, v^l) - z_j(\theta^k, v^{l'})) \geq 0.$$

It then follows that $z_j(\theta^k, v^l)$ has to be weakly increasing in v for all θ^k and j.

Now consider the local downward incentive constraints for types (θ^k, v^{l+1}) and $(\theta^k, v^{l'})$:

$$S(Z,Q) \cdot z_j(\theta^k, v^{l+1}) \cdot v^{l+1} - p_j(\theta^k, v^{l+1}) \geq S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^{l+1} - p_j(\theta^k, v^l),$$
$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) \geq S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^l - p_j(\theta^k, v^{l-1}).$$

Using the weak monotonicity of $z_j(\theta^k, v^l)$, the IC constraint above implies that:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^{l+1} - p_j(\theta^k, v^l) \geq S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^{l+1} - p_j(\theta^k, v^{l-1}).$$

Therefore:

$$S(Z,Q) \cdot z_j(\theta^k, v^{l+1}) \cdot v^{l+1} - p_j(\theta^k, v^{l+1}) \geq S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^{l+1} - p_j(\theta^k, v^{l-1}),$$

what shows that if the local downward IC constraint holds, then all downward IC constraints should hold as well. An analogous argument shows that the same is true for all upward IC constraints.

Finally, every local downward IC constraint should bind. To see why, let:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) = S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^l - p_j(\theta^k, v^{l-1}) + \varepsilon,$$

where $\varepsilon > 0$. Consider a new payment rule $\hat{P}$ that differs from P only for bidder j on types $(\theta^k, v^{l'})$ with $v^{l'} \geq v^l$, in which case $\hat{p}_j(\theta^k, v^{l'}) = p_j(\theta^k, v^{l'}) + \varepsilon$. Clearly, all IC and IR constraints are unaffected by this change, but the platform now extracts more rents from type (θ^k, v^l). This shows that all local downward IC constraints should indeed bind at the optimum. This proves necessity.

For sufficiency, consider the binding IC constraint for type (θ^k, v^l) and bidder j:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^l - p_j(\theta^k, v^l) = S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^l - p_j(\theta^k, v^{l-1}).$$

Since $z_j(\theta^k, v^l)$ is weakly increasing in v for all θ^k and j by assumption, it follows that:

$$S(Z,Q) \cdot z_j(\theta^k, v^l) \cdot v^{l-1} - p_j(\theta^k, v^l) \leq S(Z,Q) \cdot z_j(\theta^k, v^{l-1}) \cdot v^{l-1} - p_j(\theta^k, v^{l-1}),$$

what shows that upward IC constraints hold for all types and bidders. The same reasoning shows that binding local IC constraints and the monotonicity of $z_j(\theta^k, v^l)$ are sufficient for conditions (3) and (4) to hold for all bidders j and types (θ^k, v^l).■

Proof of Lemma 2: From the payoff equivalence formula (10), we know that:

$$\sum_{j=1}^{N} E\left[P_j(\mathbf{t})\right] = \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) p_j(\theta^k, v^l)$$

$$= S(Z,Q) \cdot \left\{ \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) \left[z_j(\theta^k, v^l) \cdot v^l - \sum_{l'<l} z_j(\theta^k, v^{l'}) \cdot \epsilon \right] \right\}$$

$$= S(Z,Q) \cdot \left\{ \begin{array}{c} \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) \cdot z_j(\theta^k, v^l) \cdot v^l \\ - \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) \sum_{l'<l} z_j(\theta^k, v^{l'}) \cdot \epsilon \end{array} \right\}$$

Now notice that, using summation by parts:

$$\sum_{k,l} f(\theta^k, v^l) \sum_{l'<l} z_j(\theta^k, v^{l'}) \cdot \epsilon = \sum_{k} \sum_{l=1}^{T-1} \left\{ z_j(\theta^k, v^l) \cdot \epsilon \cdot \sum_{l'>l} f(\theta^k, v^{l'}) \right\}.$$

Therefore:

$$\sum_{j=1}^{N} E\left[P_j(\mathbf{t})\right] = S(Z,Q) \cdot \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) \cdot z_j(\theta^k, v^l) \cdot v^l$$

$$- S(Z,Q) \cdot \sum_{j=1}^{N} \sum_{k} \sum_{l=1}^{T-1} \left\{ z_j(\theta^k, v^l) \cdot \epsilon \cdot \sum_{l'>l} f(\theta^k, v^{l'}) \right\}$$

$$= S(Z,Q) \cdot \sum_{j=1}^{N} \sum_{k,l} f(\theta^k, v^l) \cdot z_j(\theta^k, v^l) \cdot \left\{ v^l - \epsilon \cdot \frac{\sum_{l'>l} f(\theta^k, v^{l'})}{f(\theta^k, v^l)} \right\},$$

where $(v^{T_2+1}-v^{T_2})\frac{\sum_{l'>T_2} f(\theta^k,v^{l'})}{f(\theta^k,v^{T_2})}\equiv 0$. We can now rewrite the expression above as:

$$\begin{aligned}\sum_{j=1}^{N} E\left[P_j(\mathbf{t})\right] &= S(Z,Q)\cdot\sum_{j=1}^{N}\sum_{\mathbf{t}_{-j}}\sum_{k,l} f(\mathbf{t})\cdot Z_j(\mathbf{t})\cdot\left\{v^l-\epsilon\cdot\frac{1-F(v^l|\theta^k)}{f(v^l|\theta^k)}\right\}\\ &= S(Z,Q)\cdot E\left[\omega(\theta,v)|Z\right],\end{aligned}$$

where $\omega(\theta,v)$ is the the bidder's virtual surplus. Plugging the result above in the objective function (5) gives (11).■

Proof of Proposition 2: Lemma 3 implies that the efficient matching rule Z^{II} is always optimal for the platform. As a consequence, the optimal fee Q^{II} has to solve program (**??**) with Z evaluated at Z^{II}. For convenience, define the univariate real function $\psi(x)$ as:

$$\psi(x)\equiv\frac{G_c(E\left[\mu(\theta,v)|Z^{II}\right]-x)}{g_c(E\left[\mu(\theta,v)|Z^{II}\right]-x)}-E\left[\omega(\theta,v)|Z^{II}\right].$$

Condition (14) can be rewritten as the fixed point of this function:

$$\psi(Q^{II})=Q^{II}.$$

Now note that at the optimum $-E\left[\omega(\theta,v)|Z^{II}\right]\leq Q^{II}$, as otherwise the platform's profits are negative. Moreover, $Q^{II}\leq E\left[\mu(\theta,v)|Z^{II}\right]$, as otherwise no users join the platform. As a consequence, we can restrict attention to payment rules such that:

$$-E\left[\omega(\theta,v)|Z^{II}\right]\leq Q\leq E\left[\mu(\theta,v)|Z^{II}\right].$$

Clearly,

$$\psi(-E\left[\omega(\theta,v)|Z^{II}\right])-(-E\left[\omega(\theta,v)|Z^{II}\right])=\frac{G_c(E\left[\mu(\theta,v)+\omega(\theta,v)|Z^{II}\right])}{g_c(E\left[\mu(\theta,v)+\omega(\theta,v)|Z^{II}\right])}>0$$

and:

$$\psi(E\left[\mu(\theta,v)|Z^{II}\right])-(E\left[\mu(\theta,v)|Z^{II}\right])=-E\left[\mu(\theta,v)+\omega(\theta,v)|Z^{II}\right]<0.$$

Since $\psi(x)-x$ is a continuous function, we can apply the intermediate value theorem to conclude that there is a fee:

$$Q^{II}\in(-E\left[\omega(\theta,v)|Z^{II}\right],E\left[\mu(\theta,v)|Z^{II}\right])$$

satisfying $\psi(Q^{II})=Q^{II}$ (or, alternatively, condition (14)).

I will now argue that the solution to (14) is a local maximum. Indeed, the second-order derivative of the objective function (13) with respect to Q is:

$$\begin{aligned}\frac{\partial^2 \Pi(Z,P,Q)}{(\partial Q)^2} &= g_c'(E\left[\mu(\theta,v)|Z^{II}\right]-Q)\cdot\left(E\left[\omega(\theta,v)|Z^{II}\right]+Q\right)\\ &\quad -2g_c(E\left[\mu(\theta,v)|Z^{II}\right]-Q).\end{aligned}$$

Evaluating this condition at Q^{II} (by using (14)) leads to:

$$\begin{aligned}\frac{\partial^2 \Pi(Z,P,Q^{II})}{(\partial Q)^2} &= g_c'(E\left[\mu(\theta,v)|Z^{II}\right]-Q^{II})\cdot\frac{G_c(E\left[\mu(\theta,v)|Z^{II}\right]-Q^{II})}{g_c(E\left[\mu(\theta,v)|Z^{II}\right]-Q^{II})}\\ &\quad -2g_c(E\left[\mu(\theta,v)|Z^{II}\right]-Q^{II}),\end{aligned}$$

which is strictly negative since $G_c(\cdot)$ is log concave. This shows that any solution to (14) is a local maximum.

Because $G_c(\cdot)$ is log concave, the function $\psi(\cdot)$ is a weakly decreasing. As a consequence, it has only one fixed point, what implies that condition (14) is both necessary and sufficient for the optimum. Finally, its fixed point is weakly smaller than zero if and only if $\psi(0)\leq 0$. This is precisely condition (15).■

Proof of Proposition 3: It is immediate from equation (14) that $S(Z^E,Q^E)>S(Z^{II},Q^{II})$. The reserve prices from Claim 2 follows from Definition 2.

I will now prove Claim 3. In order to do so, take two types (θ,v) and $(\hat{\theta},\hat{v})$ such that $\mu(\theta,v)<\mu(\hat{\theta},\hat{v})$. Now let $\mu(\theta,v)+v>\mu(\hat{\theta},\hat{v})+\hat{v}$. This obviously implies that $v>\hat{v}$. Moreover,

$$\begin{aligned}\omega(\theta,v)-\omega(\hat{\theta},\hat{v}) &= v-\hat{v}-\epsilon\frac{1-F(v|\theta)}{f(v|\theta)}+\epsilon\frac{1-F(\hat{v}|\hat{\theta})}{f(\hat{v}|\hat{\theta})}\\ &= v-\hat{v}+\underbrace{\epsilon\left(\frac{1-F(\hat{v}|\hat{\theta})}{f(\hat{v}|\hat{\theta})}-\frac{1-F(\hat{v}|\theta)}{f(\hat{v}|\theta)}\right)}_{A}\\ &\quad \underbrace{+\epsilon\left(\frac{1-F(\hat{v}|\theta)}{f(\hat{v}|\theta)}-\frac{1-F(v|\theta)}{f(v|\theta)}\right)}_{B}.\end{aligned}$$

$A>0$ as a consequence of Assumption 1 (positive affiliation) and $B>0$ because of the monotone hazard rate condition. We can then conclude that:

$$\begin{aligned}\omega(\theta, v) - \omega(\hat{\theta}, \hat{v}) &> v - \hat{v} \\ &> \mu(\hat{\theta}, \hat{v}) - \mu(\theta, v).\end{aligned}$$

This implies that $E[u|Z^E] = E[\mu(\theta, v)|Z^E] \geq E[\mu(\theta, v)|Z^{II}] = E[u|Z^{II}]$.

Finally,

$$B(\theta, v) = E\left[s^E(t) - s^{II}(t)|\theta, v\right] = E\left[\epsilon \cdot \frac{1 - F(v|\theta)}{f(v|\theta)}|\theta, v\right] = \epsilon \cdot \frac{1 - F(v|\theta)}{f(v|\theta)},$$

as we wanted to show.■

Proof of Proposition 4: We only need to show that the matching rule Z^{II} is implementable under the alternative Assumption 2. Having done so, the arguments from Lemma 3 and Proposition 2 guarantee that the mechanism (Z^{II}, P^{II}, Q^{II}) is revenue-maximizing.

As we know from Lemma 1, the implementability of Z^{II} is equivalent to showing that $z^{II}(\theta, v)$ is weakly increasing in v for all θ. This, in turn, is equivalent to showing that the scoring rule $s^{II}(t) = \mu(\theta, v) + \omega(\theta, v)$ is weakly increasing in v for all θ. Since $\mu(\theta, v)$ weakly decreases in v (by Assumption 2) and $\omega(\theta, v)$ strictly increases in v (by the monotone hazard rate condition), $s^{II}(t)$ is weakly increases in v if and only if

$$\left|\mu(\theta, v^{l+1}) - \mu(\theta, v^l)\right| \leq \left|\omega(\theta, v^{l+1}) - \omega(\theta, v^l)\right| \qquad \text{for all } l \text{ and } \theta \in \Theta,$$

as we wanted to show.■

Proof of Proposition 5: From the expression (19) we know that Z^I can be described by a scoring rule of the form:

$$s_b(\theta, v) = \omega(\theta, v) + b \cdot \mu(\theta, v).$$

Denote by $Z(b)$ the matching rule associated to the scoring rule $s_b(\theta, v)$. With slight abuse of notation, define $E[\omega(\theta, v)|Z(b)]$ as the correspondence that associates to each b the set of real numbers x such that $E[\omega(\theta, v)|Z(b)] = x$ for some tie-breaking rule (that resolves ties between (θ, v) and $(\hat{\theta}, \hat{v})$ with $s_b(\theta, v) = s_b(\hat{\theta}, \hat{v})$). Clearly, $E[\omega(\theta, v)|Z(b)]$ is a continuous and weakly decreasing correspondence with empty interior.[24]

[24] A weakly decreasing correspondence is such that if $b \leq \hat{b}$, then $E\left[\tilde{v}|Z(\hat{b})\right] \leq E[\tilde{v}|Z(b)]$ in the strong set order.

Analogously, define $E[\mu(\theta,v)|Z(b)]$ as the correspondence that associates to each b the set of real numbers x such that $E[\mu(\theta,v)|Z(b)] = x$ for some tie-breaking rule. Clearly, $E[\mu(\theta,v)|Z(b)]$ is a continuous and weakly increasing correspondence with empty interior.

From the argument derived in the text, we know that Z^I is the unique fixed point that solves:

$$b \in \eta\left(E[\mu(\theta,v)|Z(b)]\right) \cdot \frac{E[\omega(\theta,v)|Z(b)]}{E[\mu(\theta,v)|Z(b)]}. \tag{30}$$

Because $G_c(\cdot)$ is log concave, it follows that $\frac{\eta(x)}{x}$ is a strictly decreasing function in x. Therefore, $\eta\left(E[\mu(\theta,v)|Z(b)]\right) \cdot \frac{E[\omega(\theta,v)|Z(b)]}{E[\mu(\theta,v)|Z(b)]}$ is a continuous and weakly decreasing correspondence in b. Since $\eta\left(E[\mu(\theta,v)|Z(0)]\right) \cdot \frac{E[\omega(\theta,v)|Z(0)]}{E[\mu(\theta,v)|Z(0)]}$ is bounded away from zero and $\eta\left(E[\mu(\theta,v)|Z(b)]\right) \cdot \frac{E[\omega(\theta,v)|Z(b)]}{E[\mu(\theta,v)|Z(b)]}$ is uniformly bounded, we can conclude that a solution to (30) exists. Uniqueness follows from the fact that $\eta\left(E[\mu(\theta,v)|Z(b)]\right) \cdot \frac{E[\omega(\theta,v)|Z(b)]}{E[\mu(\theta,v)|Z(b)]}$ has empty interior.$\blacksquare$

Proof of Proposition 6: We will show that:

$$\eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} \geq 1 \qquad \Leftrightarrow \qquad \eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]} \geq 1.$$

I start by proving sufficiency ($\Leftarrow$). I proceed by contradiction. So let's assume that $\eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} < 1$. By definition, the efficient matching rule is such that a bidder j with type (θ,v) obtains the match when a bidder $\hat{j}$ with type $(\hat{\theta},\hat{v})$ is available if and only if:

$$\mu(\theta,v) + \omega(\theta,v) \leq \mu(\theta,v) + \omega(\theta,v).$$

Hence, from the fact that $\eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} < 1$ and condition (19), we can conclude that $E\left[\omega(\theta,v)|Z^I\right] \geq E\left[\omega(\theta,v)|Z^{II}\right]$ and $E\left[\mu(\theta,v)|Z^I\right] \leq E\left[\mu(\theta,v)|Z^{II}\right]$. Since $\frac{\eta(x)}{x}$ is strictly decreasing (because $G_c(\cdot)$ is log concave), it follows that:

$$1 > \eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} \geq \eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]},$$

contradicting the maintained assumption that $\eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]} \geq 1$.

I will now prove necessity ($\Rightarrow$) by establishing the counter-positive: if $\eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]} < 1$, then $\eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} < 1$. I will proceed by contradiction. So let's assume that $\eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} \geq 1$.

It is then immediate from condition (19) that $E\left[\omega(\theta,v)|Z^I\right] \leq E\left[\omega(\theta,v)|Z^{II}\right]$ and

$E\left[\mu(\theta,v)|Z^I\right] \geq E\left[\mu(\theta,v)|Z^{II}\right]$. Since $\frac{\eta(x)}{x}$ is strictly decreasing, it follows that:

$$1 \leq \eta\left(E\left[\mu(\theta,v)|Z^I\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^I\right]}{E\left[\mu(\theta,v)|Z^I\right]} \leq \eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]},$$

contradicting the maintained assumption that $\eta\left(E\left[\mu(\theta,v)|Z^{II}\right]\right) \cdot \frac{E\left[\omega(\theta,v)|Z^{II}\right]}{E\left[\mu(\theta,v)|Z^{II}\right]} < 1$.■

Proof of Corollary 1: Since $\frac{\eta(x)}{x}$ is a strictly decreasing function in x, it follows that $\eta\left(E\left[\mu(\theta,v)|Z(b)\right]\right) \cdot \frac{E[\omega(\theta,v)|Z(b)]}{E[\mu(\theta,v)|Z(b)]}$ is a continuous and weakly decreasing correspondence of b. As a consequence, if we take two demands $G_c(\cdot)$ and $\hat{G}_c(\cdot)$ satisfying $\eta(x) \geq \hat{\eta}(x)$, it follows that the solution b to (30) associated to $G_c(\cdot)$ will be weakly greater than the solution $\hat{b}$ to (30) associated to $\hat{G}_c(\cdot)$.■

Proof of Lemma 4: Let platform B choose an arbitrary mechanism $M_B = (Z_B, P_B, Q_B)$ and assume that platform A picks a mechanism $M_A = (Z_A, P_A, Q_A)$. Now consider a new mechanism $\bar{M}_A = (Z^{II}, P_A, \bar{Q}_A)$ with the user fee $\bar{Q}_A$ chosen to satisfy:

$$E\left[\theta - Q_A|Z_A\right] = E\left[\theta - \bar{Q}_A|Z^{II}\right].$$

By the exact same arguments developed on Lemma 3, one can see that rule Z^{II} is implementable.

Note that, by construction, $S_A(\bar{M}_A, M_B) = S_A(M_A, M_B)$. I will now show that $\Pi_A(\bar{M}_A, M_B) \geq \Pi_A(M_A, M_B)$ for any mechanism M_B. For simplicity we will write ω to mean $\omega(\theta,v)$ and μ to mean $\mu(\theta,v)$. Indeed:

$$\begin{aligned}
\Pi_A(M_A, M_B) &= S_A(M_A, M_B) \cdot \left(E\left[Q_A + \omega|Z_A\right]\right) \\
&= S_A(\bar{M}_A, M_B) \cdot \left(E\left[\omega + |Z_A\right] - E\left[\mu|Z^{II}\right] + E\left[\bar{Q}_A|Z^{II}\right]\right) \\
&\leq S_A(\bar{M}_A, M_B) \cdot \left(E\left[\omega + \mu|Z^{II}\right] - E\left[\mu|Z^{II}\right] + E\left[\bar{Q}_A|Z^{II}\right]\right) \\
&= S_A(\bar{M}_A, M_B) \cdot \left(E\left[\bar{Q}_A + \omega|Z^{II}\right]\right) \\
&= \Pi_A(\bar{M}_A, M_B),
\end{aligned}$$

where the second equality follows from the construction of $\bar{Q}_A$ and the inequality (in the third line) uses the fact that the matching rule Z^{II} maximizes $E\left[\omega + \mu|Z\right]$ among all implementable rules Z (see Lemma 3). This shows that it is weakly dominant for the platform to choose a mechanism of the form (Z^{II}, P_A, Q_A). By symmetry, the same reasoning establishes the claim for platform B.■

Proof of Proposition 7: Lemma 4 states that it is a weakly dominant strategy for platforms A and B to choose the matching rule Z^{II}. As a consequence, the equilibrium payment rules Q_A^D and Q_B^D have to solve the system of best responses (25) and (26) obtained from differentiating program (24) evaluated at $Z_J = Z^{II}$ for $J \in \{A, B\}$.

From the log concavity of $G_c(\cdot)$ and $G_d(\cdot)$, it follows that $\frac{\eta_c(x)}{x}$ and $\frac{\eta_d(x)}{x}$ are strictly decreasing functions. Therefore, it is straight-forward to show that the best-reply functions $Q_A(Q_B)$ and $Q_B(Q_A)$, implicitly defined by equations (25) and (26) respectively, are strictly increasing.

Moreover, from the symmetry of the cdf $G_d(\cdot)$, we know that $\eta_c(x) = h_d(-x)$. As a consequence, the function $Q_A(Q_B)$ is the inverse of the function $Q_B(Q_A)$. This implies that any solution to the system (25)-(26) has to be symmetric.

Plugging $Q_A(Q_B) = Q_B(Q_A)$ into the system (25)-(26) leads to equation (27). By using the same arguments from the proof of Proposition 2, one can easily see that there is only one solution to (27), and that this solution is a maximum of program (24). Therefore, we can conclude that the duopoly game has a unique pure strategy Nash equilibrium which users' payment rules are given by (27).

Finally, since the bidders' reports in each matching mechanism M_A and M_B are independent, we can apply Lemma 1 to conclude that equilibrium payments for bidders are given by equation (10) evaluated at Z^{II} and Q_A.■

Proof of Proposition 8: $Z_A(Z_B)$ is the best response of platform A to Z_B. Clearly, the expression (29) evaluated at $Z_A(Z_B)$ has to satisfy $\frac{\partial \Pi_A}{\partial q}(Z_A(Z_B)) \leq 0$ for any bidder $\hat{\jmath}$ in profile $\mathbf{t}$, as otherwise the platform could strictly increase profits by matching the bidder to $\hat{\jmath}$. Therefore, according to the best reply $Z_A(Z_B)$, a bidder j with type (θ, v) obtains the match when a bidder $\hat{\jmath}$ with type $(\hat{\theta}, \hat{v})$ is available if and only if:

$$\begin{aligned} & \omega(\theta, v) + \left(\frac{\eta_c\left(E\left[u|Z_A(Z_B)\right]\right)}{E\left[u|Z_A(Z_B)\right]} + \frac{\eta_d\left(E\left[u|Z_A(Z_B)\right] - E[u|Z_B]\right)}{E\left[u|Z_A(Z_B)\right] - E[u|Z_B]} \right) \cdot E\left[\omega(\theta, v)|Z_A(Z_B)\right] \cdot \mu(\theta, v) \\ \geq \;& \omega(\hat{\theta}, \hat{v}) + \left(\frac{\eta_c\left(E\left[u|Z_A(Z_B)\right]\right)}{E\left[u|Z_A(Z_B)\right]} + \frac{\eta_d\left(E\left[u|Z_A(Z_B)\right] - E[u|Z_B]\right)}{E\left[u|Z_A(Z_B)\right] - E[u|Z_B]} \right) \cdot E\left[\omega(\theta, v)|Z_A(Z_B)\right] \cdot \mu(\hat{\theta}, \hat{v}) \end{aligned}$$

By the same reasoning, the best reply $Z_B(Z_A)$ selects a bidder j with type (θ, v) when a bidder $\hat{\jmath}$ with type $(\hat{\theta}, \hat{v})$ is available if and only if:

$$\begin{aligned} & \omega(\theta, v) + \left(\frac{\eta_c\left(E\left[u|Z_B(Z_A)\right]\right)}{E\left[u|Z_B(Z_A)\right]} + \frac{h_d\left(E\left[u|Z_A\right] - E[u|Z_B(Z_A)]\right)}{E\left[u|Z_A\right] - E[u|Z_B(Z_A)]} \right) \cdot E\left[\omega(\theta, v)|Z_B(Z_A)\right] \cdot \mu(\theta, v) \\ \geq \;& \omega(\hat{\theta}, \hat{v}) + \left(\frac{\eta_c\left(E\left[u|Z_B(Z_A)\right]\right)}{E\left[u|Z_B(Z_A)\right]} + \frac{h_d\left(E\left[u|Z_A\right] - E[u|Z_B(Z_A)]\right)}{E\left[u|Z_A\right] - E[u|Z_B(Z_A)]} \right) \cdot E\left[\omega(\theta, v)|Z_B(Z_A)\right] \cdot \mu(\hat{\theta}, \hat{v}). \end{aligned}$$

In the unique symmetric equilibrium of this game $Z_A^I = Z_A(Z_B^I) = Z_B(Z_A^I) = Z_B^I$. Making this substitution in the best responses above leads to the scoring rule in the statement of Proposition 8.■

Proof of Corollary 2: The optimal fees to users Q^{II} satisfies the fixed point equation:

$$\phi(Q^{II}) = Q^{II},$$

where the function $\phi(\cdot)$ is defined by:

$$\phi(x) \equiv \frac{1}{\frac{g_c(x)}{G_c(x)} + 2f_d(0)} - E\left[\omega(\theta, v)|Z^{II}\right].$$

Because $G_c(\cdot)$ is log concave, one can see that $\phi(\cdot)$ is a weakly decreasing function of x and a strictly decreasing function of $g_d(0)$. Therefore, the fixed point associated to $g_d(0)$ will be strictly smaller than the fixed point associated to $\hat{g}_d(0)$ whenever $g_d(0) > \hat{g}_d(0)$.■

Proof of Corollary 4: The proof is analogous to that of Corollary 1, and is therefore omitted.■

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