Making sure your vote does not count: ESG activism and insincere proxy voting

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Abstract

This paper models strategic voting on ESG proposals by blockholders with heterogeneous reputational concerns and varying levels of commitment to ESG values. ESG activists, whose public-good gains from intervention are not attenuated by selling shareholders' free-riding, rationally sponsor even long-shot proposals. Proposals that lower firm value but produce environmental benefits pass with positive, but perhaps small, probability. Our analysis leads to some non-obvious insights: neither increases in blockholders' personal commitments to ESG values nor increases in blockholder dispersion reliably increase the probability of proposal success. However, the probability of success is uniformly increased both by increasing overall reputational pressure on blockholders and by increasing the gap between the pressure faced by the most and least pressured blockholders.

1 Introduction

One of the fundamental normative questions in financial economics is whether firms' objectives *should* be limited to market value maximization or also encompass the social and environmental preferences of shareholders (Hart and Zingales, 2017). This question has no practical relevance if shareholders have no effective means for influencing firms' objectives. The most transparent means of compelling firms to prioritize social and environmental concerns is environmental, social, and governance (ESG)proxy activism. This paper examines the effectiveness of ESG proxy activism when large institutional investors' votes are decisive.

Although shareholder proposals related to social and environmental issues predate the turn of the century, ESG activism is largely a twenty-first century phenomenon. Since 2000, The number of ESG-related shareholder proposals has steadily increased. The 2022 U.S. proxy season yielded another record increase in ESG proposals, a 22% increase over 2021.[1]

These proposals attempted, sometimes successfully, to force significant changes in firms' operating policies and the environmental commitments of firms' boards. For example, in May 2021, an ESG activist fund, Engine No.1, successfully secured three "green" board seats at ExxonMobil despite owning just 0.02% of ExxonMobil's shares.[2] On the heels of Engine No.1's success, Third Point, another activist fund, submitted a proposal calling for Royal Dutch Shell to separate its oil and gas business from initiatives in renewable energy.[3] A May 2022 vote

[1] See S&P Global (2022).
[2] See The New York Times (2021a).
[3] See The New York Times (2021b).

on a greenhouse gas reduction proposal, submitted by investor activist As You Sow, was supported by 72.18% of Chubb Limited's shareholders despite Chubb board's opposition.[4] Nonetheless, shareholder support for ESG proposals is volatile. Average support almost doubled over 2015 to 2020, yet suffered a substantial decline over the last two years.

Who decides whether ESG proposals pass? The largest share blocks in many public companies in the U.S. are held by *universal owners*, i.e., large, diversified, institutional investors (Amel-Zadeh et al., 2022).[5] In fact, the three largest institutional investors, BlackRock, Vanguard, and State Street, vote 25% of S&P 500 shares (Coffee Jr, 2021). Because these universal owners frequently hold the largest share blocks, in many cases, they can determine the outcome of shareholder votes on proxy proposals. The proxy votes of institutions are publicly observable.[6] When proxy votes are related to controversial ESG issues, votes can trigger adverse reactions to the institution's vote based on social and environmental values. Because universal owners are diversified, the effects of these reactions to a universal owner's vote, e.g., withdrawal of funds from the universal owner, sanctions imposed on the universal owner by state governments, can be larger than the effects on corporate value engendered by the success or failure of the proposal.[7] Moreover, institutional investors may themselves value the ESG goals of shareholder proposals and be willing to accept some firm value reductions to further these goals.[8]

ESG proposals require a proposer. This role is frequently assumed by an ESG activist fund. These funds aim to buy shares and make proposals that further environmental and social objectives. In many ways, the problems faced by ESG activists are similar to the problems faced by non-ESG activists. They need to formulate a viable proposal, acquire shares, and launch a campaign to ensure adoption of their proposal.[9] In our analysis, ESG activists have green preferences, i.e., their utility depends both on wealth and environmental outcomes. Activists buy floating shares from atomistic shareholders and make proposals at shareholder meetings. On first inspection, it might appear that green activism, i.e., buying shares from "free-riding" shareholders and launching long-shot proposals that promise environmental benefits at the expense of reduced firm value, is not viable when activists are unwilling to incur huge monetary losses to effect environmental improvement.

[4]See Orrick (2022).

[5]Many researchers term such owners "common owners." We prefer using the term "universal owners" to avoid the miss-impression that this paper relates to the effects of institutional joint ownership on intra-industry competition. The Society of Actuaries defines "universal owners" as "institutional asset owners (pension funds, mutual funds, insurance companies, sovereign wealth funds) that own such a representative slice of the economy as to find it impossible to diversify away from large system-wide risks." See Institute and Faculty of Actuaries (2011).

[6]Our description of institutional owners applies to U.S. institutions regulated by the SEC, e.g., mutual funds. Collective Investment Funds (CIFs) are not subject SEC oversight and do not have to disclose their votes. Currently the AUM of CIFs (approx. $800 Billion) is tiny relative to AUM of mutual funds (approx. $45 Trillion). We are not aware of any research related to the voting behavior of CIFs.

[7]For an example of state government sanction threats triggered by institutional shareholder proxy votes, see Reuters (2022). For further discussion about the investor catering rationale for pro-ESG voting by institutions, see Wang (2021); Ramelli et al. (2021).

[8]Evidence suggests that, for shareholder proposals motivated by ideological beliefs such as many ESG proposals, institutional investors' votes frequently diverge significantly from the recommendations of proxy advisers, e.g., Institutional Shareholder Services (ISS) (Bolton et al., 2020). This suggests that institutions factor into their voting decisions their value system and/or the effect of votes on reputation.

[9]Many ESG proposals lie outside the scope of our model. Governance proposals typically do not attempt to alter firms' objective function. Rather, they aim to force firms to adopt value maximizing plans in the face of management opposition. Many non-management initiated climate and social proposals also do not represent "activism" under our definition. Almost half of shareholder proposals are made by religious groups or "micro-shareholders" with negligible share holdings. These proposals frequently relate to affirming generalized commitments to social causes, e.g., abortion rights, climate change, diversity, or handgun regulation. The sort of proposals we aim to model are very different: concrete, detailed plans for changing firm's operating policies or slates of alternative directors motivated by a desire to prioritize social and environmental concerns. In practice, such proposals are almost always motivated by environmental concerns.

However, the public-good nature of green benefits ensures that activism can be viable even when activism entails deadweight costs and is unlikely to succeed. The benefit from the proposal's success captured by green activists, the environmental improvement effected by the adoption of the proposal, is a public good, not contingent on the size of activists' shareholding or the price of share acquisition. In contrast to the standard Grossman and Hart (1980) setting, atomistic target firm shareholders cannot capture the activist gains by holding out for the post-activism share value. At the ask price of atomistic shareholders, ESG activists break even. The only cost borne by the ESG activist is the expenditure required to fund the campaign and develop proposals.[10] If the ESG activist targets a large firm in a carbon-intensive industry, passage of a proposal that significantly reduces the firm's carbon footprint can conceivably generate green benefits orders of magnitude greater than the activist's campaign costs. Thus, even a small probability of success can incentivize activist ESG investing.

In our model, share blocks voted by universal owners determine whether proposals pass. Residual non-institutional share holdings are held by atomistic owners. Some universal owners, *green owners*, share the environmental values of the ESG activist; other universal owners, *brown owners*, do not. Universal owners face *reputation costs*, i.e., costs engendered by voting against ESG proposals. Owners vote strategically. We consider the case where, even accounting for reputation costs, it is in the collective interest of brown owners for the green proposal to fail and in the collective interest of green owners for the proposal to pass. There is a positive, perhaps small, probability that any given universal owner is green and thus there is always a positive probability that the proposal will pass. We term the probability that a given universal owner is green the level of *green sentiment*.

If the firm has only one universal owner, the proposal fails if and only if the single owner votes against the proposal. The proposal will be supported by the universal owner if and only if the owner is green. When there are multiple universal owners, owners' voting decisions are more complex. The tension is that each brown owner would like to see the proposal fail but, because of reputation costs, would prefer not to vote against the proposal. A brown owner's vote only affects her welfare when her vote is marginal. So, each brown owner will trade off the benefit of voting yes, avoiding reputation costs, against the cost of voting yes, the value reduction produced by a yes vote when that vote is marginal.

This tension results in a voting game which has many Nash equilibria. We refine the set of Nash equilibria using a refinement approach frequently employed in economics, network analysis, and operations research, the potential game approach.[11] We show that for generic model parameters, the game has a unique potential maximizing equilibrium. Consistent with the empirical evidence (Michaely et al., 2021), in the equilibrium, brown universal owners vote strategically by insincerely supporting green proposals when it is unlikely that their votes are marginal.

[10] Interestingly, if activists had toehold stakes, they would incur an additional cost: the reduction in the monetary value of their pre-existing shareholdings. Thus, toehold stakes, rather than being necessary for successful activism as in Grossman and Hart (1980) style models, are, in fact, impediments to ESG activism.

[11] We discuss potential games and the implications of potential maximization extensively in section 5.1.

The properties of potential-maximizing Nash equilibria depend to a large extent on the level of green sentiment. When green sentiment is high, it is likely that many universal owners are green and thus will support the proposal. Hence the vote of any individual brown owner is unlikely to be marginal. For this reason, brown owners capitulate and insincerely support the green proposal. When green sentiment is moderate, the proposal can only be defeated if "all hands are on deck," so brown owners opt for extreme voting strategies, i.e., they either all vote no or all capitulate. In both of these cases, increasing green sentiment does not reduce the probability that the proposal will pass.

However, when green sentiment is low, the situation is more complex. If all brown owners resist, it is likely that the proposal will fail by a wide margin, in which case, no brown owner is likely to be marginal. Thus, the complete resistance strategy is not optimal. Instead, brown owners adopt partial resistance strategies: some brown owners, those facing the largest reputation costs, insincerely vote for the proposal while others resist. In this case, because an increase in green sentiment can increase the equilibrium level of brown resistance, increasing green sentiment can *reduce* the probability that green proposals succeed. This strategic effect of increased sentiment can dominate the positive mechanical effect of increased green sentiment on the probability of proposal success. Because increasing green sentiment does not reliably increase the pass probability for ESG proposals, this result suggests, consistent with the stylized facts, that activists concentrate their efforts on increasing reputational pressure rather than changing the environmental ethos of institutional owners.

In contrast to green sentiment, the effect of increasing the level of reputation costs on brown resistance is quite transparent: increasing reputation costs decreases brown resistance and thus increases the probability of proposal success. In general, the effect of increasing the dispersion of reputation costs on the probability of proposal success is indeterminate.[12] However, increasing the dispersion of reputation costs through a *high-low reputation cost spread*, which roughly speaking involves transferring reputation costs from universal owners with lower-than-median reputation costs to owners with higher-than-median reputation costs, increases the pass probability whenever, before the transfer, brown owners resisted the proposal.

This result identifies a novel channel, independent of well-analyzed cost-of-capital channel (e.g., Heinkel et al., 2001), through which passive retail investors affect real environmental outcomes—if green retail investors buy into only a small subset of mutual funds, the high-low reputation cost spread between institutional reputation costs increases. Thus, by channeling their investments into a few funds, green retail investors increase the reputation costs of marginal brown votes, thereby ensuring that some universal owners will vote green even if they have brown preferences. "Flipping" a few high-reputation cost owners, makes the resistance of the other brown owners less effective.

The effects of ownership dispersion on the prospects of ESG proposals and the welfare of brown owners

[12] In more precise terms, inequality reducing "Dalton transformations" (Marshall et al., 2011) of the reputation cost vector can either increase or decrease the probability of proposal success.

are somewhat subtle. The benefit a brown owner captures by insincerely supporting the proposal, avoidance of reputation costs, is private. The cost of insincere voting, an increased probability that the proposal passes, is spread across all brown owners. This free-rider effect of ownership dispersion increases the probability that proposals pass and reduces brown owner welfare.

However, there are two countervailing effects. First, dispersion can reduce the reputation costs associated with brown resistance. Each universal owner casts all of her proxy votes either in favor of or against a green proposal. If there is only one universal owner, when the proposal is defeated, all universal owner votes are cast against the proposal, far more votes than required to defeat the proposal.[13] No votes generate reputation costs. When green sentiment is low, a thin majority of brown owners' votes opposing the proposal is very likely to ensure failure. In this case, somewhat dispersed ownership supports equilibria in which some brown owners vote insincerely in favor of the proposal. Such outcomes feature significantly lower total reputation costs and entail only a negligible probability of proposal success. Hence, dispersed ownership increases the welfare of brown owners.

Second, because brown owners do not know which other owners are brown, the effect of a given brown owner's vote on the success of the proposal is uncertain. Increasing the dispersion of share ownership, through a law of large numbers effect, reduces the variance of the proportion of votes cast in favor of the proposal. When green sentiment is low, this effect reduces the pass probability associated with a fixed level of brown resistance. This effect can increase the probability that proposals fail and increase the welfare of brown owners.

Related literature

Our paper is closely related to the emergent literature on the effects of corporations' environmental and social policies on shareholder and social welfare. Like many papers in this literature, our green agents have what Gupta et al. (2022a) term "broad green preferences," i.e., part of the utility green investors derive from investing is based on the effect of their investment on environmental outcomes. Thus, in broad green preferences models like ours, green investors, per se, do not increase their utility by divesting from brown assets and green utility is not tied to the number of shares owned by the investor but rather the size of the change in the greenness of output that an investor can affect. In contrast, Goldstein et al. (2022) consider equilibrium security prices when investors have "narrow green preferences," a preference for holding shares of firms producing green output. Most of the broad green preferences literature (e.g., Jagannathan et al., 2022; Gupta et al., 2022a; Broccardo et al., 2022; Albuquerque et al., 2019) models worlds where firms are either green or brown. Green agents affect changes in policy by buying up brown firms. In contrast, we focus on the struggle for control between green and brown shareholders of a given

[13] Because voting for the proposal is a weakly dominant strategy if and only if the owner is green, it seems reasonable to assume that any proxy votes opposing proposal engender reputation costs. Vote splitting, i.e., a single fund voting some proxies for and some against a proposal has, to our knowledge, never occurred. In fact, this possibility, to our knowledge, has not even been considered in the legal literature. Fund families can recommend a yes vote to some family members and a no vote to others. However, non-uniform recommendations are purportedly based on the differences between the preferences of the fund's beneficial owners. The typical pattern is for one recommendation to be offered to traditional funds and another to "green" funds.

firm. These investors fight for control through proxy voting rather than through acquisition offers.

In this respect our paper is related to Hart and Zingales (2017) who also study corporate policy when firm actions affect owners' utility through channels other than firm value. However, in Hart and Zingales (2017), individual shareholders' green preferences directly affect corporate policies through their voting behavior. In our analysis, decisive votes are cast by universal owners.[14] The green preferences of these institutional investors only indirectly affect the strategic voting of universal owners through the reputational penalties associated with opposing green proposals. Also, unlike Hart and Zingales (2017), which focuses on how firm policy should be determined when shareholders have conflicting objectives, we focus instead on how policy is actually determined when controlling agents are large and strategic.

Our model of ownership structure is to a large extent inspired by the empirical literature documenting the rise of common ownership (Amel-Zadeh et al., 2022) and the legal literature considering the implications of universal/common ownership for securities' regulation (Coffee Jr, 2021). The effect of social pressure on investor, fund, and firm behavior incorporated in our model is motivated by a large empirical literature (e.g., Wang, 2021; Ramelli et al., 2021; Dimson et al., 2015).

Our model of activist share acquisition is quite simple and structurally quite similar to the atomistic shareholder model in Grossman and Hart (1980). However, we reach very different conclusions about the ability of share acquirers, corporate raiders in their model, activist investors in our model, to gain from share acquisitions. In Grossman and Hart (1980), acquisitions are motivated by pecuniary gain and current shareholders capture the pecuniary gain generated by acquirers' value-add plans. Thus, when acquirers lack a toe-hold stake, they cannot profit from adding value. In our setting, the small atomistic shareholders who sell to activists also sell their shares at prices that reflect the expected market value effects of activists' interventions. However activists can still gain from intervention because of the non-pecuniary utility they derive from inducing target firms to adopt green policies.

2 Structure of the model

2.1 Precís

We develop a model of activism and shareholder voting for firms controlled by universal owners. Some agents' preferences over actions are completely determined by their monetary payoffs; other agents' preferences also depend on the environmental effects of their actions. An activist fund, henceforth called the *activist*, initiates ESG activism and acquires shares. We focus on *activism equilibria*, equilibria in which activists acquire shares, attempt

[14]Empirical research (Brav et al., 2022) suggests that our framework matches current institutional practice better than the framework developed by Hart and Zingales (2017). However, institutional practice can change. Devolving index fund proxy voting to the retail investors who own fund shares has been advocated by many legal scholars (e.g., Griffin, 2019). In the US, there are legal barriers to retail shareholder devolution. However, BlackRock UK plans to permit some retail shareholder devolved voting in 2023 (Financial Times, 2022).

to identify proposals that, if adopted, will make the firms output greener, and, when such proposals are identified, submit their proposals to shareholders. Shareholders then vote on the proposal. The proxy votes of the universal owners determine whether the proposal succeeds or fails to pass.

2.2 Preferences, agents, and timings

2.2.1 Preferences

All agents are risk neutral and patient. there are two kinds of agent preferences: green and brown. Agents with *brown* preferences simply maximize their expected wealth. Agents with green preferences, using the terminology in Gupta et al. (2022b), have "wide-green preferences," i.e., they care about the greenness of the world not the greenness of their portfolios. More specifically, let a represent an action that might affect an agent's terminal wealth and the environment; let $\tilde{V}(a)$ represent the agent's random future (date 1) terminal wealth conditioned on a; let $\tilde{G}(a)$ represent the random future greenness of the environment (e.g., some decreasing function of CO_2 ppm) conditioned on a. If the agent has green preferences, the agent's utility is given by

$$\mathbb{E}\big[\tilde{V}(a)\big] + \beta\,\mathbb{E}\big[\tilde{G}(a)\big], \quad \beta > 0. \tag{1}$$

The parameter β measures the extent to which the agent is willing to sacrifice monetary payoffs to increase greenness. We call $\mathbb{E}[\tilde{G}(a)]$ the *green payoff* from action a. Equation (1) implies that an agent with green preferences weakly prefers action a'' to action a' if and only if

$$\mathbb{E}\big[\tilde{V}(a'') - \tilde{V}(a')\big] \geq \beta\,\mathbb{E}\big[\tilde{G}(a'') - \tilde{G}(a')\big]. \tag{2}$$

Green agents tradeoff the effects of their actions on their expected terminal wealth, $\mathbb{E}[\tilde{V}(a)]$, against their expected effects on the environment, $\mathbb{E}[\tilde{G}(a)]$. So, for example, a green agent who owns a firm that has an inherently large carbon footprint (e.g., a coal-fired electricity generator) cannot increase her utility by divesting from the firm through selling out to a brown competitor. If she sold out, her portfolio would be greener but the world would not. In fact, if the brown competitor planned to make the firm's carbon footprint even larger, the green owner would only divest if the brown competitor offered sufficient monetary compensation to offset the environmental effects of the control transfer.[15]

Another obvious implication of equation (2) is that, when two actions, say a' and a'', have the same environmental effects, i.e., $\mathbb{E}[\tilde{G}(a'') - \tilde{G}(a')] = 0$, green agents' and brown agents' preferences coincide; both will choose

[15] The green benefit, $\tilde{G}(a'') - \tilde{G}(a')$, is a public good whose value depends on the overall environmental impact of the proposal. Thus, regardless of whether an agent's portfolio is diversified or concentrated in a single firm, the green benefit represents all effects of the proposal on the environment, including the effects engendered by other firms modifying their policies in response to the targeted firm's adoption of the proposal.

an action that maximizes their expected terminal wealth, $\mathbb{E}[\tilde{V}(a)]$. For example, suppose an agent is considering whether to buy one share of an oil company's stock. The oil company is controlled by blockholder, whose operating decisions cannot be swayed by small shareholders. Because purchasing the share has no effect on the environment, whether the agent's preferences are brown or green will have no effect on an agent's reservation bid price.

2.2.2 Types of agents and their share endowments

There are three kinds of agents: universal owners, an activist, and a mass of atomistic small shareholders. The firm has one share outstanding. Thus, the value of the firm equals the value of the share. We refer to the number of shares held by a shareholder before trade as the shareholder's *endowment*.

Universal owners. There are K universal owners. Each universal owners holds an appreciable endowment of firm shares. Universal owners do not alter their endowment through buying or selling shares.[16] Universal owners can have either brown or green preference. We refer to universal owners with brown preferences as *brown owners* and refer to universal owners with green preferences as *green owners*. The preferences of universal owners are determined by independent draws from a Bernoulli distribution. With probability γ, the draw results in assigning green preferences to the universal owner; with probability $1-\gamma$, the universal owner is assigned brown preferences. The assignment is private information of the universal owner receiving the assignment. We refer to γ as *green sentiment* because it measures the extent to which universal owners have an inherent preference for increased greenness.

Our assumption that universal owners may have green preferences has some empirical support. Amel-Zadeh and Serafeim (2018) report that 25% of large non-ESG institutional investors responding to their survey indicated that they factor in the ESG effects of corporate policies because considering ESG effects is an ethical responsibility. Of course, more institutional investors referred to purely financial motivations for considering ESG effect. Moreover, survey evidence raises considerable selection bias concerns. However, as we will show, even levels of green sentiment an order of magnitude less than reported in Amel-Zadeh and Serafeim (2018) can generate interesting results in our setting.

The monetary payoffs to universal owners depend both on the effect that the proposal has on the value of their share endowment and on reputation costs associated with voting in a fashion that indicates that they have brown preferences. Universal owners are decisive in the sense that a proposal succeeds if and only if it is supported by the majority of universal owners.[17]

[16]Thus, we assume that the size of institutional share blocks is exogenous. This assumption is a reasonable at least as an approximation. It is doubtful that institutions optimize their holdings simply to defeat ESG proposals. Most institutional shareholdings results from indexing. Even if an institutional investor is not an indexer, fighting ESG is probably a very minor concern relative to the other considerations, e.g., mandates, rank in league tables, etc.

[17]Our analysis presumes that green owners can make voting decisions that reflect their environmental preferences. We are not aware of any

Activists. The activist (he) has green preferences and the activist's preferences are common knowledge. The activist has no endowment of firm shares and acquires shares by trading with the atomistic shareholders. In order to make a proposal at the shareholder meeting, the proposing shareholder must have a sufficient stake in the firm. We capture this restriction by requiring the activist to acquire at least $\underline{n}$ shares.[18]

Atomistic shareholders. Each individual atomistic shareholder is endowed with infinitesimal shareholding. Collectively atomistic shareholders are endowed with $n^{\text{At.}}$ shares. Atomistic shareholders can trade their endowments. For reasons discussed later when we examine the activist's problem, the greenness of atomistic shareholders will have no effect on their behavior. Thus, we impose no restrictions on the portion of activists who have green preferences.

2.2.3 Timing

The sequencing of event in the model is provided below.

Activism phase:

Initiation phase: At date 0, the activist decides whether to initiate activism. If the activist initiates, the activist attempts to acquire shares of the firm and pays an investigation cost, c.

Launch phase: At date 1, if the investigation yields a proposal, the activist decides whether to launch a campaign by submitting a proposal to shareholders; if investigation does not yield a proposal, the activist does not submit a proposal.

Voting phase:

At date 2, if the campaign is launched, shareholders vote on the proposal. If passed, the proposal is implemented.

At date 3, environmental and monetary outcomes are realized.

3 Activism phase

We initiate our analysis by considering the activism phase, the problem of an activist deciding whether to *initiate* and *launch* an activist campaign. Initiation of the campaign involves buying a stake in the firm and then investigating, i.e., attempting to come up with a concrete proposal that, if adopted, will change the firm's carbon footprint

jurisdiction that forbids owners from voting on proposals based on their environmental and social (ES) preferences. However, there are legal questions related to whether funds pursuing ES goals can be included as investment options in ERISA-qualified pension plans. Managers of ERISA-qualified pension plans must ensure that all of the funds available for employee selection make investment/voting decisions based on the objective of maximizing returns and minimizing risk. In 2020, the Trump administration's Department of Labor, through the "Prudence and Loyalty in Selecting Plan Investments and Exercising Shareholder Rights" rule, defined the scope fund ES investing and activism consistent with this objective. In 2021, the Biden administration's Department of Labor amended the rule in ways that some argue expanded the scope for ES-based activism and portfolio choice. In 2023, Congress passed a bill that reversed the Biden administration's revisions. However, the bill was vetoed by President Biden and thus did not become law (Reuters, 2023).

[18] In the US, the ownership threshold is quite modest: owning between $2,000 and $25,000 worth of firm shares depending on the length of time the shares have been held. In the UK, the threshold is much higher: a 5% ownership stake is required to compel inclusion of a proposal on the agenda of the annual general meeting.

and will be acceptable to universal owners with green preferences. If the activist comes up with a proposal, the activist then decides whether to *launch* a campaign, i.e., submit the proposal at the shareholder meeting.

Shareholder voting only affects the activist in so far as it determines the probability that the activist's proposal succeeds, i.e., is approved by shareholders. Therefore, in this section, we assume an exogenous probability that the proposal will succeed, denoted by ρ. ρ will be endogenized in Sections 4 and 5.

The activist has wealth $b+c$ and is liquidity constrained. If he initiates activism and attempts to acquire shares, he pays an investigation cost c and invests all of his remaining wealth, b, in the firm. Thus, the activist purchases b/p_0 shares from the atomistic shareholders, where p_0 is the trading price, determined in the equilibrium.

With probability π, investigation yields a proposal. With probability $1-\pi$, investigation fails to yield a proposal. If investigation yields a proposal, the value of the firm, if the proposal is submitted and succeeds, is $V(S)$, and the green payoff is $G(S)$. If the proposal fails, i.e., no proposal is produced by investigation, or a proposal is produced but not submitted, the value of the firm and the green payoff will be $V(F)$ and $G(F)$, respectively. Thus, we can think of $(V(F), G(F))$ as representing the value of the firm and green payoff under the firm's status quo policies. In order to avoid considering trivial cases, we assume that (a) there is tradeoff between value maximization and maximizing green payoffs, and (b) despite the value reduction produced by adopting the proposal, its adoption is preferred by green owners, i.e., we assume that

$$\text{(a): } G(S) > G(F) \text{ and } \mathrm{V}(F) > V(S), \tag{3}$$

$$\text{(b): } V(S) + \beta\, G(S) > V(F) + \beta\, G(F). \tag{4}$$

Equation (3) implies that, absent the reputation costs produced by opposing the proposal, brown owners prefer rejection of the proposal. Equation (4) ensures that, if the firm is owned entirely by one green owner, even absent reputational considerations, the owner prefers acceptance of the proposal. Because the green payoff does not vary with the fraction of the firm owned by a green agent, but the value of a green owner's claim on the firm is less than the value of the whole firm, equation (4) ensures that green owners will always support the proposal regardless of the degree to which universal owners' shareholdings are dispersed.

We aim to determine the conditions for the existence of an *activism equilibrium*. In an activism equilibrium, the activist plays the *activism strategy*: the activist initiates activism and, when investigation yields a proposal, launches a campaign. In an activism equilibrium, other agents' beliefs are consistent with the activist following the activism strategy. When other agents conjecture that the activist plays the activism strategy, they estimate that the proposal will be implemented with probability $\pi\rho$ and, with probability $1-\pi\rho$, will not be implemented. When the activist attempts to purchase shares from the atomistic shareholders, these shareholders will post ask prices for their shares. Like the atomistic shareholders in Grossman and Hart (1980), atomistic shareholders do not believe that the ask prices they post will have any effect on whether the activist succeeds in purchasing a stake and launching the campaign. Thus, they conjecture that green payoffs will not vary with the ask price they set. This implies, as shown by equation (2), that the ask price set by the atomistic shareholders does not depend on whether their preferences are brown or green. If an atomistic shareholder sells to the activist at ask price p_a, the monetary payoff to the shareholder equals $p_a\,\mathrm{d}n$, where $\mathrm{d}n \simeq 0$ represents the infinitesimal share endowment of an atomistic shareholder. If an atomistic shareholder does not sell, her payoff equals the conjectured value of her share endowment, $\big(\pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F)\big)\,\mathrm{d}n$. Bertrand competition among atomistic shareholders implies that the activist can purchase shares at the lowest price consistent with selling being a best response for the atomistic shareholders, i.e., the equilibrium ask price p_0, is given by

$$p_0 = \pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F). \tag{5}$$

Thus, in an activism equilibrium, the activist acquires b/p_0 shares. The activist's valuation of the firm is the same as the atomistic shareholder's valuation, namely $\pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F)$. Thus, equation (5) shows that, if the activist initiates, the expected wealth of the activist equals b. The activist's green payoff equals $\pi\,\rho\,G(S) + (1-\pi\,\rho)\,G(F)$. If the activist does not initiate, his monetary payoff equals $b+c$ and his green payoff equals $G(F)$. Thus, initiation is a best reply for the activist if and only if

$$\pi\,\rho\,\beta\,\big(G(S) - G(F)\big) \geq c. \tag{6}$$

We term this condition the *initiation condition*. Equation (6) reveals that initiation depends only on the activist's valuation of the expected green benefit, $\pi\,\rho\,\beta\,\big(G(S) - G(F)\big)$, and the cost of initiation, c. Even though the activist's utility depends on wealth, campaign initiation does not depend on the effect of the proposal on firm value. The monetary gain from activism is proportional to the difference between the activist's share valuation and the share price. The share price equals the share value assigned by atomistic shareholders, which is the same as the activist's valuation. So the monetary effect of share acquisition is zero. Of course, the monetary effects of the proposal on firm value determine atomistic shareholders' ask price and thus affect the fraction of the firm acquired by the (liquidity constrained) activist. However, the green benefit is a public good, so the activist's fractional share ownership has no effect on the size of the activist's expected green benefit. Consequently, in contrast to activism's benefit in traditional models—capital gains from increasing share value—the benefit of ESG motivated share acquisition—a greener environment—cannot be appropriated by target shareholders through the ask prices they set for their shares.[19]

If we modeled an activist with a pre-activism, "toehold" ownership stake, say n_o, the activist would factor in the effect of activism on toehold value, the difference between the value of the toehold after and before campaign

[19] See Eckbo (2009) for a survey of the literature on non-ESG control-motivated share acquisitions.

initiation (assuming no anticipation), $-n_o\,\pi\,\rho\,(V(F)-V(S))$. By assumption, $V(F)-V(S)>0$. Thus, toehold stakes would make the initiation condition harder to satisfy. Hence, for ESG activists, toeholds are impediments to activism. In contrast, in traditional models of activism, toeholds are typically facilitators of activism and sometimes necessary for activism (Eckbo, 2009).[20]

Another necessary condition is that the share acquisition by the activists is feasible, i.e., activist's demand is less than the potential supply of shares provided by the atomistic shareholders, $n^{\text{At.}}$, and the activist can acquire sufficient shares to qualify for submitting a proposal, $\underline{n}$. These constraints impose another necessary condition for an activism equilibrium which we term the ownership condition:

$$\underline{n} \le \frac{b}{p_0} \le n^{\text{At.}}, \quad \text{where } p_0 = \pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F). \tag{7}$$

The final condition for an activism equilibrium that it is incentive compatible for the activist, after acquiring shares and learning that a proposal has been developed, to launch the activism campaign by submitting a proposal. The incentive compatibility of launching is not entirely obvious because, at the time the launch decision is made, the activist, who factors monetary payoffs into his utility function, has an ownership stake. Launching the campaign will reduce the monetary value of the activist's holding. However, as we show the proof of Lemma 1, the satisfaction of the initiation and ownership conditions implies the satisfaction of the launch condition. In fact, as following lemma asserts, the ownership and initiation conditions are necessary and sufficient for the submission of a proposal that has a positive probability of passing.

Lemma 1. *An equilibrium exists in which the green proposal is adopted with positive probability if and only if the initiation condition, equation* (6)*, and the ownership condition, equation* (7)*, are satisfied.*

4 Voting phase: Single universal owner

In this section, we consider the case where there is a single universal owner, i.e., $K=1$, and a proposal has been submitted. Because only the atomistic shareholders trade with the activist, the combined holdings of the activist and the atomistic shareholders will equal the holdings of the atomistic shareholders before trade, $n^{\text{At.}}$. Because one share is outstanding, the universal owner's shareholding, which we represent with N^U, equals $1-n^{\text{At.}}$. Thus, to ensure the universal owner is decisive, we assume in this section that $n^{\text{At.}} < 1/2$. As we discuss in detail in the following section on voting by multiple universal owners, this assumption is much stronger than required to ensure universal owner control in real-world proxy contests.

[20] Also, if we extended the model by dropping the assumption that the greenness of the activist is common knowledge and posited instead that some fraction of activists are fake/pseudo greens, these pseudo greens would have an incentive to initiate campaigns, drive down the stock price, but not follow up by launching, and thereby profit from the increased value of their shareholding. Rational atomistic investors would anticipate this behavior in equilibrium. This would lead to ask prices exceeding the monetary value of shares held by truly green activists. We will discuss this scenario more in subsequent drafts of this paper.

The universal owner decides between voting yes, $v = 1$, or no, $v = 0$ on the proposal. The monetary payoff to the universal owner has two components, the value of the universal owner's stake in the firm, $N^U V$, which depends on whether the proposal succeeds, S, or fails, F, and a reputation cost, denoted by $R > 0$. This cost is incurred whenever the owner votes no on the proposal. If the owner is green, the owner's utility also contains a green payoff component, G, which also depends on the success or failure of the proposal. Thus the utilities of a green, U^G, and brown, U^B, single universal owner are given by

$$U^G(v,x) = N^U V(x) + \beta G(x) - R\,\mathbb{1}_{\{v=0\}}, \quad U^B(v,x) = N^U V(x) - R\,\mathbb{1}_{\{v=0\}}, \quad v \in \{1,0\},\ x \in \{S,F\}.$$

Because there is only one universal owner, the universal owner decides the outcome of the vote, i.e., $x = S$ if and only if $v = 1$. Again, to avoid consideration of the trivial case where the proposal is always accepted, we assume that, even factoring in the reputation penalty, a brown owner prefers proposal failure, i.e., $U^B(0,F) > U^B(0,S)$, i.e.,

$$N^U V(F) - R > N^U V(S).$$

Condition (4) ensures that the green owner prefers proposal success. The probability that the universal owner is green is given by $\gamma \in (0,1)$. Thus, when there is a single universal owner, the voting phase is trivial: the proposal succeeds with probability $\rho = \gamma$. These results are obvious but, for the sake of comparison with the multiple universal owner case, we record them below.

Result 1. When there is a single universal owner, the proposal passes if and only if the universal owner is green, which occurs with probability γ. Thus, the probability that a green proposal is adopted equals $\pi\gamma$, and the adoption probability is strictly increasing in green sentiment, γ.

5 Voting phase: Multiple universal owners

5.1 Assumptions: Ownership structure

Assume that there are K universal owners and let $\mathcal{K} := \{0,1,2,\ldots,K\}$. Let n_i^U represent the shareholdings of universal owner $i \in \mathcal{K}$. Assume that the universal owner share blocks are equal sized, i.e., $n_i^U = N^U/K$, $i \in \mathcal{K}$, where, as in the previous section, N^U represents the total shareholdings of the universal owners. The notation n_i^U is thus simplified to n^U henceforth. The assumption that block sizes are exactly equal is not essential for the analysis. However, it does yield a simple necessary and sufficient condition for the number of universal owners voting yes alone determining the effect of universal owner votes on the outcome. Weaker, but more complicated, conditions on block sizes could also ensure the number of yes votes determines the voting outcome. If block sizes varied greatly, then universal owners' effect on the voting outcome would be a function of the subsets of universal owners who vote yes. This would greatly complicate the analysis.

We assume, consistent with actual practice, the universal owners always vote their shares (Brav et al., 2022). We also assume that universal owners are decisive, i.e., whether a proxy proposal passes depends only on the votes of universal owners. Under plurality voting, the standard voting rule in corporate voting, universal owners will be decisive if and only if the number of universal owner yes votes at least equals m, where $m = \lfloor K/2 \rfloor + 1$. This condition ensures that the proposal will pass when supported by the majority of universal owners, and $n^{\text{At.}}/n^U < K - 2\lfloor K/2 \rfloor$. This condition ensures that, regardless of the votes of other shareholders, the proposal will not pass whenever less than m universal owners support the proposal.

Remark 1 (Decisiveness). These conditions will be satisfied if $n^{\text{At.}} < n^U$ and K is odd. We assume that these conditions are satisfied in the subsequent analysis. We also restrict attention to cases where no single universal owner is decisive, i.e., $m > 1$. Collectively these restrictions imply that K is odd, $m \geq 2$, and $K \geq 3$. Next, note that the fact that, K is odd implies that $K - 1$ is even, hence the threshold for success, m, equals $(K-1)/2 + 1$.

Our conditions for decisiveness are probably much stronger than required in real-world corporate voting. They ensure that even if other shareholders block vote against the majority of universal owners, they cannot affect the outcome of corporate votes. In fact, non-institutional investors do not block vote. Moreover, on average, only 30% of non-institutional shares are voted while virtually 100% of institutional shares are voted (Brav et al., 2022). Hence, because non-institutional investors hold approximately 30% of the shares of large U.S. firms, they represent about 9% of the shares voted in corporate proxy contests. Thus, although our implementation of universal owner decisiveness is quite stylized, universal owner decisiveness in proxy contests plausibly approximates many corporate votes. Because the ESG activist is one of the other shareholders, our analysis implicitly assumes that the ESG activist's stake is small and thus the ESG activist cannot affect the outcome of the proxy contest through his proxy votes. In fact, the shareholdings of ESG activist making proxy proposals are frequently quite small (Dimson et al., 2015; Barko et al., 2021; Lopez de Silanes et al., 2022).

To simplify notation, let $w(x)$, $x = S, F$, represent the value of an individual universal owner's stake in the firm conditioned on the success, S, or failure, F, of the proposal, i.e., $w(x) := n^U V(x)$, $x = S, F$. Also let $\Delta w := w(F) - w(S)$ represent increase in the value of owner i's shareholdings if the proposal fails and let $\Delta G = G(S) - G(F)$ be increase in the green payoff if the proposal passes. Let r_i be the reputation cost incurred by the universal owner i if i votes no. Let v_i represent the vote of universal owner $i \in \mathcal{K}$, where $v_i = 1$ if the vote is yes, and $v_i = 0$ if the vote is no. Let $\boldsymbol{v} := (v_1, v_2, \ldots v_K)$ represent the vector of universal owner votes.

Using these definitions and the decisiveness condition (see Remark 1), we can express the utility of green and brown owners, u_i^G and u_i^B, as follows:

$$u_i^G(\boldsymbol{v}) := w(F) + \beta\, G(F) + (\beta\, \Delta G - \Delta w)\, \mathbb{1}_{\sum_{j \in \mathcal{K}} v_j \geq m} - r_i\, \mathbb{1}_{v_i = 0}, \tag{8}$$

$$u_i^B(\boldsymbol{v}) := w(F) - \Delta w\, \mathbb{1}_{\sum_{j \in \mathcal{K}} v_j \geq m} - r_i\, \mathbb{1}_{v_i = 0}. \tag{9}$$

Remark 2. Condition (2) ensures that $\beta\,\Delta G - \Delta w > 0$. Voting for the proposal weakly increases the probability that the proposal passes and avoids the reputation cost triggered by a no vote ($v_i = 0$). Thus, equation (8) shows that voting for the proposal is a strictly dominant strategy when the universal owner is green. For this reason, the voting game is equivalent to a game where all universal owners are brown. For each universal owner, nature makes an independent draw from a Bernoulli distribution, based on this draw, with probability $\gamma \in (0,1)$, nature privately informs the universal owner that nature will vote her shares in favor of the proposal, and with probability $1-\gamma$, privately informs the universal owner that she can decide how to vote her proxies. Thus, we can focus all of our analysis on the voting strategies of brown universal owners.

As in the single universal owner case, we assume that, even net of the reputation penalty incurred by the universal owner, each brown universal owner, if she alone decided the outcome of the vote, would oppose the proposal, i.e., we assume that $r_i < \Delta w$. Because, $r_i < \Delta w$, if brown owners voted "as if pivotal," i.e., each voted as if her vote determined whether the proposal passes, all brown owners would vote no. When brown owners vote yes, they hope that the proposal will fail. Thus, brown votes in favor of the proposal will be termed "insincere" votes. Also, to further simplify notation, let $y_i := r_i/\Delta w$; y_i represents *normalized reputation cost* of voting no on the proposal incurred by universal owner i. Our assumptions imply that $y_i \in (0,1)$ for all $i \in \mathcal{K}$.

Each universal owner casts a vote, either yes or no on the proposal. Voting yes produces 1 yes vote and voting no produces 0 yes votes. Let σ_i represent the probability that a universal owner, when brown, votes yes. The probability that universal owner i casts a yes vote, which we represent with t_i, is given by $t_i = t(\sigma_i)$, where $t : [0,1] \to [0,1]$ is the function $t(\sigma) = \gamma + (1-\gamma)\sigma$, $\sigma \in [0,1]$. Thus, the vote of each universal owner i is a Bernoulli distributed random variable, $\tilde{B}_i$, equal to 1 with probability $t(\sigma_i)$ and equal to 0 with probability $1 - t(\sigma_i)$.

Let $\boldsymbol{t} := (t_1, t_2, \ldots t_K)$ represent the vector of yes-vote probabilities. Since the mixed strategies of the universal owners are jointly independent and independent of nature's brown/green type assignment, the sum of the votes, $\tilde{S}(t)$, is a Poisson-Binomial (PB) random variable, i.e.,

$$\tilde{S}(\boldsymbol{t}) := \sum_{k \in \mathcal{K}} \tilde{B}(t_k). \tag{10}$$

We will denote the sum of yes votes, $\tilde{S}(\boldsymbol{t})$, the sum excluding universal owner i with $\tilde{S}^{-i}(\boldsymbol{t})$, and the sum excluding universal owners i and j, $j \neq i$, by $\tilde{S}^{-ij}(\boldsymbol{t})$, i.e.,

$$\tilde{S}^{-i}(\boldsymbol{t}) := \sum_{k \in \mathcal{K} \setminus \{i\}} \tilde{B}(t_k), \tag{11}$$

$$\tilde{S}^{-ij}(\boldsymbol{t}) := \sum_{k \in \mathcal{K} \setminus \{i,j\}} \tilde{B}(t_k). \tag{12}$$

Note that $\tilde{S}(\boldsymbol{t}) \geq m$ if and only if universal owner i votes yes and at least $m-1$ other universal owners vote yes or universal owner i votes no, and at least m other universal owners vote yes. Hence,

$$\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = t_i \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m-1] + (1-t_i)\mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m] + t_i \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) = m-1]. \tag{13}$$

Using equation (13), it is apparent that

Lemma 2. *If $\tilde{S}(\boldsymbol{t})$ is $PB(t_1, t_2, \ldots, t_K)$ distributed, then*

$$\frac{\partial}{\partial t_i}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) = m-1],$$

$$\frac{\partial^2}{\partial t_i \partial t_j}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-ij}(\boldsymbol{t}) = m-2] - \mathbb{P}[\tilde{S}^{-ij}(\boldsymbol{t}) = m-1], \quad \textit{if } i \neq j,$$

$$\frac{\partial^2}{\partial t_i^2}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = 0.$$

5.2 Nash equilibria

The "greenness" of other universal owners is private information. Thus, a brown universal owner does not know which other universal owners are green. Having rational expectations, she conjectures each of the other universal owners is green with probability γ. Suppose that the candidate equilibrium strategy is $\boldsymbol{\sigma}$. Let $\tau : [0,1]^K \to [0,1]^K$ be the map defined by

$$\tau(\boldsymbol{\sigma}) := \big(t(\sigma_1), t(\sigma_2), \ldots, t(\sigma_K)\big).$$

The distribution of yes votes under strategy vector $\boldsymbol{\sigma}$ is Poisson-Binomial (PB) where the yes vote probability for each Bernoulli random variable is given by $t_i = t(\sigma_i)$. Hence the distribution of yes votes is $\text{PB}\big(t(\sigma_1), t(\sigma_2), \ldots, t(\sigma_K)\big) = \text{PB}(\tau(\boldsymbol{\sigma}))$. Let u_i represent the payoff to universal owner i when i is brown in the mixed strategy extension of a brown owner's payoff function defined by equation (9).[21]

The linearity of payoffs in mixed strategies implies that the payoff to a brown universal owner who plays σ_i,

[21] Because, as explained in Remark 2, the game is effectively played by the universal owner only when the owner is brown, we do not subscript or superscript the utility function with B, thereby reducing the notational burden.

given that other universal owners play $\boldsymbol{\sigma}$, is given by

$$u_i(\sigma_i|\boldsymbol{\sigma}^{-i}) = u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i\left(u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i})\right).$$

The first term in this expression represents a brown owner's payoff from voting no. The second term represents the difference between a brown owner's payoff when she votes yes and votes no. The difference between the yes and no payoffs results from two effects: (a) voting yes avoids the reputation cost but (b) increases the probability that the proposal will pass, which reduces a brown owner's payoff by Δw. The proposal will pass with i's support but not without i's support if and only if $m-1$ other universal owners vote for the proposal. Thus observations verify that

$$\begin{aligned} u_i(0|\boldsymbol{\sigma}^{-i}) &= w_F - \Delta w\,\mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) \geq m] - r_i, \\ u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i}) &= r_i - \Delta w\,\mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) = m]. \end{aligned}$$

Thus, expressed in terms of normalized reputation costs, y_i, the payoff to i from strategy σ_i given that the other brown owners play $\boldsymbol{\sigma}$ is given by

$$\begin{aligned} u_i(\sigma_i|\boldsymbol{\sigma}^{-i}) = u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i\left(u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i})\right) = \\ u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i\,\Delta w\left(y_i - \mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) = m-1]\right). \end{aligned} \tag{14}$$

Next note that $u(0|\boldsymbol{\sigma}^{-i})$ is constant in σ_i as is the term in parenthesis on the right-hand-side of the last line of equation (14). Thus, the set of best responses of i to $\boldsymbol{\sigma}$, which we represent by BR_i is given by

$$\mathrm{BR}_i(\boldsymbol{\sigma}) = \begin{cases} \{1\} & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] > 0, \\ [0,1] & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] = 0, \\ \{0\} & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] < 0. \end{cases} \tag{15}$$

The best response correspondence, BR, for the game is given by

$$\mathrm{BR}(\boldsymbol{\sigma}) := \left(\mathrm{BR}_1(\boldsymbol{\sigma}), \mathrm{BR}_2(\boldsymbol{\sigma}), \ldots, \mathrm{BR}_K(\boldsymbol{\sigma})\right). \tag{16}$$

A *Nash equilibrium of the voting game*, is a strategy vector, $\boldsymbol{\sigma}^*$, satisfying $\boldsymbol{\sigma}^* \in \mathrm{BR}(\boldsymbol{\sigma}^*)$.

5.3 The potential for the game and its properties

There are many Nash equilibria of the voting game, For reasons discussed below, we focus our attention on strategy vectors that maximize a potential function. The potential function we employ, $\Pi : [0,1]^K \to \mathbb{R}$, is defined below.

$$\Pi(\boldsymbol{\sigma}) = \Delta w \left(\sum_{k \in \mathcal{K}} \sigma_k y_k - \frac{\mathbb{P}[S(\tau(\boldsymbol{\sigma})) \geq m]}{1-\gamma} \right). \tag{17}$$

Noting that $\frac{\partial}{\partial \sigma_i} t(\sigma_i) = 1 - \gamma$ and $\frac{\partial}{\partial \sigma_i} t(\sigma_j) = 0,\ j \neq i$, we see the composition rule for differentiation and Lemma 2 imply that

$$\frac{\partial}{\partial \sigma_i} \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) \geq m] = (1-\gamma)\, \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1].$$

Thus, using the definition of Π (equation (17)) we see that

$$\frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}) = \Delta w \left(y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1]\right). \tag{18}$$

Differentiation of equation (14) and inspection of (18) imply that

$$\frac{\partial}{\partial \sigma_i} u_i(\sigma_i | \boldsymbol{\sigma}^{-i}) = \frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}). \tag{19}$$

Thus, Π is an exact potential for the voting game which implies that voting game is an exact potential game (Monderer and Shapley, 1996). The potential is not unique. Adding any function that is independent of the strategic decisions of the players to a potential function yields another potential function. Potential games are games in which all agents' gain from changing strategies, in our case, from voting no to voting yes is determined by a single function, the potential of the game, in our case Π. Brown universal owners act as if they control one component of a single function, Π, and use their control to select strategies that maximize Π. Potential maximizers are always Nash equilibria. To see this, note that first-order necessary conditions for $\boldsymbol{\sigma}^*$ being a local maximizer of Π, i.e.,

$$\begin{aligned}
&\frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}^*) > 0 \Longrightarrow \sigma_i = 1, \\
&\frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}^*) < 0 \Longrightarrow \sigma_i = 0, \\
&\frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}^*) = 0 \Longrightarrow \sigma_i \in [0,1],
\end{aligned}$$

are identical to the best response conditions for a Nash equilibrium (see equation (15)). Thus, any strategy vector, $\boldsymbol{\sigma}$, that is a local maximizer of the potential function is a Nash equilibrium strategy vector. However, because the first-order conditions are not sufficient to ensure that a strategy vector is a local maximizer of the potential, Nash equilibria need not be potential local maximizers, and *a fortiori*, Nash equilibria need not be potential maximizers. So the set of potential maximizers is subset of the set of Nash equilibria, and thus potential maximization can be viewed as a Nash equilibrium refinement (Monderer and Shapley, 1996).

In potential games, such as coordination games (Chen and Chen, 2011), congestion games (Sandholm, 2002), voting games (Bouton et al., 2021), potential maximization is commonly used to refine the set of Nash equilibria. In potential games, potential maximizers have many "nice properties" with respect to learning dynamics, stability, and robustness to perturbations of the information environment. Young (1993, 2020) shows in a noisy learning setting where agents have a vanishingly small probability of making errors, agents' strategy vectors converge to potential maximizing strategies. Carbonell-Nicolau and McLean (2014) show that the set of potential maximizers contains a strategically stable set of pure strategy equilibria and that, in generic potential games, potential maximizers are perfect and essential Nash equilibria. Ui (2001) shows that potential maximizers are robust to the introduction of incomplete information. Alós-Ferrer and Netzer (2010) show that, when agents' probabilities of choosing strategies are determined by the quantile (i.e., logit) best response function, which is frequently used to model the behavior of subjects in economic experiments (McKelvey and Palfrey, 1995), the limiting distribution of agent strategies, as the error probability converges to zero, is a potential maximizing solution.

An alternative approach to modeling voting is to model collusive solutions. However, in the context of our framework, collusive solutions seem quite hard to implement and produce predictions that are inconsistent with observed voting behavior. By definition, collusive mechanisms are not Nash and thus not self enforcing. The objective of collusion is the maximization of joint owner welfare conditioned on the actual distribution of green and brown preferences across owners. Preferences are private information. So any collusive mechanism would involve some sort of side payments between owners to ensure that type (brown or green) revelation is incentive compatible. We see little or no evidence for such side-payment mechanisms. Moreover, the efficient collusion would result in voting outcomes where either (a) exactly $m-1$ owners vote in favor of the proposal or (b) all owners vote for the proposal. Actual shareholder vote distributions do not appear to be consistent with the voting patterns implied by collusion.

In contrast, our solution concept, potential maximization, implements self-enforcing Nash equilibria. No side-payments are required for implementation. In contrast to many other Nash equilibria of the voting game, there exist long-run stochastic learning dynamics which lead to potential maximizing solutions (Alós-Ferrer and Netzer, 2010). Young (2020) asserts that Nash equilibria resulting from learning dynamics can be viewed as evolved "social conventions." In fact, as we will show in the subsequent analysis, the social convention that is potential maximizing is extremely simple and intuitive. Moreover, the shareholder vote distributions resulting from our analysis are not obviously inconsistent with observed voting patterns in proxy contests.

5.4 Potential maximization

It is well known that, in potential games, pure strategy potential maximizers always exist. In Appendix Section B we show that, for almost all parameterizations of our voting game, no mixed strategy equilibrium maximizes the potential function. Because a pure strategy potential maximizer always exists and, generically, mixed strategy maximizers do not, in the subsequent analysis, we consider only pure strategy vectors.

5.4.1 o-strategies and Π_o functions

Focusing on pure strategies considerably simplifies the analysis. Because universal owners have only two pure strategies: vote yes, $\sigma = 1$, or vote no, $\sigma = 0$, determining the set of universal owners who vote yes determines the effect of brown owners on the probability that the proposal passes. The effect of each universal owner vote is the same. However, r_i, the reputation cost saving resulting from a yes vote, varies across universal owners. Inspection of the potential function shows that its maximization requires that the set of universal owners who vote yes when brown contains the universal owners with the largest reputation costs. Thus, without loss of generality, and with a great deal of notational simplification, assume henceforth that reputation costs are weakly decreasing in the index of the universal owner, i.e.,

$$\Delta w > r_1 \geq r_2 \geq r_3 \ldots r_{K-1} \geq r_K > 0.$$

Thus assumption implies that normalized reputation costs are also weakly decreasing in the index of the universal owner.

For each $o \in \mathcal{K}$, define an o-strategy as follows:

$$o\text{-strategy}: \begin{cases} \text{if } i \in \{1,2,\ldots,o\} & \text{universal owner } i \text{ votes yes, i.e., } \sigma_i = 1, \text{ when } i \text{ is brown,} \\ \text{if } i \in \{o+1, o+2, \ldots, K\} & \text{universal owner } i \text{ votes no, i.e., } \sigma_i = 0, \text{ when } i \text{ is brown.} \end{cases} \tag{20}$$

These arguments show that one of these o-strategies is a potential maximizer. Next note when o is greater than $m-1$ but less than K, then an o-strategy is not a potential maximizer. Under such strategies, the probability that the proposal passes equals 1 yet some brown owners vote no, and thus incur a reputation penalty without affecting the outcome. Thus, when identifying the o-strategies that maximize the potential, we need not consider o-strategies where $o \in \{m+1, m+2, \ldots, K-1\}$. Hence, the set of candidate pure strategy potential maximizers is given by o strategies where $o \in \mathcal{O} := \{0, 1, 2, \ldots m-1, K\}$.

Henceforth, an o-strategy refers to o-strategy in which $o \in \mathcal{O}$. We will term the $o = K$-strategy the *capitulation strategy*, where brown owners vote for the proposal even though each brown owner is better off if the proposal fails. We call all o-strategies such that $o \neq K$, *non-capitulation strategies*. We term a non-capitulation strategy where $o \neq 0$ a *partial resistance strategy*, and term the $o = 0$ strategy the *complete resistance strategy* and the

$o = m - 1$ the *minimal resistance strategy*.

Under an o-strategy, the distribution of votes has the following properties, for $i \in \mathcal{O}$, universal owner i votes yes when brown. Because green universal owners always vote yes, the universal owners in $\{1, 2, \ldots o\}$ will always cast o yes votes. The $K - o$ universal owners in $\{o+1, o+2, \ldots, K\}$ will vote no ($\sigma_i = 0$) if they are brown and vote yes ($\sigma_i = 1$) if they are green. Thus, the sum of the universal owners' yes votes from universal owners in $\mathcal{K} \backslash \mathcal{O}$ is a Binomially distributed random variable with $N = K - o$ and success probability $t = \gamma$. Let Z_o represent this random variable. Hence, the proposal will pass if and only if $o + Z_o \geq m$, or equivalently, $Z_o \geq m - o$. Hence, probability that the proposal will pass, ρ, given that the o-strategy is played, is thus given by

$$\rho(o) := \mathbb{P}[Z_o \geq m - o] = \hat{B}(m - o; K - o, \gamma).$$

Consequently, the value of the potential if brown universal owners play strategy $o \in \mathcal{O}$, which we represent by Π_o, is given by

$$\Pi_o = \Delta w \left(\Sigma_1^o - \frac{\hat{B}(m - o, K - o, \gamma)}{1 - \gamma} \right), \quad \text{where } \Sigma_1^o := \sum_{i=1}^{o} y_i. \tag{21}$$

The arguments developed thus far establish our first basic characterization of potential maximizers.

Proposition 1. *There exists an o-strategy, $o \in \mathcal{O}$, such that o maximizes the potential, Π, i.e.,*

$$\max_{\sigma \in [0,1]^K} \Pi(\boldsymbol{\sigma}) = \Pi_o.$$

Let Π^* represent the maximum value of the potential under the o-strategies, i.e.,

$$\Pi^* := \max_{o \in \mathcal{O}} \Pi_o, \tag{22}$$

and let o^* be the argmax of Π^*, i.e., the set of o-strategies that attain the maximum payoff,

$$o^* := \{o \in \mathcal{O} : \Pi_o = \Pi^*\}.^{22} \tag{23}$$

Proposition 1 shows that Π^* is the maximum value for the potential and that any strategy $o \in o^*$ is a maximizer for the potential. Generically, there is a unique potential maximizing strategy, o^*. Thus, the probability that the proposal will pass under the potential maximizing strategy, ρ^*, is

$$\rho^* = \hat{B}(m - o^*; K - o^*, \gamma).$$

[22] When the set o^* is singleton, by a slight and very common abuse of notation, we represent o^* with the unique element in the set and call this element o^*.

5.4.2 Characterization of o strategies

In this section, we characterize how changes in o, the number of owners who vote yes if brown, affect the value of the potential. The first differences between the potential's value at adjacent non-capitulation o-strategies do not have the single-crossing property with respect to green sentiment, γ. In other words, the set of $\gamma \in (0,1)$ such that $\Pi_{o+1} - \Pi_o > 0$ is generally not an interval. The intuition for the failure of single crossing, which is formally established in the appendix (Lemma A.1), is fairly straightforward: incrementing o to $o+1$ has two effects on the potential. First, incrementing ensures that $i+1^{\text{th}}$ owner will not incur reputation cost r_{o+1}. This effect increases the potential and is independent of the level of green sentiment. Second, incrementing increases the marginal pass probability, the difference between the pass probability under the $o+1$ and o strategies. This effect decreases the potential. The increase in the marginal pass probability will be small both when γ is very small, because the proposal is quite likely to fail even if $i+1^{\text{th}}$ owner votes yes, and when γ is very large, because the proposal is quite likely to pass even if $i+1^{\text{th}}$ owner votes no. Thus, the effect of incrementing on the potential can be positive for extreme values of γ and negative for intermediate values.

Because first differences do not have the single-crossing property, it is difficult to directly characterize the monotonicity properties of the potential evaluated at different o-strategies. However, as shown in the appendix (Lemma A.1), second differences do have the single-crossing property. For this reason, it is possible to characterize the convexity/concavity of the relationship between the non-capitulation o-strategies and the value of the potential. Convexity and concavity place some restrictions on which o-strategies can maximize the potential.

Remark 3 (Sequential convexity/concavity). Convexity and concavity are defined for the sequence of o-strategies, $o = 0,1,2,\ldots m-1$ using the standard definitions of sequential convexity/concavity which are analogous to the definitions of convexity/concavity for functions defined on the real line. Thus, convexity/concavity are defined as follows: Π_o, is *concave (convex)* at o', if $(\Pi_{o'+1} + \Pi_{o'-1})/2 \leq (\geq)\, \Pi_{o'}$. Π_o is concave (convex) if for all $o \in \{1,2,\ldots,m-2\}$, Π_o is concave (convex) at o.

Exploiting the single-crossing property of the second differences, we are able to provide, in Proposition 2 below, fairly sharp characterizations of convexity/concavity of the map $o \to \Pi_o$ that defines the value of the potential at different o-strategies.

Proposition 2. *For* $o \in \{1,2,\ldots,m-2\}$ *and* $m \geq 3$,[23]

(a) Low green sentiment: $\gamma < 4/(K+3)$ *is a sufficient condition for the map* $o \to \Pi_o$ *being concave. If all universal owners* $m, m+1, \ldots, K$ *have the same reputation costs, i.e.,* $\Delta y_i := y_{i+1} - y_i = 0$, *for all* $i \in \{m, m+1, \ldots, K-1\}$, *this condition is also a necessary.*

(b) Intermediate green sentiment: If $\gamma \in [4/K+3, 1/2]$ *and the differences between the reputation costs of the brown*

[23] The excluded case, $K = 3$ and $m = 2$, is excluded simply because, in this case, there are only two non-capitulation strategies, $o = 0$ and $o = 1$, so convexity/concavity of the sequence of o-strategies cannot be meaningfully defined.

owners are constant, i.e., for some constant $c \leq 0$, $\Delta y_i = y_{i+1} - y_i = c$, for all $i \in \{m, m+1, \ldots, K-1\}$, the map $o \to \Pi_o$ is initially concave and ultimately convex, i.e., there exists no $o_1, o_2 \in \mathcal{O} \backslash \{K\}$ such that $o_1 < o_2$ and Π is strictly convex at o_1 and strictly concave at o_2.

(c) High green sentiment: $\gamma > 1/2$ is a necessary condition for the map $o \to \Pi_o$ being convex. If all universal owners have the same reputation cost, this condition is also sufficient.

Roughly speaking, the intuition for Proposition 2 is as follows: the marginal effect of decreasing resistance, i.e., incrementing o to $o+1$, is the difference between the marginal reputation cost savings benefit, i.e., the reputation costs of the $i+1^{\text{th}}$ brown owner, r_{i+1}, and the marginal pass probability cost, i.e., the increase in the pass probability caused by one more brown owner voting yes.

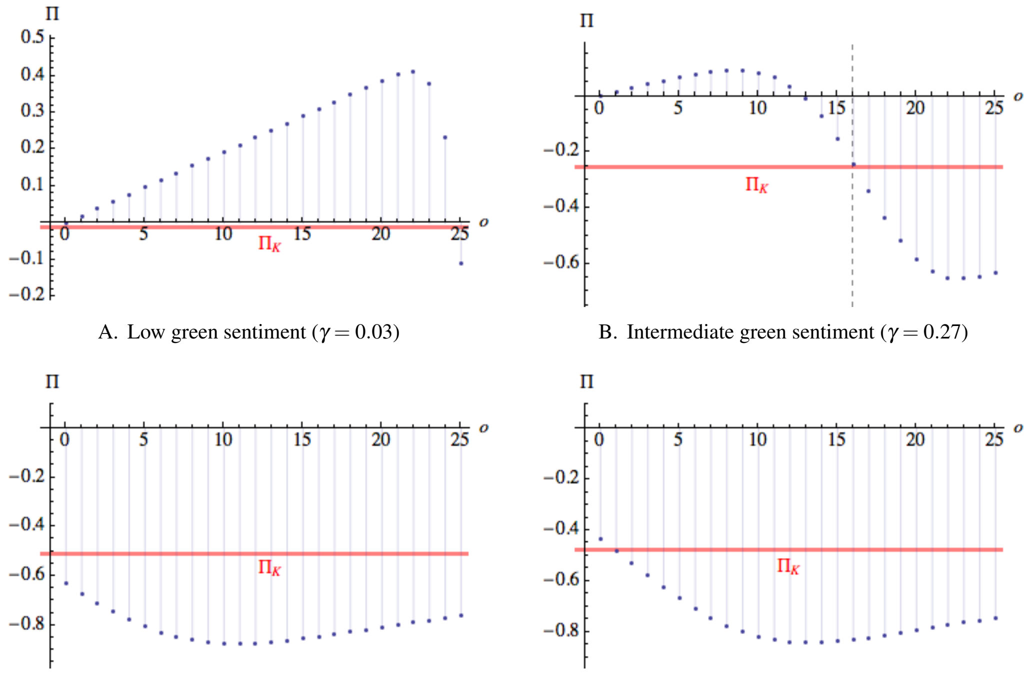
When green sentiment, γ, is low, the proposal is likely to fail even when a few brown owners vote yes. So, when the number of brown owners voting yes, o, is small, the increase in the pass probability caused by one more brown owner voting yes is small. When o is large, a brown owner yes vote is likely to be marginal; so an increase in o triggers a large increase in the pass probability. Thus, the marginal pass probability is increasing in o. Because the map $o \to r_o$ is decreasing, the marginal reputation cost saving benefit of incrementing o is decreasing in o. Thus, the marginal effect of incrementing o, the difference between the marginal reputation cost savings benefit and the marginal pass probability cost, is weakly decreasing, i.e., the map $o \to \Pi_o$ is concave. This case is characterized in part (a) of the proposition. Consequently, when green sentiment is low, potential maximization involves "fine-tuning" the marginal tradeoff between the benefits and costs of resistance. The optimal o-strategy incrementally adjusts in response to changes in the distribution of reputation costs and the level of green sentiment. This case is illustrated in Panel A of Figure 1.[24]

In contrast, when green sentiment is fairly high, the proposal is likely to fail only when very few brown owners support the proposal, i.e., o is small. In which case, incrementing o can engender a significant increase in the pass probability. In contrast, if many brown owners are voting yes, i.e., o is large, the proposal is likely to pass regardless of whether one more brown owner votes yes, i.e., o is incremented to $o+1$. Thus, the marginal pass probability is decreasing in o. Because marginal reputation cost savings are independent of green sentiment, the argument provided in discussion of the low green sentiment case shows that, in this case as well, marginal reputation costs savings are weakly decreasing in o. However, marginal reputation cost savings vary only when the reputation costs of universal owners vary. When all universal owners have the same reputation costs, marginal reputation cost savings are constant in o. Thus, the marginal effect of incrementing o, the difference between the marginal reputation cost savings benefit and the increased pass probability cost, is increasing, i.e., the map $o \to \Pi_o$ is convex. This case is characterized in part (c) of the proposition. Consequently, when green sentiment is fairly high, potential maximization involves optimizing over the two extreme resistance strategies, complete

[24] In order to make convexity/concavity easier to visually detect, the figures illustrate a case where the number of universal owners, 51, is unrealistically large.

resistance, $o = 0$, and capitulation, $o = K$. This case is illustrated in Figure 1. A case where capitulation is optimal is illustrated in Panel C and a case where complete resistance is optimal is illustrated in Panel D.

At intermediate levels of green sentiment, the map $o \to \Pi_o$ is concave when o is small and convex when o is large. In this case, the potential can either be optimized at intermediate levels of resistance, extreme resistance, or capitulation. The only definite characterization of the optimal o-strategy in this case is that it cannot lie in the interior of the region where $o \to \Pi_o$ is convex. Since this region contains the large o strategies, brown resistance, if it occurs, is fairly strong. This case is characterized in part (c) of the proposition and illustrated in Panel B of Figure 1.



A. Low green sentiment ($\gamma = 0.03$)

B. Intermediate green sentiment ($\gamma = 0.27$)

C. High green sentiment ($\gamma = 0.48$), capitulation

D. High green sentiment ($\gamma = 0.52$), complete resistance

Figure 1: *o-strategies and green sentiment.* In the figure, the number of universal universal owners supporting the proposal when brown, o, for different non-capitulation o-strategies is plotted on the horizontal axis. The value of the potential under these strategies is plotted on the vertical axis, and represented by a blue dot. The value of the potential under of the capitulation strategy, $o = K$, is represented by the red horizontal line. In all panels, $K = 51$, $m = 26$. In panels A, B, and C, $\mathbf{y} = (0.02, 0.02, \ldots, 0.02)$. In Panel D, $\mathbf{y} = (0.01, 0.01, \ldots, 0.01)$. The optimal o-strategy, o^* is the o-strategy corresponding to the highest blue dot, unless this dot lies below the red line, in which case $o^* = K$.

6 Comparative statics

In this section, we consider the effects of normalized reputation costs, green sentiment, and ownership dispersion, on the likelihood that green proposals pass and the welfare of brown owners.

6.1 Reputation costs

Level Normalized reputation costs of a given owner, say i, increase when (a) the reputation cost of voting yes, r_i, increases or (b) the value difference between the green proposal and the brown status quo, Δw, decreases. Increasing normalized reputation costs, increases the gain to brown owners, per unit of value difference, from avoiding the reputation costs that result from opposing green proposals. Thus, not surprisingly, increasing normalized reputation costs reduces brown resistance, and thereby increases the potential maximizing number of brown owners who vote yes, o^*. Because the distribution of yes-votes under strategy o'' strictly first-order stochastically dominates the distribution of yes votes under strategy o' if and only if $o'' > o'$, increasing o is equivalent to increasing the pass probability.

Lemma 3. *Suppose that $\mathbf{y}^1$ and $\mathbf{y}^2$ are two vectors representing normalized reputation costs. Then for any fixed $\gamma \in (0,1)$, if $\mathbf{y}^2 \geq \mathbf{y}^1$, then $o^*(\mathbf{y}^2) \geq o^*(\mathbf{y}^1)$.*[25] *Hence, increasing reputation costs increases the probability that green proposals pass.*

Dispersion Because we allow reputation costs to differ across universal owners, we can also examine the effect of the dispersion of normalized reputation costs on the passing probability. If dispersion is defined using the standard dispersion ordering, majorization (Marshall et al., 2011), increasing the dispersion of normalized reputation costs can either increase or decrease o^*.[26] Examples of cases where more dispersed vector of reputation costs leads to higher values of o^* and lower values of o^* are provided in Appendix Section C.

Although no general relationship holds between majorization and the proposal pass probability, intuitively it is fairly easy to see that, unless brown owners capitulate, it must be the case that the m universal owners with the lowest normalized reputation costs vote against the proposal when they are brown. Reducing the normalized reputation costs borne by these low normalized reputation cost owners and transferring those costs to the $m-1$ brown owners with the highest normalized reputation costs, who sometimes vote insincerely for the proposal, increases the gain, saved reputation costs, from insincere voting. Thus, such transfers seem to favor more insincere voting. Because such transfers increase the normalized reputation costs of the owners who already have the highest reputation costs and reduce the normalized reputation costs of the owners with the lowest normalized reputation costs, intuitively, such transfers can be viewed as increasing the dispersion of normalized reputation costs. To convert this intuition into a formal result, we first define the notion of a high-low reputation cost spread.

Definition 1. Given two normalized reputation cost vectors, $\mathbf{y}'$ and $\mathbf{y}''$, $\mathbf{y}''$ is *high-low reputation cost spread* of $\mathbf{y}'$ if (a) $\mathbf{y}'' \neq \mathbf{y}'$, (b) $\Sigma_1^K(\mathbf{y}'') = \Sigma_1^K(\mathbf{y}')$, (c) $y''_k \geq y'_k$, for all $k \in \{1,2,\ldots,m-1\}$.

[25] The ordering over normalized reputation costs vectors, $\mathbf{y}$, is the standard component wise ordering. So, $\mathbf{y}^2 \geq \mathbf{y}^1$ means that each component of $\mathbf{y}^2$ is no less than the corresponding component of $\mathbf{y}^1$. In non-generic cases where either $o^*(\mathbf{y}^1)$ or $o^*(\mathbf{y}^2)$ is not singleton set, "$o^*(\mathbf{y}^2) \geq o^*(\mathbf{y}^1)$" should be interpreted as $\max o^*(\mathbf{y}^2) \geq \max o^*(\mathbf{y}^1)$ and $\min o^*(\mathbf{y}^2) \geq \min o^*(\mathbf{y}^1)$.

[26] One vector, $\mathbf{x}'$, is majorized by another vector, $\mathbf{x}''$, if $\mathbf{x}'$ results from a series of Dalton inequality reducing transformations of $\mathbf{x}''$. See Marshall et al. (2011) for a very detailed analysis of majorization.

The next lemma confirms our intuition that high-low reputation cost spreads reduce brown resistance and thus increase the probability that green proposals pass.

Lemma 4. *Let* $\mathbf{y}^1$ *and* $\mathbf{y}^2$ *be two normalized reputation cost vectors. Suppose that* $\mathbf{y}^2$ *is* high-low reputation cost spread *of* $\mathbf{y}^1$ *and that capitulation is not a potential maximizer when* $\mathbf{y} = \mathbf{y}^1$*, i.e.,* $K \notin o^*(\mathbf{y}^1)$*, then, for any fixed* $\gamma \in (0,1)$*,* $o^*(\mathbf{y}^2) \geq o^*(\mathbf{y}^1)$*. Hence, a high-low reputation cost spread increases the probability that green proposals pass.*[27]

Lemma 4 shows that concentrating reputation costs on a few universal owners weakens resistance to green proposal. In practice, how might such concentration be accomplished? An obvious reputation cost of voting against green proposals is withdrawal of funds by green retail investors. Thus, reputation costs should depend on the greeness of the investors in a universal owner's fund. Hence, Lemma 4 suggests that simply by coordinating to investing in a few mutual funds, passive green retail investors can increase the likelihood that ESG activist campaigns shift environmental outcomes in the direction they prefer.

6.2 Green sentiment

In contrast to the effect of increasing normalized reputation costs, increasing green sentiment can actually reduce the probability that green proposals pass. This observation is formalized by the following lemma.

Lemma 5. *Holding normalized reputation costs fixed, if (a)* $\Sigma_m^K < 1$ *and (b) there exists* $\tilde{\gamma} \in (0,1)$*, such that (i) for all* $\gamma \in (0, \tilde{\gamma})$*,* $K \notin o^*(\gamma)$*, and (ii)* $o^*(\tilde{\gamma}) \neq m-1$*, then the probability of success is not monotonic in* γ*, i.e., increased green sentiment can reduce the probability that green proposals pass.*

The intuition for this result is fairly simple: Inspecting equation (21) shows that the potential's value under a given o strategy has two components: (a) a reputation cost saved component representing the reduction in reputation costs associated with o universal owners voting yes even when brown, and (b) the vote outcome component representing the effect of o universal owners voting yes on the outcome.

$$\Pi_o = \Delta w \left(\overbrace{\Sigma_1^o}^{\text{Rep. Cost Saved}} - \overbrace{\frac{\hat{B}(m-o, K-o, \gamma)}{1-\gamma}}^{\text{Vote Outcome}} \right).$$

When a change in γ induces a change in the o-strategy that maximizes the potential, the change in the strategy causes the reputation cost saved component to make a discrete jump. Because the function mapping γ into the maximized potential function, $\gamma \to \Pi^*(\gamma)$, is the maximum of a finite number of continuous functions of γ, namely the Π_o functions, the potential's value under the optimal strategy is continuous function of γ. Hence, when a change

[27] Again, in the non-generic cases where either $o^*(\mathbf{y}^1)$ or $o^*(\mathbf{y}^2)$ is not singleton set, "$o^*(\mathbf{y}^2) \geq o^*(\mathbf{y}^1)$" should be interpreted as $\max o^*(\mathbf{y}^2) \geq \max o^*(\mathbf{y}^1)$ and $\min o^*(\mathbf{y}^2) \geq \min o^*(\mathbf{y}^1)$.

in γ induces a change in the optimal o-strategy, to maintain the continuity of Π^*, the jump in the reputation cost saved component must be compensated by an equal jump in the same direction in the vote outcome component. The vote outcome component is proportional to the probability that the proposal passes at γ, $\hat{B}(m-o,k-o,\gamma)$. Thus, whenever an increase in green sentiment causes the optimal o-strategy to shift from o'' to o', $o' < o''$, the probability of proposal success jumps down. Because, for any fixed o-strategy, increasing green sentiment increases the probability of proposal success, in between the jump points, increased green sentiment increases the probability that the proposal passes.

The non-monotone relation between green sentiment, γ, and the probability that proposal passes is illustrated in Figure 2. In the figure, when green sentiment is very low, the potential is maximized by the minimal resistance strategy, $o = 2$; as green sentiment increases, resistance stiffens and the optimal resistance strategy shifts from $o = 2$ to $o = 1$ and then to complete resistance, $o = 0$. Finally, green sentiment becomes so large that the proposal will pass regardless of brown opposition, at which point, the optimal strategy shifts to capitulation, $o = 5$.

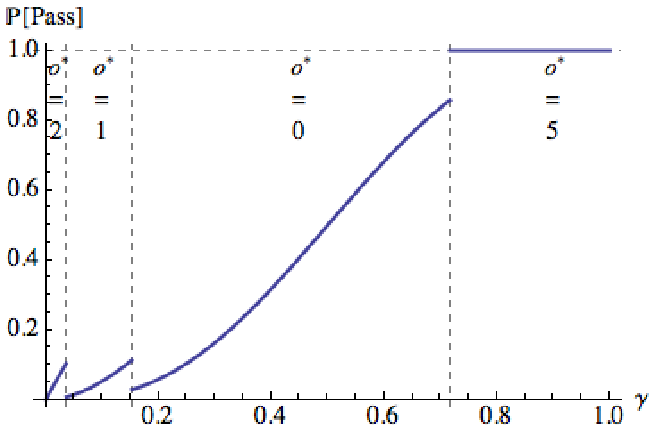


Figure 2: In the figure, $y_k = 0.10$ for all $k \in \mathcal{K}$, $K = 5$, and $m = 3$.

Lemma 5 shows that increasing green sentiment amongst universal owners does not have a reliably positive effect on the probability that green proposals pass. Efforts by activists to convince institutional investors to embrace green values might backfire, especially if these efforts are only marginally successful. Unconvinced institutional investors, upon observing these efforts, might worry that other institutional investors have been convinced and thus that, in order to block the proposal, they must eschew insincere voting and vote no. This "brown backlash" lowers the probability of proposal success. In contrast, influencing the level and distribution of reputation costs does have reliable effects on the probability of proposal success. For this reason, activists, when attempting to increase the probability of proposal success, might prefer to devote their efforts to increasing the public's (and the institutions' beneficial owners) commitment to ESG goals rather than trying to convince institutional investors to adopt green values.

6.3 Universal ownership dispersion

As shown in Section 4, if the firm is controlled by a single universal owner, that owner's preferences determine whether green proposals succeed. Thus, the probability of proposal success simply equals the level of green sentiment. As shown in Section 5, analyzing the far more realistic case of many strategic owners non-trivially complicates the analysis. This naturally raises the question of whether the strategic complications entailed by dispersed universal ownership increase or decrease the ability of brown owners to determine the voting outcome.

In this section, we answer this question. We fix total firm value effects of proposals and the total reputation cost of resistance. Ownership dispersion is increased by increasing the number of universal owners. It is obvious that, if the division of ownership is accompanied by an extremely asymmetric division of reputation costs, increasing the number of owners can significantly reduce the probability of green proposals passing. For example, if the ownership stake of a single universal owner is divided and assigned to a large number of universal owners, and all reputation costs are assigned to one of these owners, then, for all o-strategies except $o = 0$, resistance to green proposals will be costless. In the limit, as the number of universal owners increases without bound, brown owners will block all green proposals without incurring any reputation costs.

To avoid considering trivial cases like this, we assume that total reputation costs, like total universal owner shareholdings, are divided symmetrically. Thus, we consider parameterizations of the model where

$$\forall i \in \mathcal{K},\ r_i = r := \frac{R}{K}, \quad \Delta w = \frac{\Delta W}{K}, \quad \forall i \in \mathcal{K}, \quad y_i = y := \frac{r}{\Delta w} = \frac{R}{\Delta W}. \tag{24}$$

When considering shifts in the number of owners, it is convenient to parametrize the model using the threshold required for passage, m, rather than the total number of universal owners, K. Recalling that m and K are related by $K = 2m-1$ (see Remark 1), we see that we can express the potential function as follows:

$$\Pi_o^m = \left(\frac{\Delta W}{2m-1}\right)\left(o\,y - \frac{\hat{B}(m-o, 2m-1-o, \gamma)}{1-\gamma}\right). \tag{25}$$

The superscript m explicitly represents the dependence of the potential on m. Note that the case of $m = 1$ and thus $K = 1$ represents the single universal owner case.

We consider two measures of the relationship between ownership dispersion and the effectiveness of brown opposition to green proposals: the *pass probability*, i.e., the probability that proposals pass, and the *monetary payoff*, i.e., total expected monetary payoff received by brown universal owners. We show that, despite the rather obvious free-rider problem produced by ownership dispersion, for some configurations of the model parameters, ownership dispersion reduces the pass probability and increases the monetary payoffs, and thus the welfare of brown owners.

6.3.1 The pass probability

The effect of increasing the dispersion, i.e., increasing m, on the pass probability depends on both (a) the effect of dispersion on the willingness of brown owners to resist green proposals and (b) the impact of dispersion on the effectiveness of resistance. Because, potential maximizing strategies are Nash equilibrium strategies, the increase in other brown owners' welfare engendered by a given brown owner's opposition to a green proposal does not affect the potential solution. This "free-rider problem" militates in favor of dispersion increasing the pass probability.

However, dispersion also impacts the effectiveness and efficiency of resistance. Under the complete resistance strategy, $o = 0$, it is very easy to characterize this effect: suppose we increase dispersion by increasing the passing threshold, m, by one unit and thus increase the number of owners by two. If the two new universal owners turn out to be green, the pass probability increases, if both turn out to be brown, the pass probability decreases, and if one is brown and one is green, the pass probability is not changed. Thus, the effect of increased dispersion on the pass probability will be positive (negative) if universal owners are more (less) likely to be green than brown. We record this simple result below.

Result 2. Under the complete resistance strategy $o = 0$, reducing concentration by incrementing m to $m+1$ (or equivalently K to $K+2$) strictly increases (reduces) the pass probability if $\gamma > \frac{1}{2}$ ($\gamma < \frac{1}{2}$).

High green sentiment, $\gamma > \frac{1}{2}$ Result 2 has important consequences when $\gamma > \frac{1}{2}$. As shown in Proposition 2.c, when $\gamma > \frac{1}{2}$, the potential is a convex function of o for $o < K$. Thus, the potential maximizing resistance o-strategy is extreme, either maximal resistance, $o = 0$, or minimal resistance, $o = m-1$. When, as we assume in this section, normalized reputation costs are the same for all universal owners, this implies that, when $\gamma > \frac{1}{2}$, the optimal resistance strategy is maximum resistance, $o = 0$, and this strategy is optimal if its payoff is no less than the payoff of the capitulation strategy, $o = K = 2m-1$. This result is recorded below.

Result 3. If $\gamma > \frac{1}{2}$, the potential maximizing o-strategy is either complete resistance, $o = 0$, or capitulation, $o = 2m-1$.

When $\gamma > \frac{1}{2}$, Result 3 shows that brown universal owners will either completely resist or capitulate. Result 2 shows the effectiveness of complete resistance is reduced by increased ownership dispersion. Thus, increased dispersion decreases the attractiveness of complete resistance relative to capitulation and also increases the pass probability even when brown owners adopt the complete resistance strategy. Thus, when $\gamma > \frac{1}{2}$, increased dispersion increases the pass probability.

Result 4. If $\gamma > \frac{1}{2}$, increasing the number of universal owners weakly increases the pass probability.

Low to moderate green sentiment, $\gamma < \frac{1}{2}$ When $\gamma < \frac{1}{2}$, the relationship between dispersion and the pass probability is much more subtle. Result 2 shows that, in this case, the effectiveness of complete resistance strategies is increased by dispersion. This effect favors increased dispersion reducing the pass probability. However, when $\gamma < \frac{1}{2}$, the relationship between the resistance o-strategies and the value of the potential is generally not convex (See Proposition 2). Thus intermediate partial resistance o-strategies with $0 < o < m-1$ can maximize the potential. Dispersion increases the attractiveness of partial resistance and capitulation relative to complete resistance. Thus, developing a simple general comparative static in this case is not possible. However, as illustrated in the following figures, it is easy to provide examples with plausible numbers of universal owners under which multiple universal owners more effectively block proposal passage than a single universal owner. Such cases are illustrated in Figure 3.

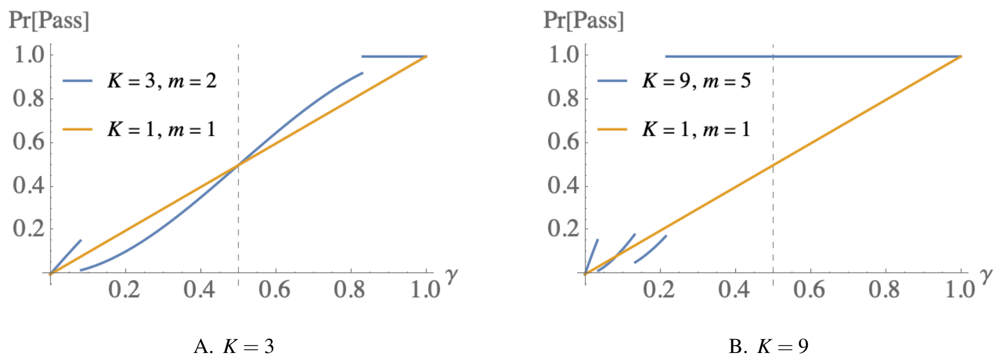


Figure 3: *Ownership concentration and the pass probability.* In both panels, the horizontal axis represents the level of green sentiment amongst universal owners, γ, and the vertical axis represents the pass probability, Pr[Pass]. The reduction in the value of the universal owners' shares produced by the proposal passing is $\Delta W = 1$. The reputation costs incurred by the universal owners if they all oppose the proposal is $\Delta R = 0.15$. For the sake of comparison, the relationship between the pass probability and green sentiment when there is a single universal owner, $K = 1$, is represented by the orange line. The blue line represents the relationship between green sentiment and the pass probability when there are K universal owners. In Panel A, $K = 3$ and in Panel B, $K = 9$.

6.3.2 Monetary payoffs

We now consider the effect of universal ownership dispersion on universal owners' monetary payoff. The monetary payoff has two components: one that captures the effect of proposal passage on the expected value of the universal owners' stake and one that captures the expected reputation costs imposed by opposing the proposal:

$$\text{Exp. Univ. Owner Value:} \quad W(F) - \Delta W \, \overbrace{\hat{B}(m-o, 2m-1-o, \gamma)}^{\text{Pr[Pass]}} \tag{26}$$

$$\text{Exp. Reputation Costs:} \quad \underbrace{(1-\gamma)(2m-1-o)}_{\text{exp. \# resisting owners}} \underbrace{\left(\frac{R}{2m-1}\right)}_{\text{Rep. cost per owner}} \tag{27}$$

Subtracting reputation (equation (27)) from value effects (equation (26)) and simplifying yields the following expression for the monetary payoff, M-payoff:

$$\text{M-Payoff}_o^m := W(F) - (1-\gamma)R + \left((1-\gamma)\frac{R}{2m-1}o - \Delta W\,\hat{B}(m-o, 2m-1-o, \gamma)\right). \tag{28}$$

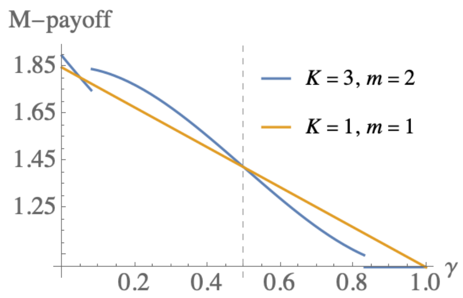
In order to facilitate comparison with the potential function, Π_o^m, we can rewrite the expression for M-payoff as follows:

$$\text{M-Payoff}_o^m := W(F) - (1-\gamma)R + (1-\gamma)\frac{\Delta W}{2m-1}\left(yo - (2m-1)\frac{\hat{B}(m-o, 2m-1-o, \gamma)}{1-\gamma}\right). \tag{29}$$

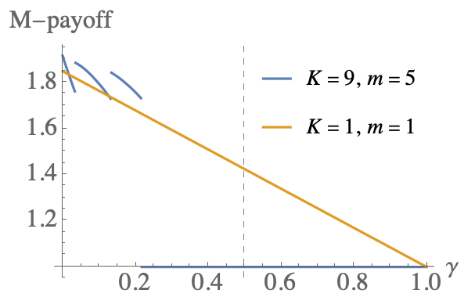
This formulation highlights the difference between the potential function, Π_o^m, defined by equation (25), and the monetary payoff function, M-Payoff$_o^m$, defined by equation (29). First, note that, for a fixed number of universal owners, the o-strategy maximizing Π_o^m and the o-strategy maximizing M-Payoff$_o^m$ depend only on the parts of these expression enclosed in the large parenthesis on the right hand side of their defining equations. The only difference between the expressions for Π_o^m and M-Payoff$_o^m$ within these parentheses is that M-Payoff$_o^m$ multiplies the probability of passage by $K = 2m-1$. Thus, when $m > 1$ and thus $K > 1$, the monetary payoff factors in the effect of proposal passage on *all* universal owners while the potential only factors in the effect on an individual universal owner. The gap between the monetary payoff and the potential (the function which determines the actual strategy played by brown owners) increases with the number of universal owners. This gap militates in favor of dispersion reducing the monetary payoff. However, there are two countervailing forces: the increased efficiency of resistance engendered by ownership dispersion when $\gamma < \frac{1}{2}$ (see Result 2) and a new force: the reputation cost savings from strategic voting. When green sentiment is sufficiently low, even if some brown owners insincerely vote in favor of the green proposal (and thus avoid incurring reputation costs), the proposal is still very likely to fail. Thus, when green sentiment is sufficiently low, strategic insincere voting can appreciably reduce total expected reputation costs while only negligibly increasing the pass probability. In this case, dispersion also increases the monetary payoff. These observations are recorded in the following result.

Result 5. (i) If $\gamma > \frac{1}{2}$, the monetary payoff is lower if ownership is divided amongst multiple owners rather than concentrated in the hands of a single universal owner, (ii) When $\gamma < \frac{1}{2}$ then, whenever (a) green sentiment $\gamma > 0$ is sufficiently small or (b) under divided ownership, the potential maximizing o-strategy is complete resistance, the monetary payoff is higher under divided ownership.

We illustrate these results in Figure 4. Note that when the number of universal owners is greater than 1 but small, $K = 3$ in Panel A, the increased resistance efficiency and reputation cost minimization effects ensure that, despite the free-riding incentives engendered by divided ownership, as long as $\gamma < \frac{1}{2}$ and reputation costs are small relative to value effects, divided universal ownership generally produces higher monetary payoffs than unified ownership. In contrast, when the number of universal owners is large, $K = 9$ in Panel B, divided ownership only produces higher monetary payoffs when green sentiment is very low.



A. $K = 3$



B. $K = 9$

Figure 4: *Ownership concentration and the monetary payoff.* In both panels, the horizontal axis represents the level of green sentiment amongst universal owners, γ, and the vertical axis represents the monetary payoff, M-Payoff. The reduction in the value of the universal owners' shares produced by the proposal passing is $\Delta W = 1$. The reputational cost incurred by the universal owners if they all oppose the proposal is $\Delta R = 0.15$. For the sake of comparison, the relationship between the monetary payoff and green sentiment when there is a single universal owner, $K = 1$, is represented by the orange line. The blue line represents the relationship when there are K universal owners. In Panel A, $K = 3$ and in Panel B, $K = 9$.

As we have seen, the effects of dispersed ownership depend primarily on two parameters, (a) the level of green sentiment, and (b) the degree to which ownership is dispersed. So, the implications of our analysis for actual corporate proxy votes depend on the typical values of these parameters in actual proxy voting contests. Empirical research does provide some guidance for assessing these parameters. Based on the survey evidence provided by Amel-Zadeh and Serafeim (2018), as discussed in Section 2.2.2, most universal owners probably do not have an inherent, non-instrumental preference for green outcomes. So, green sentiment in real-world proxy contests is almost surely less than $1/2$, and is likely to be considerably less than $1/2$. As documented by Amel-Zadeh et al. (2022), the number of universal owners, and thus the degree of universal ownership dispersion, is limited. In these typical cases, the strategic effects introduced by ownership dispersion frequently increase the influence of brown owners on the voting outcome, i.e., reduce the pass probability, and also increase the welfare of brown owners.

7 Conclusion

In this paper, we modeled the ESG activists' green campaigns and universal owners' voting in proxy contests. First we showed that, in contrast to the case of activist aiming to increase firm value, green activism is not constrained by the hold-up problem first modeled in Grossman and Hart (1980). As in Grossman and Hart (1980), the price at which the activist purchases shares fully impounds the effects of intervention on firm value. However, the ESG activist has another source of gains from share acquisition that is not appropriable by selling shareholdings, the change in the environment produced by adoption of the activist's proposal. In equilibrium, the ESG activist's payoff captures the entire "environmental return" from activism. As long as this return exceeds the cost of activism, launching a campaign is a viable strategy for the ESG activist. Thus, in a world where a subset of investors have strong pro-environment preferences, activism campaigns are not very costly relative to the potential environmental benefits of changing corporate policies, and ESG proposals have some chance of being adopted, many activist campaigns will be launched.

When voting on proposals by ESG activists, universal owners face the trade-off between reputation costs and financial value reduction. This leads to strategic voting. Brown owners tend to vote insincerely in favor of a proposal when their no votes are not likely to be required to defeat the proposal or when the proposal is likely to pass by a wide margin regardless of their vote. We find that increasing the reputation cost of no votes on green proposals and concentrating reputation costs on the universal owners most exposed to public pressure always increases the probability that green proposals will pass. However, a higher likelihood that universal owners have pro-environment, "green," preference, does not always increase the probability that green proposals pass. More green sentiment can trigger a brown backlash, i.e., more resistance from the remaining brown shareholders, making a proposal less likely to pass.

When there are multiple universal owners, the presence of reputation costs ensures that, even when green sentiment and public pressure are low, and, net of reputation costs, brown universal owners are better off when green proposals fail, there is always some, albeit small, probability that green proposals pass. Thus, if the cost of activism are small, many green proposals will be advanced and few will pass. In this case, the passing probability of green proposals is very sensitive to the firm value reduction required to affect the environmental improvement. In periods of public "moral panic" about the environment, even if green sentiment of brown owners remains quite low and reputation penalties are less than the value loss from adopting green proposals, brown universal owners capitulate. As a consequence, aggressive proposals that require considerable sacrifices of firm value to achieve environmental objectives have a significant probability of passing.

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A Appendix:Proofs and additional results

A.1 Proofs for Section 1

No proofs or derivations are in this section.

A.2 Proofs for Section 2

All derivations of result in this section are presented in the main body of the manuscript.

A.3 Proofs for Section 3

Proof for Lemma 1. First, that, as asserted in the discussion before Lemma 1, the initiation condition, equation (6) and the ownership condition, (7) imply that the launch condition is satisfied. To see this, note that conditional on initiation, the payoff to the activist from launching when a proposal has been developed is $\frac{b}{p_0}\big(\rho\, V(S) + (1-\rho)\, V(F)\big) + \beta\,\big(\rho\, G(S) + (1-\rho)\, G(F)\big)$, the payoff from not launching is $\frac{b}{p_0} V(F) + \beta\, G(F)$. Hence, using equation (5), we see that the condition for launching the campaign assuming a proposal has been developed is

$$\beta\,\big(G(S) - G(F)\big) - \big(V(F) - V(S)\big)\,\frac{b}{p_0} \geq 0, \text{ where } p_0 = \pi\,\rho\, V(S) + (1-\pi\,\rho)\, V(F). \tag{A-1}$$

Note that b/p_0 equals the number of shares acquired by the activist. The firm has one share outstanding, so the number of shares acquired by the activist, b/p_0, must be less than one to satisfy the ownership condition, (7). Equation (4) implies that $\beta\,\big(G(S) - G(F)\big) > \big(V(F) - V(S)\big)$. Therefore,

$$\beta\,\big(G(S) - G(F)\big) - \big(V(F) - V(S)\big)\,\frac{b}{p_0} > \big(V(F) - V(S)\big)\big(1 - \frac{b}{p_0}\big) > 0.$$

Next note that the initiation condition, equation (6), can never be satisfied if the probability of success, $\rho = 0$. Thus, if the initiation condition is satisfied, $\rho > 0$. Finally, note that the activism strategy is the only activist strategy that results in a positive probability of a proposal being adopted. Thus, as well as being sufficient, the initiation can ownership conditions are also necessary for activism equilibrium. □

A.4 Proofs for Section 4

All derivations of result in this section are presented in the main body of the manuscript.

A.5 Proofs for Section 5

Proof of Lemma 2. To prove (a), first note that

$$\mathbb{P}[S(t) \geq m] = t_i \mathbb{P}[S^{-i}(t) \geq m-1] + (1-t_i)\mathbb{P}[S^{-i}(t) \geq m].$$

Thus,

$$\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m] = \mathbb{P}[S^{-i}(t) \geq m-1] - \mathbb{P}[S^{-i}(t) \geq m] = \mathbb{P}[S^{-i}(t) = m-1].$$

To prove (b), note that (a) implies that

$$\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m] = \frac{\partial}{\partial t_j}\left(\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m]\right) = \frac{\partial}{\partial t_j}\mathbb{P}[S^{-i}(t) = m-1] \tag{A-2}$$

and that

$$\mathbb{P}[S^{-i}(t) = m-1] = t_j \mathbb{P}[S^{-ij}(t) = m-2] + (1-t_j)\mathbb{P}[S^{-ij}(t) = m-1].$$

Thus,

$$\frac{\partial}{\partial t_j}\mathbb{P}[S^{-i}(t) = m-1] = \mathbb{P}[S^{-ij}(t) = m-2] - \mathbb{P}[S^{-ij}(t) = m-1]. \tag{A-3}$$

and (b) follows from (A-2) and (A-3). □

A.5.1 Results for Section 5.4

Let $b(n;N,t)$ represent the probability of exactly n success realizations of a Binomial distribution with N trials and success probability t:

$$b(n;N,t) = \mathbb{P}[X = n] = \begin{cases} 0 & n > N \\ t^n(1-t)^{N-n}\binom{N}{n} & 0 \leq n \leq N \\ 1 & n < 0. \end{cases} \tag{A-4}$$

Let $\hat{B}$ represent the probability that a binomially distributed random variable X is greater than or equal to n,

$n = 0, 1, \ldots N$.[28] That is, define $\hat{B}$ as follows: For an integer n, $N \in \{0, 1, 2, 3, \ldots\}$, and $t \in [0, 1]$,

$$\hat{B}(n; N, t) = \mathbb{P}[X \geq n] := \begin{cases} 0 & n > N \\ \sum_{k=n}^{N} b(k; N, t) & 0 \leq n \leq N \\ 1 & n < 0, \end{cases} \tag{A-5}$$

The key result about the Binomial distribution that we will use in the sequel is presented below.

Fact A.1. *For integers, K, n such that $K \geq n \geq 1$ and $t \in (0, 1)$,*

$$\frac{d}{dt}\hat{B}(n; N, t) = N\, b(n-1; N-1, t).$$

A few general properties of o-strategies are presented in the following lemmas. We first consider non-capitulation strategies.

Lemma A.1. *The Π_o functions of non-capitulation strategies, $o \in \mathcal{O} \backslash \{K\}$, have the following properties:*

(a) Π_o*,* $o \in \mathcal{O} \backslash \{K\}$*, is strictly decreasing in* γ*.*

(b) If $\gamma = 0$ *or* 1*, then* $\Pi_{m-1} > \Pi_{m-2} > \ldots > \Pi_1 > \Pi_0$*.*

(c) For $o \leq m - 2$*,*

$$\Delta \Pi_o := \Pi_{o+1} - \Pi_o = \Delta w \left(y_{o+1} - b(m - o - 1; K - o - 1, \gamma)\right).$$

(d) For $o \leq m - 3$*,*

$$\Delta^2 \Pi_o := \Delta \Pi_{o+1} - \Delta \Pi_o = \\ \Delta w \big(y_{o+2} - y_{o+1}\big) + \Delta w \left((1-\gamma)^{K-m} \gamma^{m-o-2} \binom{K-o-1}{m-o-1} \left(\gamma - \frac{m-o-1}{K-o-1} \right) \right).$$

Proof. (a) When $\gamma = 0$, the green proposal will pass if and only if it is supported by at least m brown universal owners. Under all the $o \in \mathcal{O} \backslash \{K\}$ strategies, less than m owners vote yes. Hence, the proposal fails, i.e. $\hat{B} = 0$. Because, $\boldsymbol{y} > 0$, the result is apparent. When $\gamma = 1$, the proposal will pass regardless of the votes of the brown universal owners, so $\hat{B} = 1$, and the result is then apparent from inspecting the definitions.

[28] $\hat{B}$ is not equal to the survival function (i.e., complementary distribution function) of a Binomially distributed random variable. The survival function of an (N, p) binomial distribution represents $\mathbb{P}[X > n] = \mathbb{P}[X \geq n - 1]$. So if we used the survival function, the threshold for success would be $m - 1$, which might be confusing.

(b) Differentiation shows that

$$\frac{d}{d\gamma}\Pi_o = -\Delta w((1-\gamma)^{-2})\Big((K-o)(1-\gamma)b(m-o-1;K-o-1,\gamma)+\hat{B}(m-o-1;K-o-1,\gamma)\Big).$$

The terms in the parentheses are all positive for all $\gamma \in (0,1)$, so the right-hand side of the equation is negative.

(c) First note that the definition of the Π_o functions implies that

$$\Pi_{o+1}(\gamma,y)-\Pi_o(\gamma,y) = \Delta w\left(y_{o+1} - \left(\frac{\hat{B}(m-o-1;K-o-1,\gamma)-\hat{B}(m-o;K-o,\gamma)}{1-\gamma}\right)\right).$$

Noting that

$$\hat{B}(m-o,K-o,\gamma) = \hat{B}(m-o-1,K-o-1,\gamma)\,\gamma + \hat{B}(m-o,K-o-1,\gamma)(1-\gamma).$$

we see that

$$\frac{\hat{B}(m-o-1;K-o-1,\gamma)-\hat{B}(m-o;K-o,\gamma)}{1-\gamma} = \hat{B}(m-o-1;K-o-1,\gamma)-\hat{B}(m-o;K-o-1,\gamma) = b(m-o-1;K-o-1,\gamma),$$

and the result follows.

(d) Part (d). Using part (c) we see that

$$\Delta^2\Pi_o := \Delta\Pi_{o+1} - \Delta\Pi_o = \Big(y_{o+2}-y_{o+1}\Big) + \Big(b(m-(o+1)-1;K-(o+1)-1,\gamma) - b(m-o-1;K-o-1,\gamma)\Big).$$

Part (d) then follows by algebraic simplification and rearrangement of the second term in parentheses on the right hand side of the equation above.

□

Part (a) of Lemma A.1 simply shows that the potential is decreasing in green sentiment γ. This result is expected. The potential measures the effect of other brown owners' actions on each others' payoffs. As γ increases, this effect diminishes. Part (b) is more interesting. It shows that, relative to other non-capitulation strategies, the minimal resistance strategy, $o = m-1$, is attractive both when green sentiment, γ, is very high and very low. However, in these two cases, minimal resistance is attractive for different reasons. When $\gamma = 0$, brown owners are sure that the proposal will succeed only when at least m brown universal owners vote yes. Because of reputation costs, the potential is maximized over non-capitulation strategies by minimizing the number of brown owners who vote no subject to the constraint that the proposal fails. Thus, having the $m-1$ brown owners with the largest reputation costs vote yes and the remaining $K-m$ brown owners vote no, ensures the proposal will fail at minimum reputation cost. When $\gamma = 1$, the proposal will pass with certainty and so potential is only affected by reputation costs, because $m-1$-strategy features the most yes votes of any non-capitulation strategy, it is the optimal non-capitulation strategy. This result suggests that payoffs under the o-strategies will not satisfy in single-crossing property with respect to green sentiment, γ.

This suggestion is confirmed by part (c). Because, $o \leq m-1$, $m-o-1 \geq 1$, thus, the map $\gamma \to b(m-o-1; K-o-1, \gamma)$ is inverse-U shaped (↗↘). Part (c) of the lemma shows that the crossings of the potential under two adjacent o-strategies, o and $o+1$ as γ varies is determined by $b(m-o-1; K-o-1, \gamma)$. Hence, it implies that the potentials under the two strategies will either cross (i.e., transversally intersect) twice or not at all.

In contrast, as shown by part (d), second differences between adjacent o-strategies do have the single-crossing property with respect to γ. As we will show later, this single-crossing property permits determinant characterizations of effect of γ on the optimality of non-capitulation strategies.

For a non-capitulation o-strategy to be optimal, the value of the potential under o must at least equal the value of potential under the capitulation, Π_K. Thus, a non-capitulation strategy can only maximize the potential when $\Pi_o - \Pi_K \geq 0$. Some of the properties of the difference, $\Pi_o - \Pi_K$, are provided by the following Lemma.

Lemma A.2. *The differences between the non-capitulation strategies, $o \in \mathcal{O} \backslash \{K\}$, and the capitulation strategy, K, $\Pi_o - \Pi_K$, have the following properties:*

(a) When $\gamma = 0$, $\operatorname{sgn}[\Pi_o - \Pi_K] = \operatorname{sgn}[1 - \Sigma_{o+1}^K]$. When γ is sufficiently close to 1, $\Pi_K > \Pi_o$.

(b) If $o = m-1$, then $\Pi_o - \Pi_K$ is decreasing (↘) in γ.

(c) If $o < m-1$, then $\Pi_o - \Pi_K$ is inverse U-shaped (↗↘) in γ.

Proof. Part (a). First note that the definition of the Π_o functions (Equation (21)) shows that

$$\Pi_o - \Pi_K = \frac{1 - \hat{B}(m-o; K-o, \gamma)}{1-\gamma} - \Sigma_{o+1}^K. \tag{A-6}$$

When $\gamma = 0$, $1 - \hat{B}(m-o; K-o, \gamma)/(1-\gamma) = 1$ and application of LĤopital's rule shows that $\lim_{\gamma \to 1} 1 - \hat{B}(m-o; K-o, \gamma)/(1-\gamma) = 0$. Thus, the assertions in this part follow from inspection of equation (A-6) and the continuity of the Π_o functions.

Part (b). When $o = m-1$, $m-o = 1$. This observation and equation A-6 shows that

$$\Pi_{m-1} - \Pi_K = (1-\gamma)^{K-m} - \Sigma_m^K,$$

which is evidently strictly decreasing in γ.

Part (c). This is the only part of the lemma that is somewhat difficult to establish. We will used the general form of the what is frequently termed the monotone LĤopital rule.

Using equation (A-6) we can express $\Pi_o - \Pi_K$ for $o > m-1$ as follows:

$$\Pi_o - \Pi_K = \frac{1 - \hat{B}(m-o; K-o, \gamma) - (1-\gamma)\,\Sigma_{o+1}^K}{1-\gamma} := \frac{N(\gamma)}{D(\gamma)}. \tag{A-7}$$

Let $\rho = N'/D'$ and let $\tilde{\rho} = (D\rho - N)\,\mathrm{sgn}[D']$. Inspection shows that

$$\lim_{\gamma \to 1} N(\gamma) = 0 \text{ and } \lim_{\gamma \to 1} D(\gamma) = 0. \tag{A-8}$$

Equation A-7 and Fact A.1 show that

$$\rho(\gamma) = (K-o)\,b(m-o-1; K-o-1, \gamma) - \Sigma_{o+1}^K, \tag{A-9}$$

$$\tilde{\rho}(\gamma) = -\Big((1-\gamma)\,\big((K-o)\,b(m-o-1; K-o-1, \gamma) - \Sigma_{o+1}^K\big) - \big(1 - \hat{B}(m-o; K-o, \gamma) - (1-\gamma)\,\Sigma_{o+1}^K\big) \Big). \tag{A-10}$$

Because $0 \le o < m-1$, $0 < m-o-1 < K-o$, Because $m-o-1$ lies between 0 and $K-o$, the two extreme realizations of the Binomial$(\cdot; K-o, \gamma)$ distribution, the probability of $m-o-1$ first increases and then decreases in γ, i.e, $\gamma \to b(m-o-1; K-o-1, \gamma)$ is $\nearrow\searrow$ in γ; thus, inspecting equation (A-9) shows that

$$\rho \text{ is } \nearrow\searrow. \tag{A-11}$$

Because $o < m-1$, $b(m-o-1; K-o-1, \gamma) \to 0$ and (the probability the proposal fails) $1 - \hat{B}(m-o; K-o, \gamma) \to 1$ as $\gamma \to 0$. These observations applied to equation (A-10) show that

$$\lim_{\gamma \to 0} \tilde{\rho}(\gamma) = 1 > 0. \tag{A-12}$$

Proposition 4.4 in Pinelis (2002) shows that equations (A-8), (A-11), and (A-12) are sufficient N/D to be $\nearrow\searrow$. Because $N/D = \Pi_o - \Pi_K$ (see equation (A-7)), part (c) is established.

□

Part (a) of Lemma c establishes the fairly obvious result that, when green pressure is sufficiently severe, the potential is maximized by brown capitulation and that, even in the absence of green sentiment, non-capitulation is only optimal when the normalized reputation cost faced by the no-voting brown owners under the non-capitulation strategy, Σ_{o+1}^K, is less than 1, the normalized effect of the proposal passing on each brown owner's wealth.

Parts (b) and (c) show that with the exception of the $m-1$ strategy of minimal resistance, the advantage of each non-capitulation strategy over capitulation is not monotonically decreasing in green sentiment, γ. However, if $\Sigma_{o+1}^K < 1$, part (a) and the (↗↘) relationship between the advantage of non capitulation over capitulation, $\Pi_o - \Pi_K$, reported in part (c) show that $\Pi_o - \Pi_K$ crosses 0 from above as γ increases. So the region where non-capitulation is optimal is an interval with lower end point 0.

The next result uses Lemmas A.1 and Lemma A.2 to identify a simple necessary and sufficient condition for brown resistance to be a viable strategy at some level of green sentiment, γ. The proposition shows that if the sum of normalized resistance costs entailed by the minimum resistance strategy, Σ_m^K, at least equals 1, brown owners will always capitulate, if $\Sigma_m^K < 1$, brown owners will sometimes resist.

Lemma A.3.

(a) If $\Sigma_m^K \geq 1$, then $\Pi_o - \Pi_K < 0$, for all $\gamma \in (0,1)$. Hence, the unique potential maximizing strategy is capitulation, $o^* = K$.

(b) If $\Sigma_m^K < 1$, then for $\gamma > 0$ but sufficiently small, the unique potential maximizing strategy is the minimum resistance strategy, i.e., $o^* = m - 1$.

Proof of Lemma A.3

Proof of part (a). First consider the case where $o = m - 1$, When $\gamma = 0$, $\Pi_{m-1} - \Pi_K = 1 - \Sigma_m^K$ Because the normalized reputation costs are decreasing in the index, if $\Sigma_m^K > 1$ then at $\gamma = 0$, $\Pi_{m-1} - \Pi_K \leq 0$. Lemma A.2.(b) shows that, when $o = m - 1$, $\Pi_o - \Pi_K$ is strictly decreasing in γ. So, for all $\gamma \in (0,1)$, $\Pi_{m-1} - \Pi_K < 0$.

Now consider the more challenging case, $o < m - 1$. In this case, $\Pi_o - \Pi_K$ is ↗↘ in γ; so the fact that, at $\gamma = 0$, $\Pi_o - \Pi_K < 0$ does not imply that, for all $\gamma \in [0,1]$, $\Pi_o - \Pi_K < 0$.

We start by defining

$$\bar{y} := \frac{1}{K - m + 1}. \tag{A-13}$$

Because the normalized reputation costs are decreasing in the index, if $\Sigma_m^K \geq 1$ then $\Sigma_{o+1}^K \geq (K - o)\,\bar{y}$. Hence $\Pi_o - \Pi_K \leq g_o(\gamma)/(1-\gamma)$, where

$$g_o(\gamma) := (1 - \hat{B}(m - o; K - o, \gamma)) - (1 - \gamma)\,\bar{y}\,(K - o). \tag{A-14}$$

Thus, to establish the proposition for $o < m - 1$ we need only show that

$$\gamma \in (0,1) \text{ and } o \in \mathcal{O} \backslash \{K, m-1\} \Longrightarrow g_o(\gamma) < 0.$$

Using equation (A-14) we compute

$$g'_o(\gamma) = \bar{y} - (K-o)\,(1-\gamma)^{K-m}\,\gamma^{m-o-1}\binom{K-o-1}{m-o-1}, \tag{A-15}$$

$$g''_o(\gamma) = (K-o-1)\,(K-o)\,(1-\gamma)^{K-m-1}\,\gamma^{m-o-2}\binom{K-o-1}{m-o-1}(\gamma-\gamma_o), \text{ where} \tag{A-16}$$

$$\gamma_o := \frac{m-o-1}{K-o-1}. \tag{A-17}$$

Equations (A-14), (A-15), and (A-16) imply that

$$g_o(0) = 1-(K-o)\bar{y} < 0, \tag{A-18}$$

$$g'_o(0) = \bar{y}, \tag{A-19}$$

$$\operatorname{sgn}[g''_o(\gamma)] = \operatorname{sgn}[\gamma-\gamma_o]. \tag{A-20}$$

Equation (A-20) shows that g_o is concave on the interval $[0,\gamma_o]$ and is convex on the interval $[\gamma_o,1]$.

First, we show that $\max\{g_o(\gamma) : \gamma \in [0,\gamma_o]\} < 0$. To show this, note that when $\gamma \in [0,\gamma_o]$, g_o is concave (equation A-20) and thus bounded from above by its support lines and, in particular, by its support line at 0, i.e.,

$$g_o(\gamma) \leq g_o(0) + \gamma g'_o(0), \quad \gamma \in [0,\gamma_o].$$

Equation (A-19) shows that $g'_o(0) = \bar{y} > 0$. Hence,

$$g_o(\gamma) \leq g_o(0) + \gamma_o\, g'_o(0).$$

Substituting in the values of $\bar{y}$, γ_o, $g_o(0)$, and $g'_o(0)$, provided by equations (A-13), (A-17), (A-18), and (A-19), we see that

$$g_o(0) + \gamma_o\, g'_o(0) = -\frac{2K-(m+o+1)}{(K-m-1)\,(K-o-1)} < 0.$$

Thus, over the interval $[0,\gamma_o]$, $g_o < 0$.

Because g_o is strictly convex over $[\gamma_o,1]$, it attains its maximum over this interval only at the extreme points of this interval, 1 and γ_o. The definition of g_o (equation (A-14)) shows that $g_o(1) = 0$ and we have just shown that $g_o(\gamma_o) < 0$. Hence, $g_o(\gamma) < 0$ on $(\gamma_o,1)$. Combining the concave and convex cases, shows that $g_o \leq 0$ over $[0,1]$ and the result is established.

Proof of part (b). This result follows directly from Lemma A.1.(b), Lemma A.2.(a), and the continuity of the Π_o, $o \in \mathcal{O}$, functions.

□

A.5.2 Proof of Proposition 2

Proof. We consider parts (a) and (c) as the arguments supporting these parts of the lemma are interconnected.

Proof of parts (a) and (c). By definition $o \to \Pi_o$ is concave (convex) at o if $\Delta^2\Pi_{o-1} \leq (\geq) 0$ (see Remark 3).

Lemma A.1.(d) shows that

$$y_{o+1} - y_o = 0 \Longrightarrow \operatorname{sgn}[\Delta^2\Pi_{o-1}] = \operatorname{sgn}\left[\gamma - \frac{m-o}{K-o}\right]. \tag{A-21}$$

This establishes necessity condition in part (a) and the sufficiency condition in part (c).

Because of the decreasing arrangement of owners by reputation costs, it is always the case that $y_{o+1} - y_o \leq 0$.

Thus, we now need only consider the $y_{o+1} - y_o < 0$ case. Lemma A.1.(d) shows

$$\gamma - \frac{m-o}{K-o} < 0 \Longrightarrow \Delta^2\Pi_{o-1} < 0, \tag{A-22}$$

i.e., $o \to \Pi_o$ is strictly concave at o.

Again, because $y_{o+1} - y_o \leq 0$, Lemma A.1.(d) shows that, if i.e., $o \to \Pi_o$ is strictly convex at o, i.e.,

$$\Delta^2\Pi_{o-1} > 0 \Longrightarrow \gamma - \frac{m-o}{K-o} > 0. \tag{A-23}$$

Thus, equation (A-22) implies that if

$$\gamma < \min\left\{\frac{m-o}{K-o} : o \in \{1,2,\ldots,m-2\}\right\} = \frac{2}{K-m+2}.$$

then $o \to \Pi_o$ is strictly concave. Part (a) follows by noting (Remark 1) that $m = 1 + (K-1)/2$.

Equation (A-23) implies that if the map $o \to \Pi_o$ is strictly convex, it must be the case that

$$\gamma > \max\left\{\frac{m-o}{K-o} : o \in \{1,2,\ldots,m-2\}\right\} = \frac{m-1}{K-1}.$$

Part (c) follows by noting (Remark 1) that $m = 1 + (K-1)/2$.

Proof of part (b) This part follows directly from noting that the hypotheses of the this part of the lemma, and the fact that $o \to (m-o)/(K-o)$ is decreasing; these facts imply that the map $o \to \operatorname{sgn}[\Delta^2\Pi_{o-1}]$, $o \in \{1,2,\ldots,m-2\}$ is (weakly) increasing.

□

A.6 Proofs for Section 6

Proof of Lemma 3. Lemma A.1.(c) shows that $\boldsymbol{y}^2 \geq \boldsymbol{y}^1$ implies $\Delta\Pi_o(\boldsymbol{y}^2) \geq \Delta\Pi_o(\boldsymbol{y}^1)$, for all $o \in \{0, 1, \ldots, m-2\}$. If we compare any two non-capitulation strategies, o' and o'' such that $o' < o''$, we see that for an $\boldsymbol{y} \in [0,1]^K$

$$\Pi_{o''}(\boldsymbol{y}) = \Pi_{o'}(\boldsymbol{y}) + \sum_{o=o'}^{o''-1} \Delta\Pi_o(\boldsymbol{y}).$$

So, for any non-capitulation strategy

$$o'' > o' \Longrightarrow \Pi_{o''}(\boldsymbol{y}^2) - \Pi_{o'}(\boldsymbol{y}^2) \geq \Pi_{o''}(\boldsymbol{y}^1) - \Pi_{o'}(\boldsymbol{y}^1).$$

Similarly, if $o \in \mathcal{O}\backslash\{K\}$

$$\Pi_K(\boldsymbol{y}^2) - \Pi_o(\boldsymbol{y}^2) \geq \Pi_K(\boldsymbol{y}^1) - \Pi_o(\boldsymbol{y}^1).$$

So for all $o \in \mathcal{O}$, if $o'' > o'$

$$\Pi_{o''}(\boldsymbol{y}^1) - \Pi_{o'}(\boldsymbol{y}^1) \geq 0 \Longrightarrow \Pi_{o''}(\boldsymbol{y}^2) \geq \Pi_{o'}(\boldsymbol{y}^2).$$

□

Proof of Lemma 4. By hypothesis, $K \notin o^*(\boldsymbol{y}^1)$. Hence there exists some k such that $k \in \mathcal{O}\backslash\{K\}$ such that $\Pi_k(\boldsymbol{y}^1) > \Pi_K(\boldsymbol{y}^1)$. The definition of the potential and condition (b) of Definition 1 imply that $\Pi_K(\boldsymbol{y}^1) = \Pi_K(\boldsymbol{y}^2)$. Thus, $\Pi_k(\boldsymbol{y}^1) > \Pi_K(\boldsymbol{y}^2)$. Condition (c) of Definition 1 implies that $\Pi_k(\boldsymbol{y}^2) \geq \Pi_k(\boldsymbol{y}^1)$. Hence, $\Pi_k(\boldsymbol{y}^2) > \Pi_K(\boldsymbol{y}^2)$. Thus $\Pi^*(\boldsymbol{y}_2) > \Pi_K(\boldsymbol{y}^2)$ and, hence, $K \notin o^*(\boldsymbol{y}^2)$. Using condition (c) and same argument as developed in the proof of Lemma 3 establishes the result. □

Proof of Lemma 5. Part (a.i) of the hypothesis, Lemma A.1.(b) and the fact that the functions Π_o, are continuous in $\gamma \in (0,1)$ implies that for all γ in some neighborhood of 0 $o^*(\gamma) = m-1$. Now let $\bar{\gamma} := \sup\{\gamma' \in (0, \tilde{\gamma}) : \forall \gamma \in (0, \gamma'), o^*(\gamma) = m-1$.

Hypothesis, (b.i) implies that $\bar{\gamma} < \tilde{\gamma}$ and (b.ii) implies that $o^*(\gamma) \neq m-1$. Because $\gamma \to \Pi_o(\gamma)$ is a polynomial (and thus continuous) in γ and no two Π_o functions are identical, the set γ values at which any two of the $\gamma \to \Pi_o(\gamma)$ functions have the same values is discrete. Thus, there exists some interval $(\bar{\gamma}, \bar{\gamma}+\varepsilon)$, $\varepsilon > 0$, such that for all γ in this interval, $\Pi^*(\gamma) = \Pi_{\bar{o}}(\gamma)$, where $\bar{o} \neq m-1$ or K, i.e.,

$$\text{for all } \gamma \in (\bar{\gamma}, \bar{\gamma}+\varepsilon), \quad \begin{aligned} \Pi^*(\gamma) &= \Delta w \left(\Sigma_1^{o^*(\gamma)} - \frac{\hat{B}(m - o^*(\gamma); K - o^*(\gamma), \gamma)}{1-\gamma} \right) \\ &= \Delta w \left(\Sigma_1^{\bar{o}} - \frac{\hat{B}(m-\bar{o}; K-\bar{o}, \gamma)}{1-\gamma} \right) = \Pi_{\bar{o}}(\gamma). \end{aligned} \tag{A-24}$$

Similarly, the definition of $\bar{\gamma}$ there exists an interval $(\bar{\gamma}-\varepsilon,\bar{\gamma})$, such that $\Pi^*(\gamma)=\Pi_{m-1}(\bar{\gamma})$, i.e.

$$\text{for all } \gamma\in(\bar{\gamma}-\varepsilon,\bar{\gamma}),\quad \begin{aligned}\Pi^*(\gamma) &= \Delta w\left(\Sigma_1^{o^*(\gamma)} - \frac{\hat{B}(m-o^*(\gamma);K-o^*(\gamma),\gamma)}{1-\gamma}\right),\\ &= \Delta w\left(\Sigma_1^{m-1} - \frac{\hat{B}(1;K-(m-1),\gamma)}{1-\gamma}\right) = \Pi_{m-1}(\gamma).\end{aligned} \tag{A-25}$$

$$\begin{aligned}\lim_{\gamma\uparrow\bar{\gamma}}\Pi^*(\gamma) &= \Pi_{m-1}(\bar{\gamma}) = \Delta w\left(\Sigma_1^{m-1} - \lim_{\gamma\uparrow\bar{\gamma}}\frac{\hat{B}(1;K-(m-1),\gamma)}{1-\gamma}\right),\\ \lim_{\gamma\downarrow\bar{\gamma}}\Pi^*(\gamma) &= \Pi_{\bar{o}}(\bar{\gamma}) = \Delta w\left(\Sigma_1^{\bar{o}} - \lim_{\gamma\downarrow\bar{\gamma}}\frac{\hat{B}(m-\bar{o};K-\bar{o},\gamma)}{1-\gamma}\right).\end{aligned} \tag{A-26}$$

The continuity of Π^*, Π_{m-1} and $\Pi_{\bar{o}}$, in γ, and equation (A-26) imply that

$$\lim_{\gamma\uparrow\bar{\gamma}}\Pi^*(\gamma) = \lim_{\gamma\uparrow\bar{\gamma}}\Pi_{m-1}(\gamma) = \Pi_{m-1}(\bar{\gamma}) = \Pi_{\bar{o}}(\bar{\gamma}) = \lim_{\gamma\downarrow\bar{\gamma}}\Pi_{\bar{o}}(\gamma) = \lim_{\gamma\downarrow\bar{\gamma}}\Pi^*(\gamma). \tag{A-27}$$

Equations (A-26) and (A-27) imply that

$$\Sigma_1^{m-1} - \lim_{\gamma\uparrow\bar{\gamma}}\frac{\hat{B}(1;K-(m-1),\gamma)}{1-\gamma} = \Sigma_1^{\bar{o}} - \lim_{\gamma\downarrow\bar{\gamma}}\frac{\hat{B}(m-\bar{o};K-\bar{o},\gamma)}{1-\gamma}. \tag{A-28}$$

Because, $\bar{o}<m-1$, $\Sigma_1^{m-1}>\Sigma_1^{\bar{o}}$ equation (A-28) implies that

$$\lim_{\gamma\downarrow\bar{\gamma}}\hat{B}(m-\bar{o};K-\bar{o},\gamma) < \lim_{\gamma\uparrow\bar{\gamma}}\hat{B}(1;K-(m-1),\gamma). \tag{A-29}$$

Equations (A-24), (A-25), (A-27), and (A-29), imply that, at $\bar{\gamma}$, the probability that the green proposal passes, $\hat{B}(m-o^*(\gamma);K-o^*(\gamma),\gamma)$ jumps down. Hence the probability the probability that the green proposal passes is not monotonic is green sentiment, γ. □

Proof of Result 2. Given that the complete resistance, $o=0$, strategy is adopted at both m and $m+1$. The probability that the proposal passes at m is $\hat{B}(m,2m-1,\gamma)$; the probability that the proposal passes at $m+1$ is $\hat{B}(m+1,2m+1,\gamma)$. Simple algebra shows that

$$\hat{B}(m+1,2m+1,\gamma)-\hat{B}(m,2m-1,\gamma) = (2\gamma-1)\binom{2m-1}{m}\gamma^m(1-\gamma)^m.$$

□

Proof of Result 3. As shown in Proposition 2.c, when $m>2$, the map $o\to\Pi_o$ is convex for $o<K$. So the only candidate optimal resistance strategies are the two extreme strategies, $o=0$ or $o=m-1$. When $m=2$, there are only two resistance strategies, $o=m-1=1$ and $o=0$. So to show that the optimal resistance strategy is $o=0$ we need only show that $o=m-1$ is not optimal. To show this note that if $o=m-1$ is optimal than this strategy

must produce a value for the potential function at least as high as the value produced by $o = 0$ and $o = m-2$. Thus it must be the case that

$$\Pi^m_{m-1} \geq \Pi^m_K, \tag{A-30}$$

$$\Pi_{m-1} - \Pi_{m-2} = \Delta\Pi_{m-2} > 0. \tag{A-31}$$

The definition of the potential function and Lemma A.1.enum:DeltaPiOSgn show that these two conditions will only be satisfied when

$$m\,y - (1-\gamma)^{m-1} \leq 0, \tag{A-32}$$

$$y - b(1;m,\gamma) \geq 0. \tag{A-33}$$

Noting that $b(1;m,\gamma) = m\gamma(1-\gamma)^{m-1}$, we see that, because $m \geq 2$ there exists no $\gamma > \frac{1}{2}$ that can satisfy both equation (A-32) and (A-33). Thus the if resistance is optimal, the complete resistance strategy, $o = 0$, is the optimal resistance strategy. □

Proof of Lemma 4. This result is a direct consequence of Result 2 and Result 3 and the following result.

Result 6. If $\gamma > \frac{1}{2}$, and capitulation, ($o = 2m-1$) is optimal at m, then capitulation is strictly optimal at $m+1$

Proof of Result 6. For the sake of readability define, for this proof only, the probabilities of the proposal passing when $o = 0$, given passing threshold m and thus $2m-1$ universal owners.

$$p^m := \hat{B}(m, 2m-1, \gamma).$$

By hypothesis, $\Pi^m_K \geq \Pi^m_0$ We need to show that this hypothesis implies that $\Pi^{m+1}_K \geq \Pi^{m+1}_0$. To see this note that

$$\Pi^m_K \geq \Pi^m_0 \iff \left((2m-1)\,y - \frac{1}{1-\gamma}\right) - \left(0\,y - \frac{p^m}{1-\gamma}\right) \geq 0, \tag{A-34}$$

$$\Pi^{m+1}_K \geq \Pi^{m+1}_0 \iff \left((2m+1)\,y - \frac{1}{1-\gamma}\right) - \left(0\,y - \frac{p^{m+1}}{1-\gamma}\right) \geq 0, \tag{A-35}$$

Result 2 shows that, when $\gamma > \frac{1}{2}$, $p^{m+1} > p^m$ and $2m+1 > 2m-1$. Hence, we see that satisfaction of (A-34) implies the satisfaction of (A-35). □

Result 3 shows that, at m, the potential maximizing o-strategy is either complete resistance,$o = 0$, or capitulation, $o = K$. If capitulation is optimal, then Result 6 shows that the potential maximizing strategy at $o = m+1$ is also capitulation. So, at both m and $m+1$, the probability the proposal passes equals one.

If complete resistance maximizes the potential at m, then, Result 3 shows that, at $m+1$, the potential maximizing strategy is either capitulation or complete resistance. Clearly if the potential maximizing strategy is capitulation, the probability of proposal success is higher at $m+1$. If at $m+1$ the potential maximizing strategy is also complete resistance, $o = 0$, then Result 2 shows that the probability of success is higher at $m+1$. □

Proof of Result 5. Most of this proof is supplied by earlier results. To prove (i) note that the Result 2 shows that, when $\gamma > \frac{1}{2}$, increasing ownership dispersion increases the probability of passage under the complete resistance, $o = 0$ strategy. when there is one universal owner capitulation is never optimal (by our assumption that $R < \Delta W$) and resistance is always complete resistance. Thus, if the potential maximizing strategy under dispersed ownership is complete resistance, the monetary payoff of the universal owners is higher under unified ownership. If, under dispersed ownership, the potential maximizing o-strategy is capitulation, the monetary payoff equals $W(F) - \Delta W$. Under unified ownership, the single owner resists and thus the monetary payoffs equals $W(F) - \Delta W + (1-\gamma)(\Delta W - R)$. Hence, the monetary payoff is larger under unified ownership. Result 3 shows that, when $\gamma > \frac{1}{2}$, the only candidate optimal o-strategies are complete resistance or capitulation.

To prove (ii.a) note that, as shown by Lemma A.1, for γ sufficiently small, $o^* = m-1$ and the probability of the proposal passing is thus, $1-(1-\gamma)^m$. Thus, under dispersed ownership the monetary payoff equals

$$W(F) - (1-\gamma)R + \left((1-\gamma)\left(\frac{m-1}{2m-1}\right)R - \Delta W\left(1-(1-\gamma)^m\right)\right).$$

Under unified ownership, the monetary payoff equals

$$W(F) - (1-\gamma)R - \Delta W\,\gamma.$$

So, we see that, for γ sufficiently small, the monetary payoff is larger under dispersed ownership.

To prove (ii.b), Note that because under unified ownership complete resistance is the optimal strategy, if complete resistance is also the potential maximizing strategy under dispersed ownership, expected reputation costs are identical under unified and dispersed ownership. Because, by assumption, $\gamma < \frac{1}{2}$, Result 2 shows that the probability of success is less under dispersed ownership. Thus, the monetary payoff is larger under dispersed ownership. □

B Mixed Strategies

Mixed strategy vectors that maximize the potential rarely exist and strategy vectors where two or more brown owners randomize are extremely rare and are only possible when the exogenous "greenness" parameter, γ, takes one of its $m-1$ possible values on the unit interval continuum. The intuition for the Lemma is illustrated by Figure B-1. The example is symmetric strategy vector, $\boldsymbol{\sigma}^*$, in a symmetric parametrization of the game, i.e., $y_i = y$ for all $i \in \mathcal{K}$. In this equilibrium, all brown universal owners vote yes with probability $\sigma_i^* = \sigma^* = 3/25$. The graph plots the value of the potential function (on the z-axis) when σ_1 and σ_2 are allowed to vary (on the x and y-axes), holding the other brown universal owners strategies at their equilibrium values. The fact that σ^* is a best response for brown universal owners 1 and 2 is verified by the fact that moving along the red (blue) line, which leaves the strategy of the other brown owner fixed, does not increase the potential function. By definition, the derivative of the potential function with respect to σ_i equals the derivative of a brown owner's payoff with respect to σ_i. So, neither i nor j can gain by unilaterally deviating from $\boldsymbol{\sigma}^*$. Because the game is symmetric, unilateral deviations will also not increase the other brown owners' payoffs. Hence, $\boldsymbol{\sigma}^*$ is a Nash equilibrium. However, moving along the black line, i.e., in the a direction that increases (reduces) σ_1 and, at the same time reduces (increases) σ_2 by an equal amount, increases the potential function. This symmetric Nash equilibrium is thus a saddle point of the potential function.

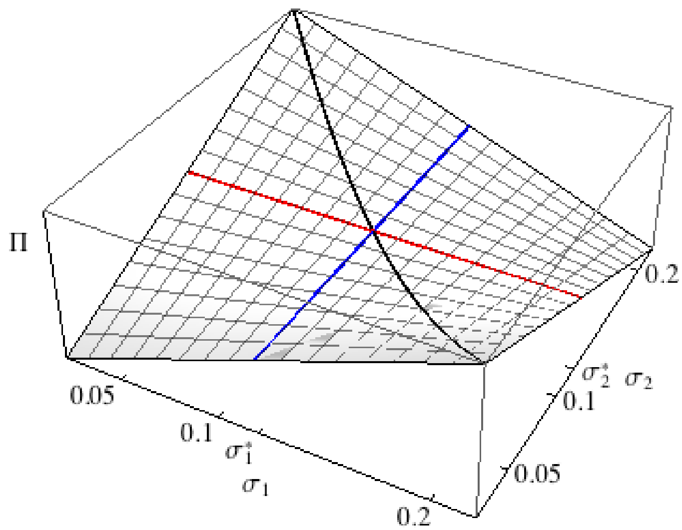

Figure B-1: Mixed strategy Nash equilibria are not potential maximizing. The figure presents the value of the potential function when brown universal owners 3, 4, ... K's strategies are fixed at $\sigma_i = 3/25$ and brown universal owners 1 and 2's strategies, σ_1 and σ_2, are allowed to vary around $3/25$. The parameters of game are $\gamma = 1/11$, $K = 5$, $m = 3$, and $y_i = 96/625$ for all $i \in \mathcal{K}$.

What is the intuition behind the example? First note that the probability of any given universal owner voting yes equals $t(\sigma^*) = \gamma + (1-\gamma)\,\sigma^* = 1/11 + (10/11) \times (3/25) = 1/5$. So, the expected number of yes votes equals $K\,t(\sigma^*) = 1$. Despite the expected number of yes votes being small relative to the passing threshold, $m = 3$, because of brown owner randomization, there is an appreciable probability the proposal will pass. The probability that the proposal passes can be reduced by reducing the dispersion of the yes-vote distribution, by increasing one brown universal owner's probability of voting yes and reducing another brown universal owner's probability of voting yes by an equal amount. Thus shift does not affect, $y \sum_k \sigma_k$, and thus will increase the potential, Π.

Lemma B.1. *A pure strategy vector (i.e. $\sigma_i \in \{0,1\}$) that maximizes the potential function, Π, always exists. Let $\bar{\boldsymbol{\sigma}}$ be a strategy vector that maximizes the potential, Π. Let $\mathcal{R}$ be the set of brown universal owners who randomize, i.e., $\mathcal{R} := \{i \in \mathcal{K} : \bar{\sigma}_i \neq 0 \text{ or } 1\}$. Then, for all $i, j \in \mathcal{R}$, $i \neq j$, $y_i = y_j$ and, if the number of randomizing universal owners, $\#\mathcal{R}$, is greater than one, then*

$$\gamma \in \left\{ \frac{m-1-j}{K-1-j} : j = 0, 1 \ldots m-2 \right\}.$$

The set of $\gamma \in [0,1]$ and $\mathbf{y} \in [0,1]^K$ such that $\boldsymbol{\sigma}$ is a potential maximizer and any universal owner plays a mixed strategy has measure 0.

Proof of Lemma B.1 This proof is established through the following Lemmas.

Lemma B.2. *The exists a pure strategy (i. e., for all $i \in \mathcal{K}$, $\sigma_i \in \{0,1\}$) maximizer of the potential.*

Proof. First note that the domain of Π is the compact set $[0,1]^K$ and Π is continuous, so a maximizer, perhaps mixed, exists. Next note the Π is multilinear so suppose that $\bar{\boldsymbol{\sigma}}$ maximizes Π and $\bar{\sigma}_i \in (0,1)$. Define the function $v : [0,1] \to \mathbb{R}$ by $v(\sigma_i) = \Pi(\sigma^i | \bar{\boldsymbol{\sigma}}^{-i})$. Since $\bar{\boldsymbol{\sigma}}$ maximizes Π, $\bar{\sigma}_i$ maximizes v. Because, Π is affine, this implies that $\sigma_i = 0$ and $\sigma_i = 1$ also maximize v. Hence, $(0|\bar{\boldsymbol{\sigma}}^{-i})$ and $(1|\bar{\boldsymbol{\sigma}}^{-i})$ also maximize Π. If, in fact $\bar{\boldsymbol{\sigma}}$ maximizes Π, we can continue in like fashion, replace all mixed components in $\bar{\boldsymbol{\sigma}}$ with 0 and 1 without affecting the value of Π. □

Lemma B.3. *If strategy vector $\boldsymbol{\sigma}$ in which at least two brown universal owners randomize, is a potential maximizer then $y_i = y_j$ for all i, j such that $i, j \notin \{0,1\}$.*

Proof. Consider a vector $\bar{\boldsymbol{\sigma}}$ in which at least two brown universal owners, say i and j randomize, i.e., $\sigma_i \in (0,1)$ and $\sigma_j \in (0,1)$. If, in fact $\bar{\boldsymbol{\sigma}}$ maximizes the potential function, then

$$\text{Max}\{\Pi(\sigma_i, \sigma_j | \bar{\boldsymbol{\sigma}}^{-ij}) : (\boldsymbol{\sigma}^i, \sigma_j) \in [0,1]^2\} = \Pi(\bar{\boldsymbol{\sigma}}).$$

First note that

$$\begin{aligned}\mathbb{P}[S(\tau(\boldsymbol{\sigma})) \geq m] = \mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) \geq m] + \\ \Big(t(\sigma_i) + t(\sigma_j) - t(\sigma_i)t(\sigma_j)\Big)\, \mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) = m-1] + t(\sigma_i)t(\sigma_j))\, \mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) = m-2], \quad \text{(B-1)}\end{aligned}$$

and that the distribution of S^{-ij} is not affected by σ_i or σ_j. So, to reduce our notational burden somewhat define.

$$\bar{s}_0 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) \geq m], \quad \bar{e}_1 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-1], \quad \bar{e}_2 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-2], \tag{B-2}$$

and define the function, $\pi : [0,1]^2 \to \mathbb{R}$ by

$$\psi(\boldsymbol{\sigma}_{ij}) := \Pi(\sigma_i, \sigma_j | \bar{\boldsymbol{\sigma}}^{-ij}), \text{ where } \boldsymbol{\sigma}_{ij} := (\sigma_1, \sigma_2)$$

Not that equation (B-1) and the definition of the potential function show that ψ can be expressed as follows:

$$\psi(\boldsymbol{\sigma}_{ij}) = \sum_{K \setminus \{i,j\}} \bar{\sigma}_k y_k + \sigma_i y_i + \sigma_j y_j - \frac{\bar{s}_0 + \Big(t(\sigma_i) + t(\sigma_j) - t(\sigma_i) t(\sigma_j)\Big) \bar{e}_1 + t(\sigma_i) t(\sigma_j) \bar{e}_2}{1-\gamma} \tag{B-3}$$

A straightforward computation, shows that the second derivative (i.e., the Hessian of π), $D^2\pi$, is given by

$$D^2 \psi(\boldsymbol{\sigma}_{ij}) = -(1-\gamma) \Delta w H, \text{ where } H = \begin{pmatrix} 0 & \bar{e}_2 - \bar{e}_1 \\ \bar{e}_2 - \bar{e}_1 & 0 \end{pmatrix}. \tag{B-4}$$

Because the first derivative of ψ (i.e. the gradient) must vanish by the first-order condition and all derivative forms higher than two vanish because D^2 is constant, the multivariate version of Taylor's Theorem shows that

$$\psi(\boldsymbol{\sigma}_{ij}) = \pi(\bar{\boldsymbol{\sigma}}_{ij}) - \frac{1}{2}(1-\gamma)\Delta w \Big(\boldsymbol{\sigma}_{ij} - \bar{\boldsymbol{\sigma}}_{ij})^T H (\boldsymbol{\sigma}_{ij} - \bar{\boldsymbol{\sigma}}_{ij}) \Big).$$

First, consider the case where $\bar{e}_2 - \bar{e}_1 \neq 0$. In this case, we see that H has two non-zero eigenvalues with opposite signs, $\bar{e}_2 - \bar{e}_1$ and $-(\bar{e}_2 - \bar{e}_1)$. Thus, inspecting equation (B-4), shows that $\mathcal{H}$ is not positive semi-definite and $(\bar{\sigma}_i, \bar{\sigma}_j)$ cannot maximize π and hence $\bar{\boldsymbol{\sigma}}$ cannot maximize the potential, Π. In fact, the eigenvectors of H are $(1,1)$ and $(1,-1)$ and thus in this case, $\bar{\boldsymbol{\sigma}}$ is a saddle point, and not a local maximizer of π.

No suppose that $\bar{e}_2 - \bar{e}_1 = 0$. If $\bar{e}_2 - \bar{e}_1 = 0$ then equation (B-3) reduces to a linear function of $\boldsymbol{\sigma}_{ij}$, i.e.,

$$\psi(\boldsymbol{\sigma}_{ij}) = \sigma_i y_i + \sigma_j y_j + \sum_{K \setminus \{i,j\}} \bar{\sigma}_k y_k - \frac{\bar{s}_0 + \Big(t(\sigma_i) + t(\sigma_j)\Big) \bar{e}_1}{1-\gamma}$$

So, $\bar{\boldsymbol{\sigma}}_{ij}$ maximizes ψ if and only if $(y_i - \bar{e}_1, y_j - \bar{e}_1) = (0,0)$. In which case all $\boldsymbol{\sigma}_{ij} \in [0,1]^2$ also maximize ψ. Thus if $\bar{\sigma}$ maximizes Π then all vectors of the form$(\boldsymbol{\sigma}_{ij} | \bar{\boldsymbol{\sigma}}^{-ij})$ also maximize the potential. Moreover, $y_i = y_j = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-1] = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-2]$. □

Lemma B.4. *If the strategy vector, $\bar{\boldsymbol{\sigma}}$ contains two or more mixed components, say $\bar{\sigma}_i \in (0,1)$ and $\bar{\sigma}_j \in (0,1)$, then σ can only be a potential maximizer when $\gamma = (m-1-j)/(K-1-j)$, where $j \in \{0,1,2,\dots m-2\}$.*

Proof. Let

$$\mathcal{O}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i = 1\}, \quad \mathcal{R}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i \in (0,1)\}, \text{ and } \mathcal{Z}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i = 0\}.$$

By assumption, $\#\mathcal{R} \geq 2$; so select two members of this set, which without loss of generality, we assume includes $i = 1,2$. Define three new strategy vectors, $\boldsymbol{\sigma}^\ell$, $\ell = 0,1,2$ as follows:

$$\sigma_i^0 = \begin{cases} 0 & 0 \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup \mathcal{R}(\bar{\boldsymbol{\sigma}}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \end{cases}, \sigma_i^1 = \begin{cases} 0 & i \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup (\mathcal{R}(\bar{\boldsymbol{\sigma}}) \backslash \{1\}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \cup \{1\} \end{cases}, \sigma_i^2 = \begin{cases} 0 & i \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup (\mathcal{R}(\bar{\boldsymbol{\sigma}}) \backslash \{1,2\}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \cup \{1,2\} \end{cases}$$

Next note that, by hypothesis $\bar{\boldsymbol{\sigma}}$ maximizes the potential and is not a pure strategy vector, implies that $\#\mathcal{O}(\bar{\boldsymbol{\sigma}}) \leq m-1$. Otherwise the proposal would pass with certainty and, in this case, the unique optimal strategy is for all brown owners to vote yes, $\sigma_i = 1$ for all $i \in \mathcal{K}$, and this strategy vector is pure. Next note that Π being multilinear and the hypotheses that $\bar{\boldsymbol{\sigma}}$ is a potential maximizer implies that

$$\Pi(\bar{\boldsymbol{\sigma}}) = \max_{\boldsymbol{\sigma} \in [0,1]^K} \Pi(\boldsymbol{\sigma}), \quad \text{and} \quad \Pi(\bar{\boldsymbol{\sigma}}) = \Pi(\boldsymbol{\sigma}^0) = \Pi(\boldsymbol{\sigma}^1) = \Pi(\boldsymbol{\sigma}^2).$$

Note that $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) < K$, for $\ell = 0,1,2$. Thus follows because, as argued above $\#\mathcal{O}(\bar{\boldsymbol{\sigma}}) \leq m-1$, and the cardinality of $\mathcal{O}(\boldsymbol{\sigma}^\ell)$ exceeds the cardinality of $\mathcal{O}(\bar{\boldsymbol{\sigma}})$ by at most two. So, $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) \leq m-1+2 = m+1$. By model assumptions, $m+1$ is less than K. Thus, if it were the case that $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) > m-1$ then $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) \in \{m, m+1, \ldots, K-1\}$. In which case σ^ℓ would not maximize the potential because the proposal would be passing with certainty and, for some i, $\sigma^i \neq 1$. But this contradicts equation B.

Note that the $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^0$ differ only in their strategy assignment to brown owner 1: under $\boldsymbol{\sigma}^0$, brown owner 1 votes against the proposal, $\sigma_1^0 = 0$ and, under σ^1, brown owner 1 votes for the proposal, $\sigma_1^1 = 1$. Note also that because both 1 and 2 randomize under $\bar{\boldsymbol{\sigma}}$, Lemma B.3 implies that $y_1 = y_2$. Let y_o represent their common value, and let $\bar{o} = \#\mathcal{O}(\boldsymbol{\sigma}^0)$ Inspection of the definition of the potential function shows that

$$\Pi(\boldsymbol{\sigma}^1) - \Pi(\boldsymbol{\sigma}^0) = 0 \Longleftrightarrow y_o - \left(\frac{\hat{B}(m-(\bar{o}+1); K-(\bar{o}+1), \gamma)}{1-\gamma} - \frac{\hat{B}(m-\bar{o}; K-\bar{o}, \gamma)}{1-\gamma} \right) = 0. \tag{B-5}$$

The argument used in the proof of part (c) of Lemma A.1, shows that

$$\frac{\hat{B}(m-\bar{o}; K-(\bar{o}+1), \gamma)}{1-\gamma} - \frac{\hat{B}(m-\bar{o}; K-\bar{o}, \gamma)}{1-\gamma} = b(m-(\bar{o}+1); K-(\bar{o}+1), \gamma) \tag{B-6}$$

From equations (B-5) and (B-6) we see that

$$\Pi(\boldsymbol{\sigma}^1) - \Pi(\boldsymbol{\sigma}^0) = 0 \Longleftrightarrow y_o = b(m-(\bar{o}+1); K-(\bar{o}+1), \gamma). \tag{B-7}$$

An identical argument shows that

$$\Pi(\boldsymbol{\sigma}^2) - \Pi(\boldsymbol{\sigma}^1) = 0 \Longleftrightarrow y_o = b(m-(\bar{o}+2); K-(\bar{o}+2), \gamma). \tag{B-8}$$

Hence,

$$b(m-(\bar{o}+1); K-(\bar{o}+1), \gamma) = b(m-(\bar{o}+2); K-(\bar{o}+2), \gamma)$$

Algebraic simplification shows that

$$b(m-(\bar{o}+1); K-(\bar{o}+1), \gamma) = b(m-(\bar{o}+2); K-(\bar{o}+2), \gamma) \Longleftrightarrow \gamma = \frac{m-1-\bar{o}}{K-1-\bar{o}}.$$

Thus, if at least two shareholders play mixed strategies, it must be the case that γ satisfies

$$\gamma = \frac{m-1-\bar{o}}{K-1-\bar{o}}, \text{ where } \bar{o} \in \{0, 1, \ldots m-2\}.$$

□

Lemma B.5. *The set of $\gamma \in [0,1]$ and $\boldsymbol{y} \in [0,1]^K$ such that $\boldsymbol{\sigma}$ is a potential maximizer and any universal owner plays a mixed strategy has measure 0.*

Proof. We have seen (Lemma B.4) that the set of γ that support an equilibrium in which two brown owners randomize is finite and thus clearly measure 0 in $[0,1] \times [0,1]^K$. Now consider the set of $(\gamma, \boldsymbol{y}) \in [0,1] \times [0,1]^K$ such that one universal owner randomizes. For a strategy vector featuring randomization by one owner, say owner j, to maximize the potential when other owner's play pure strategies, it must be the case that at least two pure strategies for the other owners, say $\boldsymbol{\sigma}_1^{-j}$ and $\boldsymbol{\sigma}_2^{-j}$ produce the same payoff for all $\sigma_j \in [0,1]$. For any fixed $\boldsymbol{y}$, these pure strategies (i.e. $\sigma_i \in \{0,1\}$) are polynomials in γ. Because they are polynomials, the polynomial that represents their difference has only a finite number of zeros, and thus, for any fixed $\boldsymbol{y}$, the measure of $\gamma \in [0,1]$ such that the two strategies have the same payoff equals 0. Using Fubini's Theorem to integrate these zero measure sets over $[0,1]^K$, shows that the measure of the set $(\gamma, \boldsymbol{y}) \in [0,1] \times [0,1]^K$ such that a potential maximizer features one owner randomizing has measure 0. □

Lemmas B, B.3, B.4, and B.5 establish Lemma B.1. □

C Majorization and Optimal Resistance Strategies

In this section, we provide numerical counterexamples of majorization and optimal voting strategies. These examples are able to show that more dispersed vector of reputation costs can either lead to higher values of o^* or lower values of o^*. Correspondingly, the level of dispersion of reputation costs can be either positively or negatively correlated with the success probability of the proposal.

As a benchmark, assume the total number of universal owners $K = 7$. In this case, the threshold is $m = 4$. When normalized reputation costs are symmetric and denoted by $\bar{\boldsymbol{y}} = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$, we can find a feasible level of γ (relatively small, e.g., can be $\gamma = 0.1$), at which the potential maximizing strategy is $\bar{o}^* = 2$. In this benchmark case, the pass probability is $\bar{\rho} = 8.15\%$.

Example 1. Consider the more dispersed normalized reputation vector produced by transferring all reputation costs to owner one, i.e., $\boldsymbol{y}_1 = (0.7, 0, 0, 0, 0, 0, 0)$. Then

(i) $\boldsymbol{y}_1$ majorizes $\bar{\boldsymbol{y}}$,

(ii) the potential maximizing solution features $1 = o_1^* < \bar{o}^*$, and

(iii) the pass probability features $1.59\% = \rho_1 < \bar{\rho}$.

Example 2. Consider the more dispersed normalized reputation vector produced by transferring all reputation costs more or less uniformly to owners one, two, and three, i.e., $\boldsymbol{y}_2 = (0.3, 0.2, 0.2, 0, 0, 0, 0)$. Then

(i) $\boldsymbol{y}_2$ majorizes $\bar{\boldsymbol{y}}$,

(ii) the potential maximizing solution features $3 = o_2^* > \bar{o}^*$, and

(iii) the pass probability features $34.39\% = \rho_2 > \bar{\rho}$.