Information Acquisition and Response in Peer-effects Networks

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Abstract

Recently the network games literature turned its attention to strategic interactions under incomplete and asymmetric information. The canonical assumption takes players' signal qualities to be exogenous. This paper allows players to privately acquire costly payoff-relevant information prior to simultaneous play. The presence of peer effects, captured by the economy's network structure, implies that information externalities assume a rich and nuanced form. When pairwise peer effects are symmetric, asymmetries in acquired information are inefficiently low relative to the utilitarian benchmark. And with information privately acquired, all players face strictly positive gains to overstating their informativeness as to strategically influence the beliefs and behaviors of neighbors. If strategic substitutes in actions are present and significant, low centrality players move against their signals in anticipation of their neighbors' actions. A blueprint for optimal policy design is developed. Applications to market efficiency in financial crises and two-sided markets are discussed.

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1 Introduction

Many environments involve individuals acquiring and using information toward both learning more about the world and inferring the information of others. This ubiquitous dual role of information plays out in financial markets, labor markets, social networks and trends, as well as in professional communities. Relevant to each of these examples, individuals commonly face asymmetric incentives to invest in costly information depending on their identity, market position, and social ties. And when information is used to infer the observations of influential players, the strategic response to signals establishes a crucial component to the private value of information. This paper studies the role of such peer effects in shaping the incentives to acquire and strategically respond to information. It examines both the positive and normative implications of the resulting disparities in acquired information qualities.

An example embodying this duality while in the presence of directed peer effects is given with the following vignette. At some point in time, an independent research institute develops and patents a novel drill technology. The new drill potentially means that a large, previously untapped field of deep-sea oil deposits can now be safely resourced. The institute advertises the promise of the drill, and is willing to lease out the rights to operate the technology on a per-drill basis. A three-player market consists of two petroleum firms (firms A and B) comprising a competitive duopoly and a lobbyist for the petroleum industry. The three players pursue their own due diligence as to confirm or refute the drill's value. We can capture the resulting network of relationships with the following figure.

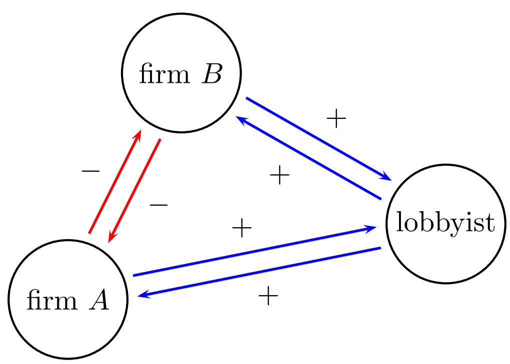

Figure 1: An oil industry and political lobbyist network

The sign and direction of links emanating from each individual capture the competitive and supportive influences that others' investment choices have on their private incentive to adopt the drill.

Peer effects in technology adoption feed into the incentives to acquire and respond to private information in the following ways. With both firms simultaneously researching the technology,[1] any acquired information regarding the drill's value brings with it the knowledge of greater competition. That is, a firm that learns the drill is effective also learns that they are likely to face stiff competition

[1] Hendricks and Porter (1996) [37] provide evidence of non-cooperative exploration in these industries.

when drilling. This is precisely because information regarding the drill's efficacy can also be used to infer the competition's observations and subsequent investment in the drill. The lobbyist, on the other hand, will decide whether to utilize her resources promoting subsidies toward the employment of the drill technology or focus her efforts elsewhere. Choosing the optimal agenda to pursue requires her own due diligence. And as a function of the connections that she has with firms A and B, her incentives to acquire information will depend on how informed she can expect the firms will be. This is because upon learning of the drill's value, the more she subsequently promotes the drill the more she will need the firms to follow suit and utilize the technology. For the firms, the more efficient the drill appears the more likely they can expect subsidization in the near future – *if* the lobbyist is also expected to do her research.

Crucially, the clarity in any individual's inference of others' observations depends on the equilibrium extent of research undergone by the others in the market. Those who learn of the technology's value also learn that other highly informed individuals observe its value. Put succinctly, the collective incentives of firms A, B and the lobbyist to acquire information intricately depend on each individual's expectation of the information acquisitions, observations and subsequent actions of the others. The in-equilibrium incentives to invest in information will ultimately depend on the strategic interdependencies that each player's market position entails. Those in highly competitive positions in the market (e.g. competitive firms) will, ceteris paribus, face less value to information than those in supported or complimented market roles (e.g. lobbyists and experts).

With weighted, directed, and signed peer effects pushing and pulling equilibrium incentives, what are the welfare implications of equilibrium information acquisition? Precisely, who over invests and who under invests in information relative to the utilitarian benchmark? And, do players carry incentives to distort other's beliefs regarding their acquired information qualities? While a rich literature studying coordination games with endogenous information[2] broadly focusing on symmetric beauty-contests has offered a number of results relevant to these questions,[3] the following network setup offers a novel platform toward assessing inefficiencies in more diverse economies.

First, the essential structural property that drives the direction of inefficiencies is the extent of symmetry in pairwise relationships. Symmetric networks, in which pairwise peer effects are identical, provide generalizations to many features obtained in the coordination games with endogenous information literature. For example, in symmetric beauty contests under/over acquisition of information in equilibrium has been shown to accompany strategic complements/substitutes in the second stage. In symmetric *networks*, a more general *bunching* in acquired information qualities obtains. For example, those facing a majority of strategic complements acquire the most informa-

[2] This literature is commonly referred to as "*global games with endogenous information*".

[3] To list a few examples, Morris and Shin (2002) [47] and later Myatt and Wallace (2009) [49] illustrate how strategic effects in actions can influence information choice. Vives (1988) [61], (2008) [62] and Hellwig and Veldkamp (2009) [34] show how strategic complements (substitutes) can directly spill into complements (substitutes) in information acquisition, and in turn derive inefficient under (over) acquisition in equilibrium. And, Colombo et al. (2014) [19] provide an encompassing analysis of the inefficiencies that arise from the strategic use of private and public information, casting equilibrium play against both the efficient acquisition *and* efficient use of information. Section 5.3 further discusses relation to this literature.

tion in equilibrium but also *under* acquire relative to the utilitarian benchmark. Those facing a majority of strategic substitutes acquire the least but *over* acquire. Departing from these results, the direction of inefficiencies *reverse* upon introducing sufficient anti-symmetry in pairwise relationships. Precisely, when pairwise peer effects exhibit opposing signs, acquired information exhibits inefficient *spreading* in equilibrium.

A second novelty unique to network settings is the introduction of players strategically moving against their signals. Under sufficient network irregularity and for players occupying adverse positions in the network (i.e. facing significant strategic substitutes), the endogenous choice to invest in costly information and strategically move against signal realizations arises. Put crudely, players may *short the network*. Inefficiencies naturally arise with this behavior, with the direction of these inefficiencies continuing to be driven by pairwise symmetry. In symmetric networks, the equilibrium extent to which these players acquire and move against their signals is inefficiently *low*. Precisely, the rationality in this equilibrium behavior is valued by the very neighbors that invoke it. And consistent with the preceding message, this value reverses when peer effects are anti-symmetric. That is, those moving against their signals impose a net cost on those they influence.

An important question arises when considering such environments comprised of a finite number of strategically informed players. What would happen if players could influence others' beliefs? With signal qualities privately acquired, players face a marginal cost due to their inability to directly influence others' perceptions of their expertise. In reality, firms in an array of industries are commonly observed marketing the qualities of their research departments. Lobbyists are found promoting the extent of their expertise in their given industry or interest. While such marketing may serve a number of goals, this paper taps into a common impetus for this behavior, found within equilibrium information acquisition and response. Once again, the strength and direction of this force ultimately depends on the network's extent of symmetry among pairwise relationships.

Elaborating on this, our three-player petroleum market is seen to display symmetry in each pair's peer effects. In this environment, the marginal value derived from the strategic use of information takes on a uniformly-positive orientation, regardless of players' positions in the network. If firm A, for example, is able to influence firm B's beliefs by acquiring additional information, this discourages firm B's strategic responsiveness. For the lobbyist, firms A's additional informativeness only encourages her corresponding behavior. Both of these effects work in firm A's favor. A similar story holds for firm B. For the lobbyist, her additional informativeness encourages the actions of both firms. And if the firms consequently acquire additional information, the value that the lobbyist obtains from her own research, which allows her to infer the observations and subsequent actions of the firms, only increases. In other words, *everyone* carries the incentive to exaggerate the quality of their acquired information. As will be seen, the extent of *connectedness* to others in the network drives the magnitude of the strategic incentives to information acquisition.

To study these heterogeneous environments in a reduced form while maintaining scope, the following model employs a familiar quadratic-payoffs setup under the general linear peer-effects pioneered by Ballester et al. (2006) [5]. Incorporating incomplete information, the model captures players' information investments in an initial stage. Signals are observed, informing players of their marginal values to second-stage action. When correlation between payoffs is introduced, signals begin to inform of the likely observations of neighbors. In line with the above vignette, the clarity of this inference is a function of the signal's quality as well as the qualities that neighbors are expected to acquire. An *information-response game* is derived and characterized, played on the same network of peer effects but transformed by the equilibrium correlation in signals. Here, players choose the extent to which their strategies respond to their information. The resulting equilibrium profile of strategic responses defines players' *informational centralities* in the game.

As a function of the unique linear equilibrium of the information-response game, the incentives to acquire information across players are derived. Marginal values to information are shown to scale with the square of each player's responsiveness. The scaling of marginal values with absolute informational centralities carries with it the potential for players moving against their signals. As such, information acquisition takes on a U-shaped non-monotonicity in networks. Acquisition at the bottom *decreases* with centrality in the information-response game, with the least central players investing in high levels of information as to move against the anticipated actions of neighbors.

After characterizing equilibrium behaviors and addressing the welfare and strategic implications of information acquisition, we turn to optimal policy design. A hypothetical *neutral player* is designated. Though an active member in the network, this player behaves as though she is in isolation, without peer influences. Then given a symmetric network, players that respond more so than the neutral player under-acquire information. Those responding less so but positively to their signal realizations over acquire information. And those moving against their signals under acquire information. With positive strategic values to information throughout the network, allowing players to publicly observe the information investments of the most central players as well as those moving against their signals increases aggregate welfare. As these players internalize the strategic value to information acquisition, the network collectively adjusts information investments efficiently. Importantly, this alignment in strategic values and informational externalities for these two sets of players persists in anti-symmetric networks. Thus together, the origin (i.e. no information acquisition) and the extent of acquisition and response of the neutral player provide a normalized yardstick useful for designing optimal transparency-based interventions, portable across network structures.

Applications of the model are then considered. The incorporation of both strategic substitutes and complements into the analysis affords a high level of flexibility and scope. Our three player network of firms A, B and our lobbyist provides one industrial organization incorporating both strategic substitutes and complements. Supply chains may also embody an array of both positive peer effects (e.g. between vertically positioned firms) and negative peer effects (e.g. between horizontal competing firms).[4] Section 5.1 further explores two more applications: financial markets under liquidity crises and two-sided markets. Both of these examples call on networks with both

[4]Ostrovsky (2008) [54] and Kotowski and Leister (2014) [40] study the tension between vertical strategic complements and horizontal strategic substitutes in competitive supply chains.

positive and negative links, with the former also exhibiting anti-symmetric relationships.

The implications for markets in crises are as follows. In liquidity flush markets, with traders unconstrained in their asset positions, strategic substitutes in asset demand implies strategic substitutes in information acquisition. Market crowding between firms' information investments parallels the market's informational inefficiency derived in rational expectations, as in the seminal work of Grossman and Stiglitz (1980) [30]. From a welfare perspective, the strategic use of costly information implies *over* investment of information in the market. The application then move beyond competitive markets to explore the implications of firms facing severe funding constraints during a liquidity crises. À la the type of liquidity spirals studied in Brunnermeier and Pedersen (2009) [11], a subset of firms are assumed to exhibit upward sloping demands, with high market prices allowing them to retain inventories and avoid unwanted liquidations. As the proportion of constrained firms to unconstrained firms grows large, firms throughout the market under acquire information. Constrained firms impose positive externalities on each other as they collect information, and aim to coordinate on high market liquidity outcomes. While unconstrained firms impose negative externalities on each other, they fail to internalize the sizable value that their information investments provide to constrained firms.

Taking job-search networks as a tangible example of a two-sided market, industry insiders and workers researching job opportunities compete with those within their group while complimenting the investment choices in the counterpart group. With these networks exhibiting extensive symmetry amongst pairwise relationships, the shorter, less competitive side of the market (insiders, commonly) under invests in information. The longer, competitive side of the market (workers) over acquires information. Here, insiders fail to internalize the value that their expertise endows workers, while workers over exert themselves researching job opportunities.

The organization of the paper is as follows. Section 2 provides the model's setup and discusses the optimal information acquisition and response problem of a single, isolated player. Section 3 then defines and characterizes equilibria in general networks. It discusses and derives the information-response game, and corresponding ex ante incentives to invest in information. It then offers a number of revealing examples describing the potential for equilibrium multiplicities and negative signal responses. Section 4 formalizes the welfare and strategic considerations discussed above under moderately sized peer effects. Welfare and strategic information acquisition for players moving against their signals are then addressed. A more general analysis of optimal policy design is then developed. Finally, Section 5 discusses applications, covers basic extensions of the model, returns to related literature, and concludes. A Supplemental Section S after the appendix more closely explores the relationship between network structure and information costs.

2 Model Setup

Time is discrete with two periods $t = 1, 2$. Period $t = 1$ gives the information acquisition game (*first stage*). Period $t = 2$ gives a Bayesian game in which N players simultaneously act in response to their information (*second stage*). For the second stage we adopt the bilinear payoffs studied by Ballester et al. (2006) [5]. We extend their setup to incorporate incomplete information regarding the marginal benefits of action $x_i \in \mathbb{R}$ for each player $i \in \{1, \ldots, N\}$.

The following notation is used. Each player i directly cares about state $\tilde{\omega}_i := \gamma\omega + \sqrt{1-\gamma^2}\omega_i$, a mixture of a player-specific state ω_i with a common (shared) state ω, each drawn from $\Omega \subset \mathbb{R}$.[5] The loading $\sqrt{1-\gamma^2}$ on ω_i merely normalizes the variance of $\tilde{\omega}_i$, simplifying the following analysis. A more general treatment is addressed in Section 5.2 with minor modification to the following. The respective state pairs (ω, ω_i) for each i and (ω_i, ω_j) for each i and $j \neq i$ are taken as jointly independent. Together, γ and ω scale the public alignment in preference shocks. $\gamma\omega$ should be interpreted as a publicly-shared but commonly-unknown component to the marginal value to adopting some technology in the second stage. $\sqrt{1-\gamma^2}\omega_i$ gives the corresponding idiosyncratic component.

All information is learned after the second stage, with each player i realizing her payoff:

$$u_i\left(\mathbf{x}|\omega, \omega_i\right) = \left(a_i + \tilde{\omega}_i\right) x_i - \frac{1}{2}\sigma_{ii}x_i^2 + \sum_{j \neq i} \sigma_{ij}x_i x_j.$$

a_i scales i's publicly-known average marginal gain to x_i, or her expected predisposition for second-stage action. It incorporates her average marginal value to action x_i, leaving residual uncertainty to be captured by the state $\tilde{\omega}_i$. σ_{ii} gives a positive constant scaling the concavity in her utility, capturing diminishing returns to x_i. σ_{ij} measures the influence that j's action x_j has on i's marginal gain to x_i (j's *peer effect* on i) and takes values in $\mathbb{R}$. Positive σ_{ij} will correspond to strategic complements, negative values to strategic substitutes, with $\sigma_{ij} = 0$ designating that j lies outside of i's *neighborhood*. Σ will be used to denote the square matrix $[\sigma_{ij}]$ with 0's along the diagonal. The sizes of the elements a_i and σ_{ij} for each $j \neq i$ relative to σ_{ii} determine the responsiveness of i's ideal action to the second-stage actions of her neighbors.

Each player i does not directly observe any component of $\boldsymbol{\omega}$. However, at $t = 2$ i does receive information (θ_i, e_i), giving signal realization $\theta_i \in \Theta \subset \mathbb{R}$ of quality $e_i \in [0, 1]$ informing her of $\tilde{\omega}_i$. i does not observe (θ_j, e_j) for each $j \neq i$. Thus, i is free to choose private information-contingent second-stage strategy $X_i\left(\cdot|\cdot\right) : \Theta \times [0, 1] \rightarrow \mathbb{R}$ mapping privately observed signal θ_i to an action in $\mathbb{R}$ given her quality e_i.

In the first stage each i privately invests in the signal quality e_i. The cost of quality (i.e. information acquisition effort) is given by the convex function $\kappa \in \mathcal{C}^2$ satisfying: $\kappa(0) = 0$ and $\kappa'(e_i), \kappa''(e_i) \geq 0$ for each $e_i \in [0, 1]$. Beyond these standard conditions we assume the following:

Assumption 1. *$\kappa \in \mathcal{C}^3$ satisfies: $\kappa'(0) = 0$, $\kappa'''(e_i) \geq 0$ for every $e_i \in [0, 1]$, and there exists an unique $e^\dagger \in (0, 1)$ solving $e^\dagger = \kappa'\left(e^\dagger\right)$.*

[5] While here $\boldsymbol{\omega}$ will denote the vector of states $(\omega, (\omega_i)_{i=1}^N)$, bold symbols will generally be used to denote profiles (vectors) of respective parameters and variables, with components for each $i \in \{1, \ldots, N\}$. We can consider ω and ω_i to follow standard normal distributions, though the more general properties in players' expectations required in the analysis are given below with E1-E4.

$\kappa'(0) = 0$ implies that the marginal cost to the lowest quality information is negligible. $\kappa'''(e_i) \geq 0$ implies that the convexity in information qualities are non-decreasing, and primary serves as a technical condition sufficing for existence of a first-stage equilibrium. Uniqueness of a solution to $e^\dagger = \kappa'\left(e^\dagger\right)$ will be seen to yield a unique interior solution to any isolated player's information acquisition problem.

Without significant loss of generality we normalize $\sigma_{ii} = 1$ for each i and scale other terms as needed.[6] Again, Section 5.2 discusses extensions incorporating heterogeneous σ_{ii}, individual costs functions κ_i, as well as additional idiosyncrasies into $\tilde{\omega}_i$. All of these extensions preserve the following analysis and results.

Together, the couple (e_i, X_i) defines a pure strategy for each i in the two-stage game. As players do not directly observe quality investments of others, $\mu_{ij} : [0,1] \to \mathbb{R}_+$ will denote the $t = 2$ belief held by player i regarding j's first-stage quality investment e_j. Thus, the initial period $t = 2$ expected payoff of player i as a function of the vector of other players' strategies $\mathbf{X}_{-i}$, private information (θ_i, e_i), and beliefs μ_i can be written:

$$u_i\left(x_i, \mathbf{X}_{-i} | \theta_i, e_i, \mu_i\right) = \left(a_i + \mathbb{E}_i\left[\tilde{\omega}_i | \theta_i, e_i\right]\right) x_i - \frac{1}{2} x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i \mathbb{E}_i\left[X_j\left(\theta_j | e_j\right) | \theta_i, e_i, \mu_i, \right]. \tag{1}$$

This yields a second-stage linear best response:

$$BR_i\left(\mathbf{X}_{-i} | \theta_i, e_i, \mu_i\right) = a_i + \mathbb{E}_i\left[\tilde{\omega}_i | \theta_i, e_i\right] + \sum_{j \neq i} \sigma_{ij} \mathbb{E}_i\left[X_j\left(\theta_j | e_j\right) | \theta_i, e_i, \mu_i\right]. \tag{2}$$

That is, each i responds to her conditional expectation of $\tilde{\omega}_i$ and to what her information informs her of the observations and actions of neighbors.

States and signals may be taken to be joint-normally distributed. The following requires only that priors be centered about the origin and posteriors be linear-in-qualities:

E1. $\mathbb{E}_i[\omega] = \mathbb{E}_i[\omega_i] = \mathbb{E}_i[\theta_i] = 0$,

E2. $\mathbb{E}_i\left[\tilde{\omega}_i | \theta_i, e_i\right] = e_i \theta_i$ for each $e_i \in [0,1]$,

E3. $\mathbb{E}_i\left[\theta_i^2 | e_i\right] = 1$ for each $e_i \in [0,1]$, and

E4. $\mathbb{E}_i\left[\theta_j | \theta_i, e_i, \mu_i\right] = \int_{[0,1]} \mu_{ij}\left(e_j\right) \gamma^2 e_j e_i \theta_i de_j$.

As is common to model information investment as a number of costly draws of a normally distributed signal of given precision, Appendix A.1 applies this particular structure to derive properties E1-E4 directly. Information structures with two states also easily satisfy E1-E4.[7] Together, these give the essential properties used through the following analysis.

Conditions E1 and E2 together imply $\theta_i = \tilde{\omega}_i$ at $e_i = 1$. Condition E3 requires a normalization obtained by the appropriate increasing affine transformation to signals. The factor $\gamma^2 e_j e_i$ in

[6]Setting $\sigma_{ii} = \sigma_{jj}$ for each i and j does carry the implication that all players face common total variation in their payoffs. This allows the network of peer effects to drive all variation in equilibrium information acquisition.

[7]Additional examples incorporating an arbitrary finite number of states can be constructed.

condition E4 gives the correlation of the signals θ_i and θ_j.[8] Noting that any strictly-monotonic transformation does not change the informational content of signals,[9] conditions E1-E3 merely simplify the following analysis. Loss of generality does come with the linear-multiplicative separability of condition E4. The following analysis and results hinge only on multiplicative separability, however. All qualitative properties remain intact under the more general (non-linear) extension $\mathbb{E}_i[\theta_j|\theta_i, \mu_i, e_i] = \int_{[0,1]} \mu_{ij}(e_j)\gamma^2\eta(e_j)\eta(e_i)\theta_i de_j$ for any non-negative and strictly monotone $\eta \in \mathcal{C}^1$. Finally, as the following will consider pure first-stage strategy profiles $\mathbf{e} \in [0,1]^N$, sequential rationality in beliefs requires $\mu^*_{ij}(e_j) = 1$ for each i and j. Therefore, condition E4 reduces in equilibrium to $\mathbb{E}_i[\theta_j|\theta_i, e_i, \mu^*_i] = \gamma^2 e_j e_i \theta_i$.

Though this paper's focus is on the role of general peer effects in equilibrium information acquisition, to help fix ideas the following example solves the information acquisition and optimal response problems of a single, isolated player.

Example. [isolated player's problem] *Consider the information response problem of a single player i having chosen quality e_i in period $t = 1$, and now maximizing the following period $t = 2$ objective:*

$$u_i(x_i|\theta_i, e_i) = (a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i, e_i])\, x_i - \frac{1}{2}x_i^2 = (a_i + e_i\theta_i)\, x_i - \frac{1}{2}x_i^2.$$

The first order condition to her problem, conditioning on information (θ_i, e_i), yields:

$$\frac{\partial}{\partial x} u_i(x_i|\theta_i, e_i) = (a_i + e_i\theta_i) - x_i = 0,$$

which gives:

$$X^*(\theta_i|e_i) = a_i + e_i\theta_i.$$

That is, i responds to her realized signal by an amount equal to the qualities of the signal, e_i. This yields period $t = 1$ expected (indirect) utility:

$$\mathbb{E}_i[u_i(X^*(\theta_i|e_i)|\theta_i, e_i)|e_i] = \mathbb{E}_i\left[(a_i + e_i\theta_i)(a_i + e_i\theta_i) - \frac{1}{2}(a_i + e_i\theta_i)^2 |e_i\right] = \frac{1}{2}\left(a_i^2 + e_i^2\right),$$

which uses condition E3: $\mathbb{E}_\theta\left[\theta_i^2|e_i\right] = 1$. Then, the period $t = 1$ first-order condition for any interior $e^\dagger \in (0,1)$ is given with:

$$e^\dagger = \kappa'\left(e^\dagger\right). \tag{3}$$

Under Assumption 1, a unique $e^\dagger \in (0,1)$ solving (3) obtains. Further, as the above holds for all values of a_i, we see that without peer effects the isolated player (i) acquires a nonzero amount of

[8] With $\kappa(\cdot)$ a function of the quality of information that is used directly to infer $\tilde{\omega}_i$, we can interpret the efforts of i to be focused toward information sources most relevant to her particular qualities or tastes. For example, a firm's inference of the value of a production technology requires acquiring information of the technology's particular attributes most consequential to the firm's marginal product. The most important attributes should depend on the firm's specific qualities, preexisting input profile, and compatibility between coexisting technologies. Thus, κ should be interpreted as a general cost to research.

[9] Precisely, such a transformation merely rescales the value of θ_i for each $\tilde{\omega}_i$.

information and (ii) responds positively to her information ($\frac{\partial}{\partial \theta_i} X^* (\theta_i | e_i) \geq 0$).

As seen in the example, the value of information exhibits a natural convexity, even when a player i acts in isolation at $t = 2$. This is because more precise information increases i's posterior belief that her response to her signal is in the optimal direction, while holding the size of her response fixed. Then, additionally allowing her to optimally increase the size of her response provides additional value. These two effects multiply each other, yielding an increasing marginal value to signal quality.

3 Equilibrium information acquisition and response

3.1 Equilibrium definitions

The following equilibrium notions are presented backward inductively.

Definition 1. [second-stage equilibrium] *Given profile of qualities* $\mathbf{e}$ *and beliefs* $\boldsymbol{\mu}$*, an* <u>information response equilibrium</u> *(IRE) is a profile of strategies* $\mathbf{X}^* := (X_1^*, \ldots, X_N^*)$ *given as a Bayesian Nash equilibrium of the second stage game:*

$$X_i^* (\theta_i | e_i) \in \arg\max_{x \in \mathbb{R}} \mathbb{E}_i \left[u_i \left(x, \left(X_j^* (\theta_j | e_j) \right)_{j \neq i} | \omega, \omega_i \right) \middle| \theta_i, e_i, \mu_i \right],$$

for each $\theta_i \in \Theta$ *and* $i \in N$*. Expectation* $\mathbb{E}_i$ *is taken over* $\tilde{\omega}_i$*,* $\boldsymbol{\theta}_{-i}$ *and* $\mathbf{e}_{-i}$ *using beliefs* μ_i*, taking other players' strategies* $\mathbf{X}_{-i}^*$ *as given.*

Given private information (θ_i, e_i), each player i best responds to her signal by investing in her action, taking the profile of all other players' actions $\mathbf{X}_{-i}$ as fixed. Her information is relevant to learning about both $\tilde{\omega}_i$ and what other players observe and do at $t = 2$.

The first-stage equilibrium for given second-stage equilibrium $\mathbf{X}^*$ and beliefs $\boldsymbol{\mu}$ is defined as follows.

Definition 2. [first-stage equilibrium] *Given IRE* $\mathbf{X}^*$ *and beliefs* $\boldsymbol{\mu}$*, an* <u>information acquisition equilibrium</u> *(IAE) is a profile of qualities* $\mathbf{e}^* := (e_1^*, \ldots, e_n^*)$ *given as a Nash equilibrium of the first stage game:*

$$e_i^* \in \arg\max_{e \in [0,1]} \mathbb{E}_i \left[u_i \left(X_i^* (\theta_i | e), \left(X_j^* (\theta_j | e_j) \right)_{j \neq i} | \omega, \omega_i \right) \middle| e, \mu_i \right] - \kappa (e),$$

for each i*, where expectation* $\mathbb{E}_i$ *is taken over* $\tilde{\omega}_i$*,* $\boldsymbol{\theta}$ *and* $\mathbf{e}_{-i}$ *using beliefs* μ_i*, taking strategies* $\mathbf{X}^*$ *as given.*

That is, each player i optimally invests in the quality of her signal θ_i at cost $\kappa(e_i^*)$, anticipating second-stage play as given by $\mathbf{X}^*$. Together, IAE $\mathbf{e}^*$, IRE $\mathbf{X}^*$ and sequentially rational beliefs $\boldsymbol{\mu}^*$ define a weak perfect Bayesian equilibrium of the two-stage game.

The following begins by characterizing equilibrium information acquisition and response under our general network setting. Section 3.3 then provides a number of examples exploring the breadth of equilibrium behaviors.

3.2 Equilibrium characterizations

Here, we characterize IRE and interior IAE of the two-stage game. As displayed below, an ex ante expected equilibrium $\boldsymbol{\alpha}^* \in \mathbb{R}^N$ can be obtained to yield average second-stage actions by averaging over realized signals.[10] A key innovation, however, is that in addition to this expected game played on a network, players play an information-response game on the same network. However, the network of peer effects is *transformed by the correlation in signals*, which is *induced by qualities acquired in the first stage*. Information now tells players not only about their marginal gain to action (i.e. the relevant state of the world $\tilde{\omega}_i$) but also about what to expect neighbors will see and do at $t = 2$. Accordingly, the relative responsiveness of each player i's strategy to their signal θ_i will depend not only on their quality of information e_i, but also on each neighbor j's equilibrium information investment and corresponding strategic responsiveness to their own signal, θ_j. Crucially, the resulting intricate interdependence of information responses is introduced precisely when players' payoffs are correlated through the common state ω: when $\gamma > 0$.

Formalizing the discussion, define the correlation adjusted adjacency matrix as:

$$\begin{aligned} \Sigma^c &:= \left[\gamma^2 e_i \sigma_{ij} e_j\right]_{i,j;i\neq j} \\ &= \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}, \end{aligned} \tag{4}$$

where $\mathbf{I}_{\boldsymbol{\phi}}$ denotes the diagonal matrix with entries given by (generic) vector $\boldsymbol{\phi}$.[11] Then, when $(\mathbf{I} - \Sigma)$ and $(\mathbf{I} - \Sigma^c)$ are invertible[12] the following unique linear second-stage solution obtains.

Theorem 1. [linear IRE] *For any* $\mathbf{e}$ *and sequentially rational* $\boldsymbol{\mu}^*$ *there exists a unique linear IRE of the form:*

$$\begin{aligned} \mathbf{X}^* &= (\mathbf{I} - \Sigma)^{-1} \mathbf{a} + \mathbf{I}_{\boldsymbol{\theta}} (\mathbf{I} - \Sigma^c)^{-1} \mathbf{e} \\ &= \left[\alpha_i^* + \beta_i^* \theta_i\right], \end{aligned} \tag{5}$$

denoting:

$$\begin{aligned} \boldsymbol{\alpha}^* &:= (\mathbf{I} - \Sigma)^{-1} \mathbf{a}, \\ \boldsymbol{\beta}^* &:= (\mathbf{I} - \Sigma^c)^{-1} \mathbf{e}. \end{aligned}$$

Note that $\boldsymbol{\alpha}^*$ is independent of $\mathbf{e}$, while $\boldsymbol{\beta}^*$ is a function of the vector of qualities chosen in the

[10] $\boldsymbol{\alpha}^*$ corresponds to the solution of Ballester et al. (2006) [5] but in expectation.

[11] $\mathbf{I_e}\Sigma\mathbf{I_e}$ is referred to as a Hadamard product of $[e_i e_j]$ with Σ, named after Jacques Salomon Hadamard (1865 –1963).

[12] Assumption A1 in Appendix A.2 provides a weak sufficient condition for this to hold.

first stage. As shown in Appendix A.2 with the theorem's proof, IRE $\mathbf{X}^*$ is the unique equilibrium in a broad class of strategies that yield convergent higher-order expectations across players in the network.

A valuable interpretation of Theorem 1 utilizes the notion of *weighted Bonacich centrality* (Bonacich (1987) [7]). Formally, for given $N \times N$ graph $\mathbf{G} := [g_{ij}]$ of interaction terms (with zero diagonal) and weighting vector $\boldsymbol{\phi} \in \mathbb{R}^n$, the weighted Bonacich centrality measure is defined as:

$$\begin{aligned} \mathbf{b}(\mathbf{G}, \boldsymbol{\phi}) &:= (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\phi} \\ &= \sum_{\tau=0}^{\infty} \mathbf{G}^{\tau} \boldsymbol{\phi}. \end{aligned}$$

This measure is well defined provided $(\mathbf{I} - \mathbf{G})$ is invertible. Each i'th component of $\mathbf{b}(\mathbf{G}, \boldsymbol{\phi})$ gives an aggregation of the total number of paths starting from player i, with sub-paths emanating from each player j weighted by ϕ_j.[13] While the matrix $\mathbf{G}$ provides a benchmark network of bilateral relationships, the components of $\boldsymbol{\phi}$ capture each player's relative prominence within the network.

Placing this centrality concept into the context of Theorem 1, we see that ex ante expected actions are proportional to players' Bonacich centrality on Σ weighted by the vector of constants $\mathbf{a}$, $\mathbf{b}(\Sigma, \mathbf{a})$. The strategic response of each player's strategy to her private information also depends on her centrality. However, the centrality of interest is now (i) adjusted for the correlation of players' signals, and (ii) weighted by the vector of signal qualities $\mathbf{e}$. For the former, scaling down links by signal correlations adjusts for the inference of neighbors' second-stage actions. For the latter, weighting the resulting Bonacich centrality measure by $\mathbf{e}$ accounts for the value that information carries toward directly inferring the payoff-relevant state, $\tilde{\omega}_i$. The resulting alternative measure of centrality, or *informational centrality*, resonates with the unweighted Bonacich centrality $\mathbf{b}(\Sigma, \mathbf{1})$ directly derived from the network Σ. $\mathbf{b}(\Sigma^c, \mathbf{e})$ instead offers an adjusted measure of player position in the information-response game.

In light of Theorem 1 and as a technical note, scaling γ is analytically equivalent to uniformly scaling each term in Σ, via the product $\gamma^2 e_i \sigma_{ij} e_j$ in (4). Much of the following analysis will consider small or bounded values of γ. Thus, with γ directly scaling links in Σ^c, this can be aptly interpreted as taking moderately sized peer effects in the information-response game.

As seen with (5), the ways in which players respond to their information in the unique linear IRE depends in an intricate way on the network of peer effects and on the acquired signal qualities of neighbors.[14] The following begins to characterize the collective incentives to acquire information, providing a necessary condition for any interior first-stage strategy.

Theorem 2. [IAE and information response] *Given signal quality profile $\mathbf{e}_{-i}$, player i's private marginal gain to signal quality e_i is given by β_i^2/e_i, yielding the necessary condition for any*

[13] Other variations of this centrality measure are defined with weighted walks starting from neighbors (see Jackson (2008) [39]), while this definition's weighting begins at the originating node.

[14] Or more precisely, the sequentially rational beliefs regarding the signal qualities of others.

IAE $\mathbf{e}^*$*:*

$$\frac{\beta_i^{*2}}{e_i^*} = \kappa'\left(e_i^*\right), \tag{6}$$

for each i *with* $e_i^* \in (0,1)$.[15]

With $e_i\kappa(e_i)$ an increasing function in e_i, $\mathbf{e}^*$ is thus ordered with respect to the size of players' informational centrality, $|\beta^*|$. Intuitively, we should expect the responsiveness of each player's strategy to their signal to be proportional –in some way– to the quality of her information, regardless of the presence of peer effects. Theorem 2 affirms this intuition.

Next, Corollary 1 ties player degree with their incentives to acquire information under moderate peer effects. It describes the speeds and directions that players diverge from $e^\dagger$ as peer effects are introduced.

Corollary 1. *Under Assumption 1, the following limit obtains:*

$$\lim_{\gamma\to^+0} \frac{\partial e_i^*}{\partial(\gamma^2)} = \frac{e^{\dagger 2}\sum_{k\neq i}\sigma_{ik}}{\kappa'(e^\dagger)-1}. \tag{7}$$

As γ departs from zero, or as peer effects are introduced, players with the highest degree depart upward away from quality $e^\dagger$ relatively faster than those with lower degree. The speed at which players adjust their qualities decreases in the concavity of $\kappa(e^\dagger)$, which measures the sensitivity in marginal gains to information around $e^\dagger$. This speed increases in $e^{\dagger 2}$, which measures the initial marginal informational content that signals provide toward inferring neighbors' second-stage observations.

As will be observed in Sections 3.3, the degree-wise ordering in $\mathbf{e}^*$ that is implied by Corollary 1 may not persist as γ is further increased. That is, while degree describes players' initial incentives to acquire quality, it does not fully determine these incentives when peer effects are more pronounced. The ease and extent to which this ordering may be violated will intimately depend on both the network's structure and the shape of κ. Supplemental Section S further explores this relationship, and develops network properties that allow player degree to persistently order the equilibrium extent of information acquisition.

3.3 Examples

The following examples illustrate the breadth of equilibrium properties in this setting. The first example illustrates the potential for multiple IAE, even under the unique IRE given with Theorem 1. Multiplicity can arise under either strategic complements or strategic substitutes.

Example 1. *For this and subsequent examples we consider the following strictly convex information cost function:*

$$\kappa(e) = K\frac{e^{\eta_1}}{(1-e^2)^{\eta_2}},$$

[15]The existence of an IAE is established with Proposition S.1 in Supplemental Section S.

where $\eta_1 \geq 2$ and $\eta_2 > 0$. It can be shown that Assumption 1 is satisfied under these bounds, yielding isolation quality $e^\dagger$. This functional form provides a standard family of convex costs functions that asymptote as $e_i \to^- 1$. It also allows for a broad range of convexities. Crudely, increasing η_1 increases convexity at higher values of e_i while increasing η shifts convexity toward lower values of e_i.

First, consider any regular network in which each player i is connected to four other players with symmetric peer effects $\sigma_{ij} = \sigma_{ji} = p > 0$ (for neighbor j). Normalize $\gamma = 1$, and set $\eta_1 = 3$, $\eta_2 = .5$ and $K = .312$. Figure 2(a) provides the set of symmetric equilibria of the information acquisition game as we increase the size of p above. In these examples, information responses are given by the increasing relationship $\beta_i^* = \sqrt{e_i^* \kappa'(e_i^*)}$ from Theorem 2.

Under second stage strategic complements, first stage information acquisition reinforces itself. The added convexity in the value of information introduces potential coordination problems. For values of p above $5/22$ the players can coordinate on low, medium, or high levels of information. Higher levels of information further incentivize acquisition as the players' signals correlate with each other.

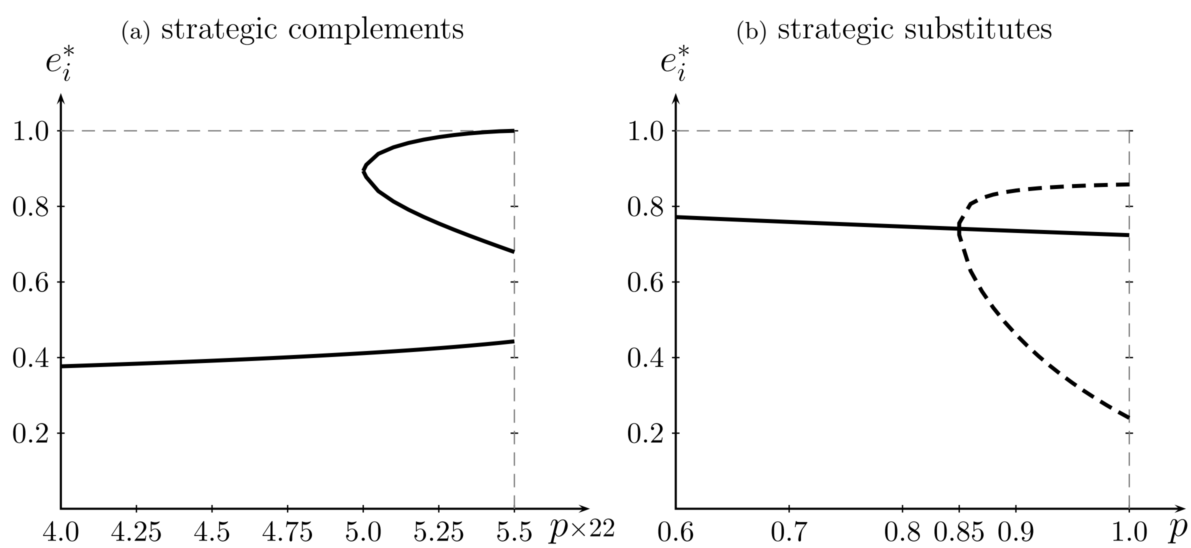

Figure 2: [Example 1] equilibrium multiplicity

Next, consider the two player network of players 1 and 2, with symmetric negative peer effect $\sigma_{12} = \sigma_{21} = -p \leq 0$. Set $\eta_1 = 2$, $\eta_2 = 1$, with $K = .03$.[16] Now, the propensity for an asymmetric equilibrium arises, as seen in Figure 2(b). For values of p below .85 the symmetric equilibrium of the information acquisition game gives the unique IAE (solid line). Information responses are again given using $\beta_i^* = \sqrt{e_i^* \kappa'(e_i^*)}$. Above $p = .85$ there also exists an asymmetric equilibrium in which one player acquires a highly precise signal while the other acquires an imprecise signal

[16] K is adjusted down with the new values of η_1 and η_2 to obtain interior solutions, with the latter set so that the qualitative properties of the equilibrium are well displayed.

(dashed lines). It can be verified that in this equilibrium the low-quality player rather prefers the symmetric equilibrium, while the high-quality player strictly prefers her equilibrium informational advantage.

The next example exhibits the potential for players to move against their information given substantial negative peer effects. Precisely, for players facing enough negative influence from others, the incentives to acquire information may increase with the size of these influences, but with these players moving against their signals in anticipation of their neighbors' actions. Strikingly, this non-monotonicity can be quite significant, with the incentives to acquire information quickly falling to zero and abruptly restoring itself at more extreme influences.

Example 2. *Again, normalize $\gamma = 1$. Take the wheel and spoke network with center player 1 and peripheral players $i \in \{2, 3, 4\}$, as depicted in Figure 3(left).*

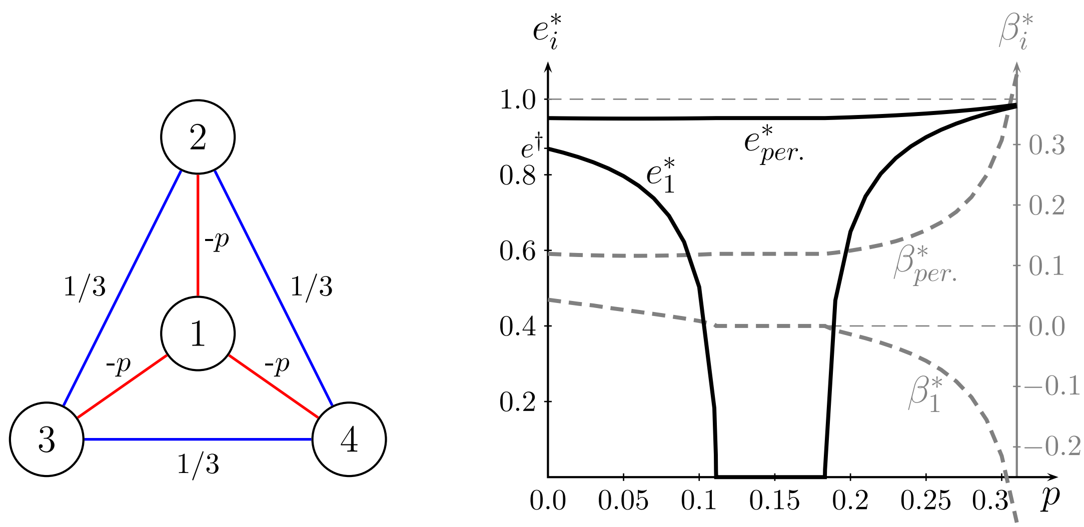

Figure 3: [Example 2] unique equilibrium with negative signal response

Each player imposes a symmetric negative externality on 1: $\sigma_{1i} = \sigma_{i1} = -p \leq 0$ for each $i \in \{2, 3, 4\}$. Finally, the peripheral players are symmetrically linked in a circle with weights $\sigma_{ij} = 1/3$ for each pair $i, j \in \{2, 3, 4\}$. Take the information cost function given in Example 1, setting $\eta_1 = 2$ and $\eta_2 = 1$ with $K = .03$.

Figure 3(right) plots qualities e_i^* and responses β_i^* in the unique equilibrium of the information-response game symmetric across the peripheral players 2-4 ('per.'), over a range of p values. As p departs from zero, information acquisition drops slightly for the center, as the negative externalities between the peripheral clique and the center increase. For values of p between 0.111 and 0.183 the center acquires no information. Then, for values of p above 0.183 the incentives to acquire information are quickly restored. However, now the center moves against the network, with 1 acquiring information and moving opposite to her signal (i.e. $\beta_1^* < 0$) in anticipation of the actions of the periphery. As negative externalities become more acute, further information is incentivized, with 1's behavior further reinforcing information acquisition throughout the network.

Example 2 highlights the potential for non-monotonicity in information acquisition. The responsiveness of each player to her private information in an IRE $\mathbf{X}^*$ remains unbounded at the origin. Players with intermediate centrality in the information-response game face moderate incentives to acquire information in period $t = 1$. And as illustrated in Figure 3, such non-monotonicity need not be gradual, but rather the incentives to acquire information can quickly vanish for players with particularly low centrality. Then, for ever lower levels of centrality the incentives to acquire information may be restored to great extent, but now with information used to infer and move against neighbors' second-stage actions.

Both multiplicity and negative signal responses come as interesting equilibrium incarnations. None the less, the following establishes sufficient conditions for the exclusion of these cases. Under moderate peer effects, a unique equilibrium in which players move in the direction of their signals always obtains.

Proposition 1. *Under Assumption 1:*

1. *there is some* $\gamma^u > 0$ *such that if* $\gamma \in [0, \gamma^u)$ *a unique IAE exists, and*
2. *there is some* $\gamma^p > 0$ *such that if* $\gamma \in [0, \gamma^p)$ *all players acquire qualities in* $(0, 1)$ *and respond positively to their signals (i.e.* $\beta_i^* > 0$ *for each* i*) in equilibrium.*

4 Equilibrium welfare and the strategic value to information

The welfare analysis takes the following approach. First, we will see that when allowing players' to respond optimally to signal realizations, correlation in signals is necessary for the presence of inefficiencies in information acquisition. That is, externalties and strategic motives arise in the first stage only when players can use their information in the second stage to infer the observations of neighbors. Departing from the case of zero correlation (i.e. $\gamma = 0$), strong welfare statements are derived given moderately sized peer effects (i.e. small γ). These results address the directions of both (i) the equilibrium profile of information qualities when information investments are publicly observed and (ii) the utilitarian solution relative to the equilibrium described in Theorem 2. We then turn to more significantly sized peer effects, incorporating the welfare implications of players moving against their signals and under the potential for multiple information acquisition equilibria.

In the following welfare benchmark, we take the second-stage information response equilibrium $\mathbf{X}^*$ –a function of qualities $\mathbf{e}$– as given. Further, given quality profile $\mathbf{e}$ we impose sequential rationality in beliefs throughout: $\mu_i^{j*}(e_j) = 1$ for each i and j. That is, the planner is free to publicly announce the information qualities that she prescribes. This prevents inefficiencies derived from inconsistent beliefs. Incorporating these elements into our benchmark leaves first-stage behavior as the sole endogenous (potential) source of inefficiencies.

As a function of the realized quality profile $\mathbf{e}$, and given IRE $\mathbf{X}^*$ and sequentially rational beliefs

$\boldsymbol{\mu}^*$, players' ex ante values reduce as follows:

$$\begin{aligned}\boldsymbol{\nu}(\mathbf{X}^*|\mathbf{e}) &:= [\mathbb{E}_i[u_i(\mathbf{X}^*|\theta_i, e_i, \mu_i^*)|e_i, \mu_i^*] - \kappa(e_i)] \\ &= \frac{1}{2}\left(\mathbf{I}_{\boldsymbol{\alpha}^*}\boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\beta}^*}\boldsymbol{\beta}^*\right) - \kappa(\mathbf{e}). \end{aligned} \tag{8}$$

This reduction to quadratic payoffs is easily shown in Appendix A.3. Then taking $\nu(\mathbf{X}^*|\mathbf{e})$ we can define the following Pareto problem:

$$\max_{\mathbf{e}\in[0,1]^N} \sum_k \lambda_k \nu_k(\mathbf{X}^*|\mathbf{e}), \tag{9}$$

for non-negative Pareto weights $\boldsymbol{\lambda}$ taken from the $(N-1)$-simplex. First order conditions yielding the planner's solution $\mathbf{e}^{po}(\boldsymbol{\lambda})$ are given for each $i \in N$ by:

$$\sum_k \lambda_k \frac{\partial}{\partial e_i}\nu_k(\mathbf{X}^*|\mathbf{e}) = 0. \tag{10}$$

The following establishes correlation in the players' payoffs as necessary for any equilibrium inefficiencies that may arise.

Proposition 2. *At $\gamma = 0$ we have $e_i^* = e_i^{po}(\boldsymbol{\lambda})$, and $e_i^* = e^\dagger$ under Assumption 1, for each i.*

Proof. With $\boldsymbol{\alpha}^*$ independent of $\mathbf{e}$, (10) can be written:

$$\begin{aligned}\sum_k \lambda_k \frac{\partial}{\partial e_i}\nu_k(\mathbf{X}^*|\mathbf{e}) &= \lambda_i \frac{\partial}{\partial e_i}\nu_i(\mathbf{X}^*|\mathbf{e}) + \sum_{k\neq i}\lambda_k \frac{\partial}{\partial e_i}\nu_k(\mathbf{X}^*|\mathbf{e}) \\ &= \lambda_i\left(\left(\frac{\partial}{\partial e_i}\mathbb{E}_i[u_i(\mathbf{X}^*|\theta_i, e_i, \mu_i^*)|e_i, \mu_i^*] - \kappa'(e_i)\right) + \beta_i^* \sum_{k\neq i}\gamma^2 e_i e_k \sigma_{ik}\frac{\partial}{\partial e_i}\beta_k^*\right) + \sum_{k\neq i}\lambda_k \beta_k^* \frac{\partial}{\partial e_i}\beta_k^*,\end{aligned}$$

where the first term in brackets takes $\boldsymbol{\beta}_{-i}^*$ fixed. The first order condition of i's IAE problem is given by setting this term to zero. Now, $\boldsymbol{\beta}^* = \mathbf{b}([0], \mathbf{e}) = \mathbf{e}$ when $\gamma = 0$, and thus $\frac{\partial}{\partial e_i}\beta_k^* = 0$ for each $k \neq i$. Thus, when $\gamma = 0$ the Pareto optimal and IAE solutions align. Finally, $e_i^* = e^\dagger$ for each i under Assumption 1 follows from $\boldsymbol{\beta}^* = \mathbf{e}$ at $\gamma = 0$ and Theorem 2. □

Under our general treatment of peer effects, it does not come surprisingly that equilibria are not generally Pareto efficient. We next begin to more completely describe the nature of inefficiency in the model. The following measures for the strategic value to information and informational externalities are required.

First, a loss in value to information is realized by each player i who, in equilibrium, is unable to directly influence others' beliefs regarding her information investment. Precisely, with qualities privately chosen at $t = 1$, incentive compatibility constrains each i when weighing the costs and benefits of acquiring information quality. If instead i could publicly invest in quality e_i and directly influence others' beliefs, she may derive additional value from acquiring more or less quality than in equilibrium (holding $\mathbf{e}_{-i}^*$ fixed). Informational externalities, on the other hand, are directly imposed on i's neighbors. Also derived from the influences that e_i has on neighbors' responses, these externalities are instead measured by the effect that e_i has on *neighbors'* welfare.

Formalizing this, consider the following utilitarian problem, given from (9) by setting $\boldsymbol{\lambda} = \frac{1}{N}\mathbf{1}$:

$$\max_{\mathbf{e}\in[0,1]^N} \sum_k \nu_k \left(\mathbf{X}^*|\mathbf{e}\right). \tag{11}$$

The partial derivative of aggregate welfare with respect i's quality is given by:

$$\begin{aligned} \frac{\partial}{\partial e_i}\sum_k \nu_k\left(\mathbf{X}^*|\mathbf{e}\right) &= \beta_i^* \frac{\partial \beta_i^*}{\partial e_i} + \sum_{k\neq i}\beta_k^*\frac{\partial \beta_k^*}{\partial e_i} - \kappa'\left(e_i\right) \\ &= \underbrace{\underbrace{\underbrace{\left(\frac{\beta_i^{*2}}{e_i} - \kappa'\left(e_i\right)\right)}_{=0 \text{ in IAE } \mathbf{e}^* \text{ f.o.c.}} + \beta_i^* \sum_{k\neq i}\gamma^2 e_i e_k \sigma_{ik}\frac{\partial}{\partial e_i}\beta_k^*}_{=0 \text{ in public acquisition equilibrium } \mathbf{e}^{pb} \text{ f.o.c.}} + \sum_{k\neq i}\beta_k^*\frac{\partial}{\partial e_i}\beta_k^*}_{=0 \text{ in planner's solution } \mathbf{e}^{pl} \text{ f.o.c.}}. \end{aligned} \tag{12}$$

In IAE $\mathbf{e}^*$, where qualities are privately acquired, the first term in brackets is set to zero by i in her optimization problem.[17] That is, the term β_i^{*2}/e_i is given by β_i^* multiplied by the marginal influence of i's quality on her own response $\partial\beta_i^*/\partial e_i$, while setting the marginal influence of i's information quality on others' responses $\partial\beta_k^*/\partial e_i$ to zero. This corresponds with Theorem 2. The middle sum adjusts for the marginal effect that i's information quality imposes on each k's response in IRE $\mathbf{X}^*$, *when* the acquisition of e_i is directly observed by each $k \neq i$. This term is excluded in IAE under i's incentive compatibility constraint, again where e_i^* is chosen fixing μ_k and in turn β_k^* for each $k \neq i$. If instead i were free to publicly choose her signal quality in the first stage she would internalize this influence. The term captures i's *strategic* incentive toward influencing her neighbors' information responses. Setting these terms in (12) to zero for each i yields the *public* information acquisition equilibrium $\mathbf{e}^{pb}$. Finally, in the planner's problem the direct marginal influence that i's quality carries for *others'* payoffs is also accounted for. That is, when total marginal gains to welfare from i's quality is set to zero, we obtain one of N first order conditions that determine the *planner's* solution $\mathbf{e}^{pl} := \mathbf{e}^{po}(\frac{1}{N}\mathbf{1})$.[18] Together, the final two terms adjust for the effect that i's quality has on welfare that is not internalized by i in the first stage.

[17] Note that the private information acquisition benchmark is equivalent to a one-stage game in which players simultaneously choose information qualities and information contingent strategies.

[18] This planner's benchmark $\mathbf{e}^{pl}$ is commonly referred to as the "*second-best team*" solution, with the "*first-best team*" or "*team-efficient*" solution determined when the planner can also control how players use their information. See Burguet and Vives (2000) [12] or Vives (2008) [62] chapter 6 for discussions.

Thus, we can define the following measures:

$$\xi_i^{st}(\mathbf{e},\mathbf{X}^*) := \beta_i^* \sum_{k\neq i} \gamma^2 e_i e_k \sigma_{ik} \frac{\partial}{\partial e_i}\beta_k^*, \tag{13}$$

$$\xi_i^{ex}(\mathbf{e},\mathbf{X}^*) := \sum_{k\neq i} \beta_k^* \frac{\partial}{\partial e_i}\beta_k^*, \tag{14}$$

$$\xi_i(\mathbf{e},\mathbf{X}^*) := \xi_i^{st}(\mathbf{e},\mathbf{X}^*) + \xi_i^{ex}(\mathbf{e},\mathbf{X}^*). \tag{15}$$

We refer to $\xi_i^{st}(\mathbf{e},\mathbf{X}^*)$ as i's marginal *strategic value* to information at quality profile $\mathbf{e}$. At IAE $\mathbf{e}^*$, it informs us in which direction i would deviate in the first stage *if* her quality investment were publicly observed by $t = 2$. In addition, it tells us how much i would be willing to pay (in utils) per unit of quality if she could directly influence her neighbors' beliefs of e_i. We refer to $\xi_i^{ex}(\mathbf{e},\mathbf{X}^*)$ as i's marginal *informational externalities* at quality profile $\mathbf{e}$. This marginal cost would not be internalized in the event that i could publicly choose her information quality. Taking these marginal costs together, $\xi_i(\mathbf{e},\mathbf{X}^*)$ gives the sum of i's marginal strategic value and informational externalities. We term this the marginal *public value* from quality e_i (at $\mathbf{e}$). When evaluated at IAE $\mathbf{e}^*$, the vector $\boldsymbol{\xi}(\mathbf{e}^*,\mathbf{X}^*)$ evaluates the equilibrium *gradient* of $\sum_k \nu_k(\mathbf{X}^*|\mathbf{e})$, pointing in the direction of the social planner's optimal deviation from the equilibrium quality profile $\mathbf{e}^*$.

Closed forms of these measures are derived in Appendix A.3. In accordance with Proposition 2, all equate to zero at $\gamma = 0$, when private signals are uninformative of others' second-stage observations and responses. When $\gamma > 0$ this no longer holds. For moderate peer effects, $\xi_i^{st}(\mathbf{e},\mathbf{X}^*)$ and $\xi_i^{ex}(\mathbf{e},\mathbf{X}^*)$ are proportional to the following network measures.

Lemma 1. [limiting marginal inefficiencies] *The following limits obtain:*

$$\lim_{\gamma\to^+0} \frac{\partial \xi_i^{st}(\mathbf{e}^*,\mathbf{X}^*)}{\partial(\gamma^4)} = 2e^{\dagger 5}\sum_{k\neq i}\sigma_{ik}\sigma_{ki}, \tag{16}$$

$$\lim_{\gamma\to^+0} \frac{\partial \xi_i^{ex}(\mathbf{e}^*,\mathbf{X}^*)}{\partial(\gamma^2)} = 2e^{\dagger 3}\sum_{k\neq i}\sigma_{ki}. \tag{17}$$

That is, the rate of increase of i's marginal strategic value as γ^4 departs from zero is proportional to $(e^\dagger)^5$ multiplied by the sum-of-products of i's peer effects (i.e. the sum of her out-links multiplied by their respective in-links). The rate-of-increase of i's marginal externalities as γ^2 departs from zero is approximately $(e^\dagger)^3$ multiplied by i's in-degree.

The intuition behind these limits goes as follows. Each i's marginal strategic value as the network of peer effects is pronounced in the information-response game depends on both i's outward and inward directed links. Outward links measure the extent to which i *cares* about each of her neighbor's second-stage actions. Inward links measure the influence that i's quality has on each neighbors' payoffs. Together, the neighbor-wise product of links scale i's marginal strategic value to her information. Marginal externalities, on the other hand, depend only on the influence that i's quality has on others. Precisely, marginal externalities scale by the sum of influences imposed on the network. For moderately size peer effects, this is propotional to i's in-degree.

As in (7), the sizes of these limiting derivatives depend on the initial level of information acquisition, $e^{\dagger}$. $e^{\dagger}$ scales the initial strategic responsiveness of strategies, as well as the initial extent to which players can infer others' signal realizations from their own signals. With strategic values involving the additional inference by neighbors of i's signal realization, (16) scales with an additional factor of $e^{\dagger 2}$.

To explore the broader implications of Lemma 1, we next focus in on the set of symmetric networks. This family of network architectures offers a broad and flexible class of familiar environments.

4.1 Symmetric pairwise peer effects and welfare

Here we further describe the nature of inefficient information acquisition by focusing on symmetric network structures. This is primarily done as symmetry is commonly observed in many real-world peer-effects environments. Be them competitive or cooperative, most relationships in society tend to be reflexive, in both direction and size. Competitors tend to be competitors, while collaborators can find a sometimes delicate balance of cooperative synergies. Symmetric peer-effects networks represent environments in which individual pairs can be either competitive or cooperative, and at various extents. As will be shown, such networks carry with them a natural tendency for strategic information acquisition.

First, we show that marginal strategic values borne by players interacting under symmetric and moderate peer effects are positive. This holds regardless of other network details. When influences between player pairs balance with each other, revealing and even exaggerating one's signal quality (if this were feasible) unambiguously increases private payoff. Remarkably, both positive and negative links reinforce this effect.

Secondly, we show that in these environments, the equilibrium response to the network of peer effects is *weak* relative to the utilitarian benchmark. This is manifested as an inefficient dispersion in $\mathbf{e}^*$. When both positive and negative links are present, this can imply that the most informed players under acquire information while the least informed players over acquire.

Formally, we consider the following family of network structures.

Assumption 2A. *Σ is symmetric: $\sigma_{ij} = \sigma_{ji}$ for each $i \neq j$.*

Taking (16) under the symmetry of Assumption 2A, each i's marginal strategic value positively scales with her *sum-of-squared* degree: $\sum_k \sigma_{ik}^2$. As such, $\xi_i^{st}\left(\mathbf{e}^*, \mathbf{X}^*\right)$ is strictly positive in symmetric, connected networks under moderate peer effects. Both positive and negative links reinforce the size of i's marginal strategic value to information. And with (17), each i's marginal externalities positively scale with her in-degree, which under Assumption 2A equates with her out-degree. With these measures taking on clear directions under symmetric, moderately-sized peer effects, the following can be shown.

Proposition 3A. [symmetric, moderate peer effects] *For symmetric Σ, there exists some γ^w with $0 < \gamma^w \leq \min\{\gamma^m, \gamma^s\}$ such that if $\gamma \in (0, \gamma^w)$ and for $\mathbf{e}^*$, $\mathbf{e}^{pb}$ and $\mathbf{e}^{pl}$ we have*[19]*:*

1. *$\mathbf{e}^{pb} > \mathbf{e}^*$, with $({e_i}^{pb} - {e_i}^*) > ({e_j}^{pb} - {e_j}^*) > 0$ for any i and j with $\sum_{k \neq i} \sigma_{ik}^2 > \sum_{k \neq j} \sigma_{jk}^2$,*
2. *$e_i^* > e_j^*$ and $({e_i}^{pl} - {e_i}^*) > ({e_j}^{pl} - {e_j}^*)$ for each i and j with $\sum_{k \neq i} \sigma_{ik} > \sum_{k \neq j} \sigma_{jk}$.*

From 1., players are *disincentivized* to acquire information as a result of incentive compatibility constraints. If players' could convincingly persuade others of their first-stage actions, they would always exaggerate their informativeness. The relative strength of this incentive scales with each player's sum-of-squared degree. With 2., the planner's optimal deviation from IAE $\mathbf{e}^*$ entails increases to signal qualities to higher degree players that are no less than increases prescribed to players with lesser degree. With equilibrium qualities similarly ordered according to player degree (for moderate peer effects), the *asymmetry in acquired information qualities are inefficiently low* as a result of marginal externalities. That is, the players' equilibrium information qualities are "*bunched*". And if all links in the network of peer effects are non-negative (non-positive), then $\xi_i^{ex}(\mathbf{e}^*, \mathbf{X}^*)$ will be non-negative (non-positive) with the most informed players imposing the greatest externalities. When both strategic complements and substitutes exist in the network, the ordering provided in Proposition 3A.2 establishes the more general result.

The economic interpretation of parts 1. and 2. in Proposition 3A are more broadly described as follows. For part 1., consider a player i with both positive and negative links with other players. If i could publicly acquire additional quality, this would encourage the responsiveness of positively linked neighbors, and simultaneously discourage the responsiveness of her negatively linked neighbors. Such directed influences are precisely due to the correlation in signals: learning that ω is likely high also informs i's neighbors that i likely observes similar information and will respond accordingly. These directional influences strictly work in i's favor regardless of the sign of her link with j. The symmetry in each pair's relationship implies a clear direction in these incentives. Thus, a player's *connectedness* in a symmetric network determines the size of the marginal strategic value to her information.[20]

With part 2., the network of peer effects can more broadly be interpreted as simultaneously quantifying the sizes and directions of externalities in the economy (in-links), as well as the sizes and signs of network effects imposed on each player (out-links). Externalities and network effects balance in symmetric networks. Thus, those that respond most positively to the network –through their information investments– are precisely those that endow the most value upon others from acquiring their information. And those that respond most negatively are precisely those that impose the most negative externalities upon others. Thus with respect to the utilitarian benchmark, players collectively under respond to a symmetric network of peer effects.

All of the above equilibrium properties are illustrated with the following example.

[19] For 1., we assume Σ to have no isolated players: $\sigma_{ij} \neq 0$ for some j for every i.

[20] Hauk and Hurkens (2001) [33] obtain a similar under acquisition in homogenous Cournot markets. In the network setting, a player i's connectedness –sum-of-squared degree– scales the size of her under acquisition arising from the privacy of e_i.

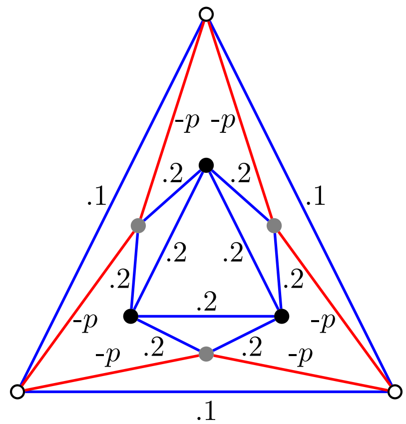

Figure 4: [Example 3] a network with three classes of players. Solid nodes give class x, gray nodes give class y, white nodes give class z.

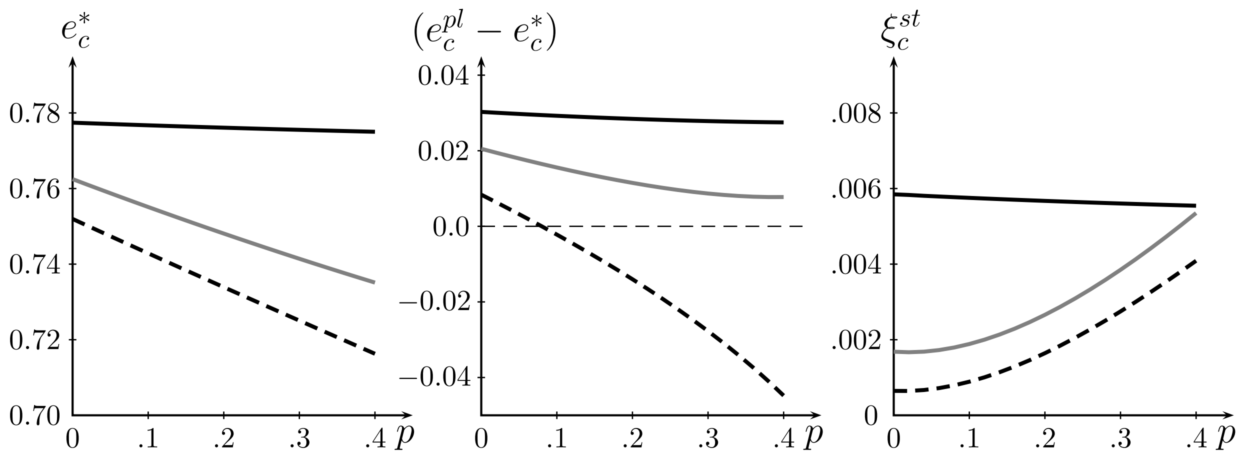

Figure 5: [Example 3] **Left**: equilibrium qualities. **Middle**: absolute welfare difference. **Right**: marginal strategic value. **All**: solid lines give class x, gray lines give class y, dashed lines give class z.

Example 3. *Take the network structure given in Figure 4, having three classes of players comprised of the center triad (class "x"), outer triad (class "z"), and three players bridging the two triads (class "y"). A general definition of player classes is provided in Supplemental Section S. Here, γ is set to* 1.

Taking the cost function from Example 2 with $\eta_1 = 2$, $\eta_2 = 1$, and $K = .1$, we consider the unique equilibria symmetric across players within each class. Equilibrium qualities e_c^*, differences $(e_c^{pl} - e_c^*)$, and marginal strategic value $\xi_c^{st}(\mathbf{e}^*, \mathbf{X}^*)$ are provided in Figure 5 for each class $c \in \{x, y, z\}$ over a range of p values. At $p = 0$ peer effects include only complements. Accordingly, externalities remain positive for all classes over a range of small p. As competition between classes y and z heightens, class z's $(e_z^{pl} - e_z^*)$ drops below zero. Marginal strategic value, on the other hand, unambiguously rises for classes y and z as these players place additional weight on each other.

With negative links representing inter-player competition, the incentives of low informational centrality players to distort the beliefs of more central neighbors –as to discourage their information responses– only heighten with great inter-class competition. While marginal externalities derive the majority of the marginal public value to e_i, marginal strategic value continues to capture and describe the incentives to distort beliefs. If strategic substitutes are significant for some players, the miss-orientation between the planner's and these players' preferences magnifies with greater competition.

4.2 Network asymmetries and welfare

Here we further explore the ramifications of network symmetry. We first consider analogues of the above results in networks with anti-symmetric pairwise peer effects. These *anti-symmetric* networks provide the opposite extreme to symmetric networks. As illustrated with the application to financial markets in liquidity crises of Section 5.1, such anti-symmetry in pairwise relationships may pervade a market when traders face asymmetric constraints in the second stage.

Formally, consider the following condition on Σ:

Assumption 2B. *Σ is anti-symmetric: $\sigma_{ij} = -\sigma_{ji}$ for each $i \neq j$.*

That is, for each peer effect the opposite-pointing effect gives the opposite-signed relationship. We refer to these pairwise relationships as *anti-symmetric*. Here, the natural analogue to Proposition 3A obtains.

Proposition 3B. [anti-symmetric, moderate peer effects] *For anti-symmetric Σ, there exists some γ^w with $0 < \gamma^w \leq \min\{\gamma^m, \gamma^s\}$ such that if $\gamma \in (0, \gamma^w)$ and for $\mathbf{e}^*$, $\mathbf{e}^{pb}$ and $\mathbf{e}^{pl}$ we have:*

1. *$\mathbf{e}^{pb} < \mathbf{e}^*$, with $0 < ({e_i}^{pb} - {e_i}^*) < ({e_j}^{pb} - {e_j}^*)$ for any i and j with $\sum_{k \neq i} \sigma_{ik}^2 > \sum_{k \neq j} \sigma_{jk}^2$,*
2. *$e_i^* > e_j^*$ and $({e_i}^{pl} - {e_i}^*) < ({e_j}^{pl} - {e_j}^*)$ for each i and j with $\sum_{k \neq i} \sigma_{ik} > \sum_{k \neq j} \sigma_{jk}$.*

In this setting, players face opposite strategic incentives. They now face the incentives to *understate* their informativeness: to *"play dumb"*. IAE now exhibit over-dispersion under moderate peer effects. In these networks, the most informed players will tend to over acquires while the least informed players will under acquire.

But, what if the network is neither purely symmetric nor anti-symmetric? With the strategic use of information taking extremes under symmetric and anti-symmetric networks, their manifestation in networks with both symmetric and anti-symmetric relationships may be less pronounced. The following example explores this more general setting.

Example 4. *First consider the two-player directed network where player 1 faces strategic substitutes in 2's action, $\sigma_{12} = -p < 0$, while player 2 faces strategic complements in 1's action of equal size, $\sigma_{21} = p$. Then, one can derive an exact expression for marginal strategic values:*

$$\xi_i^{st}\left(\mathbf{e}^*, \mathbf{X}^*\right) = -\gamma^4 2 \frac{\beta_i^{2*}}{e_i^*} p^2 e_1^{*2} e_2^{*2},$$

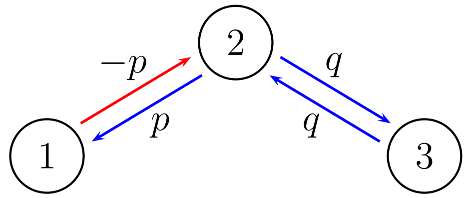

Figure 6: [Example 4] an asymmetric network

for $i = 1, 2$. That is, both players face the incentive to understate their information investment, in accordance with Proposition 3B.1. Precisely, player 1 has the incentive to understate her quality as to encourage 2's information investment. On the other hand, player 2 faces a similar incentive, but rather in order to discourage player 1's information investment.

Now consider the extended network in which player 2 is positively and symmetrically influenced by a player 3: $\sigma_{23} = \sigma_{32} = q > 0$. The structure of peer effects is offered in Figure 6. One can similarly derive:

$$\begin{aligned}\xi_1^{st}(\mathbf{e}^*, \mathbf{X}^*) &= -\gamma^4 2\beta_1^{2*} p^2 e_1^* e_2^{*2}, \\ \xi_2^{st}(\mathbf{e}^*, \mathbf{X}^*) &= \gamma^4 2\beta_2^{2*} e_2^* \left(q^2 e_3^{*2} - p^2 e_1^{*2}\right), \\ \xi_3^{st}(\mathbf{e}^*, \mathbf{X}^*) &= \gamma^4 2\beta_3^{2*} q^2 e_3^* e_2^{*2}.\end{aligned}$$

Thus, player 2 may no longer face significant marginal strategic value to her acquired information if $q^2 e_3^{*2} \approx p^2 e_1^{*2}$ in IAE $\mathbf{e}^*$.

We see that environments that couple symmetric and anti-symmetric relationships carry ambiguous strategic motives. When positive strategic values induced by symmetric relationships counterbalance negative strategic values induced by asymmetric relationships, players may be left without a unidirectional motive to influence others' beliefs. The private investment of information simultaneously imposes positive and negative strategic motives behind information acquisition. The net result is left as a function of each particular player's position in the networks of directed peer effects.

Next we address welfare implications when peer effects are large, incorporating negative information responses and multiple equilibria.

4.3 General peer effects and welfare

This section extends our welfare analysis to include more significant peer effects, and incorporates the potential for negative signal responses and multiple equilibria. As we will see, the observed U-shaped non-monotonicity in the incentives to invest in information carries over to externalities. As suggested throughout the preceding sections, the essential structural property driving the direction of the utilitarian optimum will be the extent of symmetry or anti-symmetry in pairwise relationships. We continue by taking Assumptions 2A and 2B as extremal benchmarks to pairwise symmetry and anti-symmetry (resp.) through the network. While clearly most real-world networks may not align exactly with one of these two cases, the following welfare properties can be applied by considering the extent of symmetry at a local level for sub-components of an observed peer-effects network.

To derive Lemma 1, Appendix A.3 takes the geometric expansions of the closed forms of $\boldsymbol{\xi}^{st}$ and $\boldsymbol{\xi}^{ex}$, respectively. Then taking their leading terms –which dominate their respective sums for small γ– the limits (16) and (17) are established. While affording formal proofs of Propositions 3A and 3B under moderate peer effects, these leading terms remain useful in assessing the directions of informational externalities and strategic values in the network. As derived in Appendix A.3, the approximations to $\boldsymbol{\xi}^{st}$ and $\boldsymbol{\xi}^{ex}$ for symmetric networks are given as:

$$\xi_i^{st}(\mathbf{e}^*, \mathbf{X}^*) \approx 2\frac{\beta_i^{*2}}{e_i^*}\gamma^4 \sum_{k \neq i} e_i^{*2}\sigma_{ik}^2 e_k^{*2}, \tag{18}$$

$$\xi_i^{ex}(\mathbf{e}^*, \mathbf{X}^*) \approx 2\frac{\beta_i^*}{e_i^*}\left(\beta_i^* - e_i^*\right), \tag{19}$$

for each i. And for anti-symmetric networks the negations of these corresponding approximations obtain.

In symmetric networks, we see that (18) is strictly positive for $\beta_i^* \neq 0$, consistent with Proposition 3A.1. We can assess (19), on the other hand, using Figure 7(a). In the top panel e_i^* is graphed against β_i^*. The exact form of this relationship is implicitly defined with expression (6) of Theorem 2. For any given Σ and in any IAE $\mathbf{e}^*$, the players will be spread across the domain at various points, yielding each i's e_i^*. Below this, the approximation (19) is plotted. With the exact form of marginal externalities scaling with signal response β_i^*/e_i^*, these marginal costs always pass through the origin. When $\beta_i^* = e_i^* = e^\dagger$, (19) again obtains a value of zero.

Thus, we obtain a *reversal* in the sign of marginal externalities when players move against their information. Non-monotonicity in the private value of information extends to the public value of information. For $\beta_i^* < 0$, the second-stage optimality of i's negative response implies that the value she derives from strategically moving against her signal outweighs the value from inferring and responding with her expectation of $\tilde{\omega}_i$. This is precisely because in IRE $\mathbf{X}^*$, the network imposes significant cost to i if she moves in the direction of her information. In symmetric networks and when $\beta_i^* < 0$, this cost translates to value imparted to i's competitors: to each j with $\sigma_{ji} < 0$ *and* $\beta_j^* > 0$. And with i failing to internalize this positive externality, she *under* acquires information relative to the efficient benchmark.

This reversal in the direction of the utilitarian solution relative to e_i^* can be illustrated with Example 2. p again gives the size of the negative links connecting the center player 1 to the peripheral players $\{2, 3, 4\}$. For p values below 0.111 player 1 moves in the direction of her signal realization, for values between 0.111 and 0.183 she acquires no information, and for values above 0.183 she moves against her signal in anticipation of the periphery's second-stage actions.

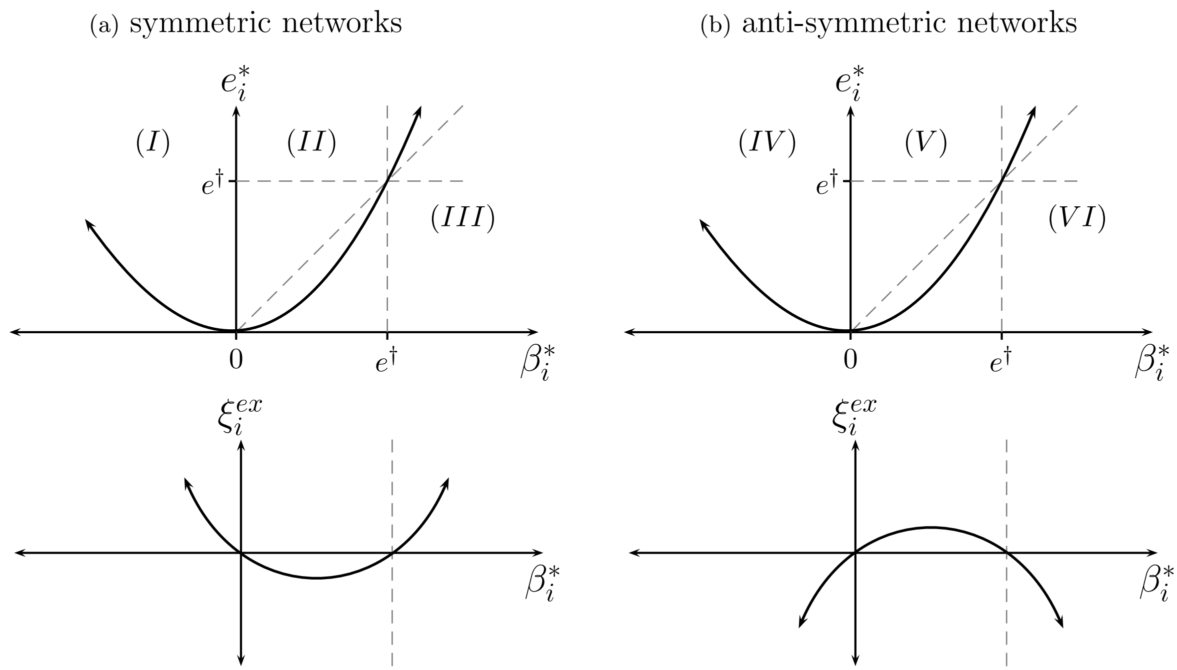

Figure 7: [Directional inefficiencies] leading terms of marginal externalities.

Figure 8 provides the planner's solution e_1^{pl} for the center (dashed line) along with IAE e_1^*. Below $p = 0.111$ player 1 over acquires information while facing positive marginal strategic values, as consistent with Proposition 3A. Internalizing marginal externalities on the periphery (as well as 1's marginal strategic values), the planner sends i's quality to zero *early*. Then for $p > .145$, $(e_1^{pl} - e_1^*)$ becomes positive with the planner setting β_1^* to be negative. Thus, player 1 under acquires information, and moves against her signal *late*. When player 1 finally starts moving against her signal (for $p > 0.183$) the gap between the planner's solution e_1^{pl} and e_1^* drops. Thus, the reversal in ξ_1^{ex} as β_1^* crosses the origin translates to a leftward horizontal shift in e_1^{pl}. While the corresponding figures for the periphery are omitted, $(e_{per.}^{pl} - e_{per.}^*)$ and $\xi_{per.}^{st}$ remain strictly positive and vary only mildly over the range of p values shown.

The economic message is noteworthy. When players acquire and move against their information in symmetric networks, the direction of this strategic behavior is socially efficient from a utilitarian perspective. But, the equilibrium extent to which these players invest in information is inefficiently *low*. The periphery now benefits from 1's negative response, and is only further encouraged to respond positively to their own private information. The value that such players create for others by acquiring and moving against their information is, once again, not internalized in equilibrium.

Returning to (18) and Figure 7(a), if ξ_i^{ex}'s leading term plays a dominant role in its sum, the exact form will shadow the depicted quadratic form. Inclusion of second order terms, or of marginal strategic values giving ξ_i –both of which will be positive away from the origin– give a more accurate approximation to the gradient of the utilitarian function. This higher-order approximation will (i)

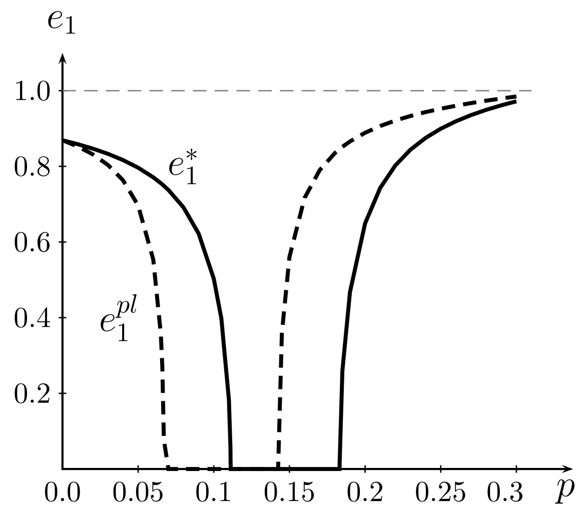

Figure 8: [Example 2] welfare inefficiency of player 1 equilibrium signal quality

continue to cross the origin, with higher-order terms also scaled by β_i^*/e_i, and (ii) cross the β_i^*-axis (again) at some point to the left of $\beta_i^* = e^\dagger$.

The exact point at which ξ_i crosses the β_i^*-axis to the right of the origin designates the lower bound of the set of players that exhibit positive margin public values to e_i^*, while setting $\beta_i^* > 0$. This includes all i that set $\beta_i^* > e^\dagger$: region (III) in the figure. Players setting $\beta_i^* \in (0, e_i^*)$ in region (II) face negative marginal externalities up to some β_i^* left of $e^\dagger$. Finally, for players moving against their information, for $\beta_i^* < 0$ giving region (I), marginal externalities once again switch positive.[21]

For the hypothetical "*knife-edge*" player that sets $\beta_i^* = e_i^* = e^\dagger$, such an i must satisfy the equilibrium condition: $\sum_{k \neq i} \sigma_{ik} e_k^* \beta_k^* = 0$. That is, the sum of i's inferred network effects –the weighted sum of expected neighbors' responses– equals zero. Responding as an informed player within the network, such an i continues to use her information to infer the actions of neighbors. However, on net, i's incentives to strategically respond by adjusting her signal response upward or downward from $e^\dagger$ *perfectly* balance. From the outside observer, i behaves as though she acts in isolation. But in actuality, the net influence that the network imposes on her behavior equates with zero. And given the symmetry of the network, so must her total externality imposed on others. We term such an i the "*neutral player*".

Figure 7(b) provides the corresponding functions under an anti-symmetric network (Assumption 2B). While the equilibrium relationship providing e_i^* as a function of β_i^* remains unaltered, the corresponding approximations to marginal externalities and marginal strategic values invert. Now, players face negative marginal strategic values. The resulting influence –either up or down– on others' incentives to acquire information from understating their informativeness always works in their favor. In region (IV) we now see players that significantly move against their information

[21]While marginal externalities and marginal strategic values are zero at $\beta_i^* = 0$, we see from Figure 8 that the planner's solution can depart from zero information. Though the gradient of the utilitarian function is fixed at zero at the origin, this does not imply that the planner and IAE solutions align: $\beta_i^* = 0$ may give an inflection point to the planner's objective.

imposing negative externality on the network. Their strategic behavior only reduces the incentives of more central players to acquire information. Players in region (V), moving in the direction of their signals but less so than the neutral player, under acquire information. The very peer influences that induce them to respond less to their information add value, on net, to the network. Those in regions (VI), to the right of the neutral player, face additional incentive to acquire information, which translates to negative marginal externalities.

4.4 Policy design

A number of policies could be implemented that nudge the economy in the direction of an efficient outcome. A tax and transfer policy gives an invasive but effective approach. If ξ_i is negative for at least some i and positive for others, a revenue neutral plan taxing the information investments of the former while subsidizing the latter could be at least partially effective. When all links are non-positive or non-negative, subsidy-only and tax-only plans, respectively, would be required.

A less invasive policy geared toward acquisition transparency provides an alternative design. Public certification of the information investments of targeted individuals give one example. Centralized verification and publication of research, or policies that physically display the efforts of individuals within the network give others. All of these examples involve targeting selected positions within the information-response game.

The preceding section suggests a more descriptive design for each of these policy types. Let us focus on symmetric networks, leaving the natural analogue for anti-symmetric networks. For tax and transfer policies, players moving against their information or who set $\beta_i^* > e^\dagger$ should be incentivized (subsidized) to acquire additional information, while those with $\beta_i^* \in (0, e^\dagger)$ should be discouraged (taxed). If links are non-negative and the network resides in region (III), then a natural policy multiplier is realized. Each dollar publicly offered to encourage the acquisition activities of highly central players in the information-response game feeds through to influence the acquisition activities of less central players. Upon introducing negative peer effects as well, such an incentive scheme continues to feed through to others' incentives. Less central players exhibiting $\xi_i < 0$ will be discouraged from acquiring information: an aggregate welfare enhancing effect. And conversely, taxing the acquisition activities of these low-centrality players will tend to encourage the acquisitions of the most central players.

For policies enhancing first-stage transparency, players with $\beta_i^* > e^\dagger$ or $\beta_i^* < 0$ should be targeted. Under only positive links or when negative links are also present, such policies again exhibit a natural multiplier. The incorporation of marginal strategic values into the objectives of the targeted players further encourages others in regions (I) and (III), and discourages those in region (II). In both Figures 7(a) and 7(b), we see a preservation of the property that players to the left of the origin and right of the neutral player tend to exhibit marginal strategic values that are aligned with their marginal externalities, while for those just right of the origin these measures miss-align. Thus, increasing transparency of players with responses outside of the interval $(0, e^\dagger)$ remains a robust and simple rule-of-thumb for these designs, regardless of the extent of symmetry or anti-symmetry in pairwise peer effects.[22]

Certification-based policies will be most feasible in symmetric networks for the following reason. Implementation for players facing positive marginal strategic values requires only a one time certification of their information investments. For those facing negative marginal strategic values (i.e. anti-symmetric peer effects), nothing prevents these players from acquiring additional information subsequent to certification. With other players rationally anticipating this behavior, one-time certifications in anti-symmetric networks may be unimplementable.[23]

A few empirical challenges must also be addressed in any of these designs. First, unless data on information responses in the market can be obtained, retrieval of the peer effects network Σ will be necessary in order to derive equilibrium $\boldsymbol{\beta}^*$. Further, understanding of the costs of information κ is needed to elicit the value of $e^\dagger$. Players may also face their own idiosyncratic information costs in real-world peer-effects environments. Section 5.2 addresses extensions that incorporate heterogeneous information costs.

5 Discussion

In this section we explore applications to financial markets in crises and to two-sided markets. Then, basic extensions of the model incorporating further heterogeneity across players' preferences are developed. The model's broader relation to the literature is discussed before concluding.

5.1 Applications

MARKET EFFICIENCY and CRISES. Here we apply the above setup to financial markets and crises. The above welfare properties are cast against the Efficient Market Hypothesis, and applied toward equilibrium information acquisition during financial crises. For the latter, this will require a mixture of both symmetric and anti-symmetric pairwise peer effects.

First consider the following stylized model of a competitive, liquidity-flush market. N traders comprise nontrivial shares of a market for a risky asset. The market price in the second stage is an increasing function of the total of their chosen holdings: $\phi(\bar{x}) = A + B\bar{x}$, where x_i gives i's holding of the asset, $\bar{x} := \sum_{i=1}^N x_i$, and $A, B > 0$. Then, as a function of the asset's fundamental value ω, each i's payoff is given by:

$$\begin{aligned} u_i(\mathbf{x}|\omega) &= (\omega + p_i\phi(\bar{x}))\, x_i - x_i^2 \\ &= (p_iA + \omega)\, x_i + (p_iB - 1)\, x_i^2 + x_ip_iB \sum_{k\neq i} x_k, \end{aligned} \tag{20}$$

where here we set $p_i < 0$ capturing a downward sloping demand from each trader. Then, the $t = 2$ expectation $\mathbb{E}_i[\omega + p_i\phi(\bar{x})|\theta_i, e_i]$ in i's best response (2) captures her long-term expected

[22] An even more precise design to the above proposals would target players with positive marginal public values to their information, $\xi_i > 0$, which includes some players in region (II). However, with second-order effects and marginal strategic values shifting the intercept to the left, targeting all i with $\beta_i^* > e^\dagger$ can be taken as a conservative design.

[23] When feasible, continuous monitoring of players below 0 and above $e^\dagger$ could insure policy compliance.

return to her investment, a decreasing function of the expected market price at which assets are purchased. The quadratic term $-x_i^2$ captures decreasing returns to holding inventory, derived from the opportunity costs of funds.

The market price $\phi(\bar{x})$, which here traders do not condition on when choosing second-stage actions, is meant to capture the strategic value that players derive from private information in the market. We can think of each trader i's final holding x_i to be realized by i placing some market order (buy or sell) in the second stage, without complete knowledge of the transaction price ultimately realized.[24] As seen in (20), the sensitivity of the asset's price to others' demands scales by B, which will depend on the total size of the N traders in proportion to the broader market. The larger is B and p_i (in size) the more i cares about the short-term demands of the other $N-1$ traders in the market. And the larger the size of $p_iB < 0$, the more i will strategically avoid highly demanded assets. Thus in reduced form, this stylized setup captures the strategic uses of private information in financial markets under monopolistic competition.

The application can be placed against the Efficiency Market Hypothesis, as follows. As first characterized by Grossman and Stiglitz (1980) [30], when prices are observed and used to infer the private information acquired by others, the asset's price can not both fully and rationally reveal all information of the asset's underlying value.[25] Precisely, if costly private information is fully transmitted through observation of the asset's market-clearing price, then the ex ante incentives to acquire the information are compromised. Here, through the strategic use of information, the shear presence of competing traders similarly reduces the incentives to acquire private information. This is now due to the inference of others' observations and equilibrium actions: privately observing that the asset has high long-term value also informs traders of high short-term market prices. The traders continue to crowd each other as they compete for valuable assets.

The application elicits an important distinction between the informational efficiency versus the welfare efficiency of the market. While the incentives to acquire information display strategic substitutes, the extent of crowding out that ensues is *inefficient*. With each peer effect taking a negative value, each i will obtain $\xi_i < 0$ with $e_i^* > e^{pl}$. In other words, the market will reside in region (II) of Figure 7(a). The informational *in*efficiency of the market is *efficient* from a utilitarian perspective, but to an inefficient extent. In other words, the traders over exert themselves in competition as they research the asset's long-term value.[26]

We can further apply the model to yield similar statements on inefficiencies during financial

[24] This is akin to Kyle (1984a) [41] and (1985) [42], where an insider's market order is a function only of the asset's value and not the market-clearing price. In rational-expectations equilibrium, Kyle (1989) [43] allows traders to submit demand schedules over market prices. The strategic use of information comes in the form of inference of *market depth*: each informed trader i submits her demand schedule given her information, inferring (i) the private observations of other informed traders, and thus (ii) their submitted demand schedules and the extent of noise traders in the market, and ultimately (iii) the sensitivity of the asset's price to i's demand.

[25] Hellwig and Veldkamp (2009) [34] also highlight a similar kinship with Grossman and Stiglitz (1980) [30].

[26] Sanford Grossman and Joseph Stiglitz [30] close with an open question of '*whether it is socially optimal to have 'informationally efficient markets*',". The above model thus provides one answer, and that is "*no*". When price discovery is introduced, complementarity in information acquisition may also arise, pushing in the opposite direction of over acquisition while reinforcing the under acquisition during crises by all N traders, as described below.

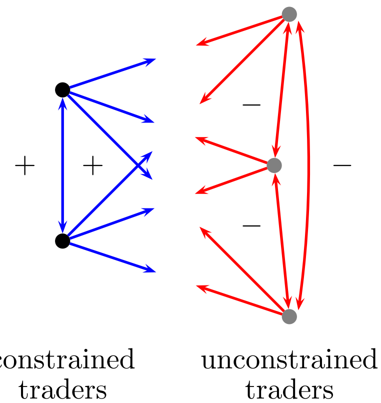

Figure 9: Market with liquidity-constrained traders

crises. Consider some subset of the traders undergoing severe funding constraints. Precisely, while these traders carry asset holdings prior to the second stage, their abilities to retain their inventories will depend on the market price $\phi(\bar{x})$. If liquidity is sufficiently thin amongst these traders, liquidity spirals may ensue.

Brunnermeier and Pedersen (2009) [11] provide a theoretical framework of liquidity spirals during crises. Their model captures the dynamic interdependence of market prices and traders' funding constraints. With an initial fall in the asset's perceived fundamental value, speculators' funding constraints force liquidity-starved traders to sell off inventory. As the market price drops, margin calls force these traders to further liquidate, causing a further drop in the asset's market price. This only further constrains the traders, and the spiral worsens.[27]

In effect, these severely constrained traders' demand functions exhibit an upward sloping form.[28] As a reduced representation, we capture this by setting $p_i > 0$ for each of these traders. How exactly would the market look? Figure 9 illustrates the network architecture for the N traders. Liquidity constrained traders, facing positive and directed peer effects, will be the most central players in the information-response game.[29] With respect to Figure 7, unconstrained traders will lie to the left of the neutral player, while highly constrained traders will lie to the right.

The stakes are high for constrained traders. If ω is high, this implies both that the traders can expect a large returns on their holdings, but more importantly, that the current market price will remain high. This is crucial, as the availability of market liquidity is necessary for them to maintain their holdings without the burden of funding constraints.[30]

[27]Here, the strategic component of information to constrained firms is even more evident as private information may allow them to forecast market prices and infer the potential for constraints to bind over the short term.

[28]See also Gennotte and Leland (1990) [26], Angeletos and Werning (2006) [2] and Gárleanu et al. (2014) [25] for models with inverted equilibrium demand functions of constrained traders in crises.

[29]In the language of Supplemental Section S, these firms' weighting functions lie strictly above those of unconstrained traders, and thus there will always exist an equilibrium in which they acquire more information. While marginal values to information may be higher for these traders, so too may their marginal costs if the opportunity costs of funds to these traders are large. This can be captured using idiosyncratic κ_i: see Section 5.2.

[30]One can either model information as directly acquired by the traders' funders, or by the banks but with signal

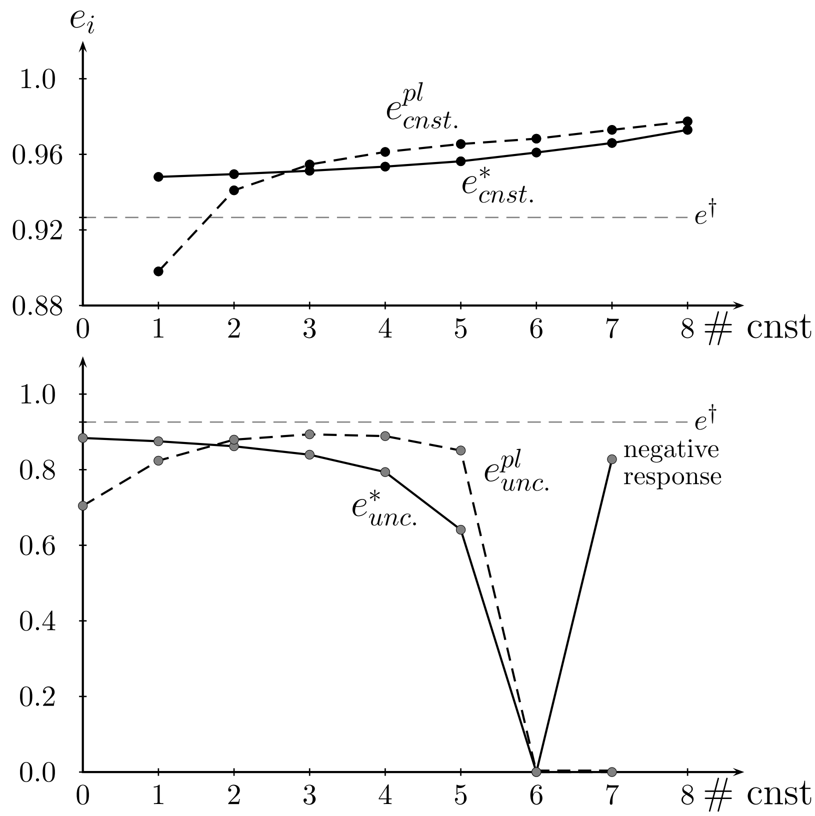

Figure 10: [Efficiency and liquidity crises] Unique equilibrium information qualities versus number of constrained traders (# cnst.) out of eight traders. All links are of size .1, $\eta_1 = 2$, $\eta_2 = 1$, and $K = .01$ for the cost function in Example 1, giving $e^{\dagger} = 0.927$.

As illustrated in Figure 10 for a market of eight traders, equilibrium welfare exhibits a paradigm shift as the extent of liquidity through the market declines. This shift is driven by the orientation (i.e. symmetry or anti-symmetry) in the local peer effects that each market participant faces. First, when most traders are unconstrained (i.e. "normal" times) the market takes on one similar to the competitive market described above. For the few constrained traders in the market, the majority of their relationships will be anti-symmetric. Residing in region (VI) of Figure 7(b), these traders will over invest in equilibrium. Responding intensely to the news of a high ω, their impact on the market price only crowds the market activities of unconstrained traders. Then, as the proportion of constrained traders grows, these traders face more symmetric and positive peer effects while unconstrained traders face more anti-symmetric relationships. When liquidity problems pervade the market, the constrained traders enter region (III) in Figure 7(a), with unconstrained traders moving to region (V) of Figure 7(b). When the number of constrained traders grows to three or more, *all* traders under invest in information. Those under significant funding constraints face positive externalities from the information investments of others. Their informativeness allows the constrained traders to coordinate on asset retention in high market-liquidity outcomes (i.e. high

realizations verifiable to the funders.

$\phi(\bar{x})$), which tend to occur when the asset is "good" (i.e. high ω).

A striking set of equilibrium behaviors arise among the few unconstrained traders during a crisis. Their acquisition activities impose positive externalities on constrained traders. Moving with their information in region (V) in Figure 7(b), unconstrained traders' informativeness further aids constrained traders to coordinate on high market-liquidity outcomes. They thus under acquire in equilibrium. When six or seven constrained traders pervade the market, the few unconstrained traders acquire zero information in the planner's solution. When the number of constrained traders rises to seven, however, the lone unconstrained trader moves to region (IV) and finds additional value to acquiring information, inferring and moving against the actions of others in the market.

Observation. *As the extent of funding constraints across traders increases, the market equilibrium shifts from being over informed to under informed from a welfare perspective. Crucially, in severe liquidity crises as constrained traders attempt to coordinate on high market-liquidity outcomes, both constrained and unconstrained traders do not internalize the positive externalities their information imposes on the constrained side of the market. In extreme crises, unconstrained firms acquiring and moving against the market do so at a cost to aggregate welfare.*

One can also introduce additional network irregularity by applying this framework to over-the-counter markets. As in Babus and Kondor (2014), if a network designating feasible trades constrains the market, and with each bilateral transaction assigned its own clearing price, Figure 9 would take on a more incomplete network structure. Only trading pairs would be linked in the corresponding peer-effects network, with the sign of out-links determined by the extent of available liquidity to the corresponding trader. The above observation extends. Precisely, highly connected traders that are liquidity deprived may significantly over acquire information relative to the utilitarian benchmark *if* their neighbors are generally unconstrained. Traders that are unconstrained but have many constrained neighbors will, again, under acquire in equilibrium.

Finally, the above policy discussion applies to the application as follows. Competition amongst firms in normal times suggests that certification-based policies may be unimplementable. During crises, however, constrained banks face positive strategic values. Stress tests, coupled with the certification of identified constrained firms, offers a simple and implementable policy intervention. Constrained banks' anticipation of being identified and certified encourages them to internalize their strategic use to information. As they acquire additional information, the market is pushed in the direction of the utilitarian solution.

With marginal strategic values to these firms scaling with their quality-weighted sum-of-squared degree (see expression (18) above), one can verify that such transparency-based policies will be most effective in incomplete network structures under large pairwise peer effects (e.g. in over-the-counter markets). In these networks, marginal strategic values can be sizable in proportion to marginal externalities.[31] With a few constrained neighbors imposing large positive externalities on each

[31] To formalize the statement in the context of Section 5.2, trader i will have large strategic values relative to externalities when $\gamma_i \sigma_{ij}$ is large relative to σ_{ii} for each neighbor j.

trader, and vice versa, the effect of internalized strategic values moves the market farther toward the utilitarian solution than in completely but weakly connected network structures.

TWO-SIDED NETWORKS. As an example of a two sided network, consider a job-search market with network structure depicted in Figure 11. Here, a pool of insiders, which may include head hunters or industry professionals, link to workers searching for a job. A particular firm to whom each insider has ties posts a number of open positions. The quality of the firm as an employer (culture, benefits, job security and growth, etc.) are captured by an unknown common state ω. At $t = 2$, each insider i exerts resources x_i toward filling the firm's vacancies with workers they know. Each worker j invests time and effort x_j tailoring their resumes to align with the firm's qualifications and formally applying to relevant positions through their acquainted insiders. The optimal second-stage actions of each player will depend on the expected quality of the firm as an employer, as well as the anticipated actions of neighbors. The workers compete with each other to fill job vacancies, while the insiders compete with each other to connect the workers with the firm and collect value in the form of commission or gained social capital.

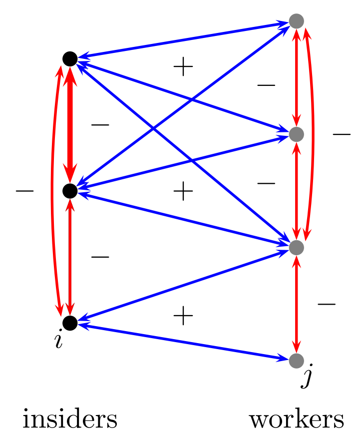

Figure 11: A job-search network

Abstracting away from variability in the size of counterpart links, the network will generally be symmetric with the welfare properties depicted in Figure 7(a) applying. If the insiders out number workers, facing more positive links across the two groups than negative links amongst other competing insiders, then they will generally reside in region (III). This places the workers in region (II). In this case, insiders face greater incentive to research the firm and will under acquire information in equilibrium. This is because they fail to internalize the positive externalities that their expertise provide their clientele. The less informed workers will over acquire information and over exert themselves researching .

Strategic substitutes within each side and complements across each side of the market introduce clear network irregularities. However, additional heterogeneity may exist across peer effects within either side of the market. As seen in Figure 11, insider i enjoys only two links with workers, which pushes against her centrality in the information-response game. However, i faces the luxury of being the only insider connected to worker j. On the other hand, while the other two insiders enjoy high connectivity with workers, they face stiff competition between each other as their clienteles highly overlap. More generally, the insiders most central in the information-response game will be those that strike an ideal balance between their connectedness (i.e. degree) and the centrality of the workers they connect with (here, client exclusivity).[32]

While this example lends itself well to job-search networks, an array of two-sided markets should adopt similar welfare properties. Entrepreneurs and venture capital investing in new platforms or technologies, Hospitals and pharmaceutical sales firms investing in new medicines or medical technologies, or any other two-sided market in which all players acquire information regarding a fundamental common state will apply.

Observation. *Two-sided markets in which the shorter side of competing insiders matches competing workers with value-creating transactions exhibit under acquisition amongst experts and over acquisition amongst workers. Experts fail to internalize the positive externalities that their information impose on workers, and workers fail to internalize the negative externalities their information impose on other workers.*

5.2 Basic Extensions

A number of generalizations to the basic model can be considered. As suggested by footnote 6, setting $\sigma_{ii} = \sigma_{jj} = 1$ comes with loss in generality in the degree of variation in players' payoffs. To account for idiosyncrasies in this variation, one can rather define $\tilde{\omega}_i = \gamma_i \omega + \iota_i \omega_i$ for $\gamma_i, \iota_i \in \mathbb{R}_+$, and take the ex post payoff structure:

$$u_i(\mathbf{x}|\omega, \omega_i) = (a_i + \tilde{\omega}_i) x_i - \frac{1}{2}\sigma_{ii} x_i^2 + \rho \sum_{j \neq i} \sigma_{ij} x_i x_j,$$

where $\rho \in \mathbb{R}_+$ directly scales the size of peer effects. The corresponding second-stage linear best response is:

$$BR_i(\mathbf{X}_{-i}|\theta_i, e_i, \mu_i) = \frac{a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i, e_i]}{\sigma_{ii}} + \rho \sum_{j \neq i} \frac{\sigma_{ij}}{\sigma_{ii}} \mathbb{E}_i[X_j(\theta_j|e_j)|\theta_i, e_i, \mu_i].$$

Such a generalization comes with only two necessary modifications to the model's primitives, made to conditions E2 and E4 to give $\mathbb{E}_i[\tilde{\omega}_i|\theta_i, e_i] = v_i e_i \theta_i$ and $\mathbb{E}_i[\theta_j|\theta_i, \mu_i, e_i] = \int_{[0,1]} \mu_{ij}(e_j) \gamma_j e_j \gamma_i e_i \theta_i de_j$, respectively, where $v_i := \sqrt{\gamma_i^2 + \iota_i^2}$ gives the variance in i's relevant state $\tilde{\omega}_i$.

Inline with these generalizations, an updated correlation-adjusted adjacency matrix $\Sigma^c := [\gamma_i e_i \sigma_{ij} \gamma_j e_j]$ can be defined. With v_i scaling i's interim expectation of $\tilde{\omega}_i$, informational centralities are now further weighted by the extent of variation in players' relevant states:

[32] Supplemental Section S discusses this further.

$$\beta^* = (\mathbf{I} - \Sigma^c)^{-1}\, \mathbf{I_v e}.$$

The analogue to network symmetry incorporates an adjustment to peer effects:

Assumption 2C. $\mathbf{I}_{\boldsymbol{\sigma}}^{-1}\mathbf{I}_{\boldsymbol{\gamma}}\Sigma\mathbf{I}_{\boldsymbol{\gamma}}$ *where* $\boldsymbol{\sigma} := [\sigma_{ii}]$ *is symmetric:* $\frac{\gamma_i}{\sigma_{ii}}\sigma_{ij} = \frac{\gamma_j}{\sigma_{jj}}\sigma_{ji}$ *for each* $i \neq j$*.*

This generalization of Assumption 2A comes with a natural interpretation. Players with low σ_{ii} possess relatively high propensities to choose high actions in the second stage, on average, as well as to acquire information and respond highly to their signal realizations, ceteris paribus. These are exactly the players that have significant influence in the information-response game. Thus, Assumption 2C requires that these influential players have proportionally greater impact on the preferences of those with less influence.[33] The weighting by γ_i and γ_j adjusts for the loading that each player places on the shared state ω. That is, the players' impacts scale directly with the extent that their preferences correlate with others' preferences. The corresponding assumption for anti-symmetric networks can also be defined and applied in a similar way.

Finally, one is free to introduce further idiosyncrasies through the curvature of information costs by providing $\kappa_i(e_i)$ for each i. With these extensions, all of the above results are preserved. With all of these modifications, we obtain the identical expression to (6): $\beta_i^{*2}/e_i^* = \kappa_i'\left(e_i^*\right)$ for any interior e_i^*. Players that are most "*central*" in the information-response game are now those with the right combination of being (i) central in the updated network Σ^c, and (ii) having a natural propensity to acquire information, as determined by the relative sizes of $\sqrt{\gamma_i^2 + \iota_i^2}$ and σ_{ii} and the extent of convexity in κ_i.

Analogous limit results are easily obtained, with partials taken with respect to ρ rather than γ^2, and by sending $\rho \to^+ 0$. The corresponding expressions to (7), (16), and (17) are respectively:

$$\begin{aligned}
\lim_{\rho \to^+ 0} \frac{\partial e_i^*}{\partial \rho} &= \frac{\gamma_i e_i^\dagger}{\kappa_i'(e_i^\dagger) - 1} \sum_{k \neq i} \gamma_k e_k^\dagger \frac{\sigma_{ik}}{\sigma_{ii}}, \\
\lim_{\rho \to^+ 0} \frac{\partial \xi_i^{st}\left(\mathbf{e}^*, \mathbf{X}^*\right)}{\partial (\rho^2)} &= e_i^\dagger (\gamma_i e_i^\dagger)^2 \sum_{k \neq i} (\gamma_k e_k^\dagger)^2 \frac{\sigma_{ik}}{\sigma_{ii}} \frac{\sigma_{ki}}{\sigma_{kk}}, \quad \text{and} \\
\lim_{\rho \to^+ 0} \frac{\partial \xi_i^{ex}\left(\mathbf{e}^*, \mathbf{X}^*\right)}{\partial \rho} &= \gamma_i e_i^{\dagger 2} \sum_{k \neq i} \gamma_k e_k^\dagger \frac{\sigma_{ki}}{\sigma_{kk}}.
\end{aligned}$$

As one may anticipate Proposition 3A maintains, but with $\sum_{k \neq i} \gamma_k e_k^\dagger \frac{\sigma_{ik}}{\sigma_{ii}}$ defining each player i's *effective* degree for moderate peer effects:[34]

Proposition 3C. [symmetric, moderate peer effects] *For symmetric* Σ*, there exists some* γ^w *with* $0 < \gamma^w \leq \min\{\gamma^m, \gamma^s\}$ *such that if* $\gamma \in (0, \gamma^w)$ *and for* $\mathbf{e}^*$*,* $\mathbf{e}^{pb}$ *and* $\mathbf{e}^{pl}$ *we have:*

[33] Taking the inter-bank network as an example, The Bank of America's expected extent of information acquisition should carry proportionally greater influence on the preferences of smaller banks than do the information investments of these banks on the incentives of The Bank of America.

[34] The results of Supplemental Section S also maintain, with our notions of degree centrality and weighting function defined in terms of normalized peer effects $\frac{\gamma_i}{\sigma_{ii}}\sigma_{ij}$ for each i and j.

1. $\mathbf{e}^{pb} > \mathbf{e}^*$, *with* $(e_i{}^{pb} - e_i{}^*) > (e_j{}^{pb} - e_j{}^*) > 0$ *for any* i *and* j *with* $\sum_{k \neq i} \left(\gamma_k e_k^\dagger \frac{\sigma_{ik}}{\sigma_{ii}} \right)^2 > \sum_{k \neq j} \left(\gamma_k e_k^\dagger \frac{\sigma_{jk}}{\sigma_{jj}} \right)^2$,
2. $e_i^* > e_j^*$ *and* $(e_i{}^{pl} - e_i{}^*) > (e_j{}^{pl} - e_j{}^*)$ *for each* i *and* j *with* $\sum_{k \neq i} \gamma_k e_k^\dagger \frac{\sigma_{ik}}{\sigma_{ii}} > \sum_{k \neq j} \gamma_k e_k^\dagger \frac{\sigma_{jk}}{\sigma_{jj}}$.

And, the analogues to Assumption 2B and Proposition 3B that incorporate these extensions can similarly be constructed.

Crucially, the basic message of Figure 7 will continue to hold. Approximations to ξ_i^{st} and ξ_i^{ex} are derived with γ_i and $1/\sigma_{ii}$ scaling each peer effect σ_{ij}. Thus, the corresponding figures can be produced for each individual player. Under Assumption 1, exactly where each i falls on their β_i^*-axis relative to the origin and their respective $e_i^\dagger$ continues to be driven by the network of peer effects. In symmetric networks (now, Assumption 2C), those in their corresponding region (III) choosing β_i^* above $e_i^\dagger$ underinvest, those to the right of the origin falling in region (II) tend to overinvest, and players moving against their signals in region (I) again underinvest. All players to the right of the origin moving in the direction of their signal realizations continue to face positive marginal strategic values.

Thus, one can view the above model's homogenous setup in the first-stage information-acquisition game –outside of network effects– as simplifying the analysis, allowing the network to "speak clearly". None the less, the extent of symmetry in pairwise peer effects coupled with players' informational centralities continue to play crucial roles shaping equilibrium inefficiencies in a much broader set of economies.

5.3 Related Literature

Here related papers are discussed, along with a number of potential avenues for future research. The paper relates to a family of papers studying information games with communication on networks. In Calvó-Armengol and de Martí (2007) [13], (2009) [14], and Calvó-Armengol et al. (2009) [15] the network is defined by the exogenous correlation matrix between signals. In Calvó-Armengol et al. (2011) [16] this network is endogenized through a communications device, and the authors study the relative extent of active and passive communication in equilibrium (i.e. "speaking" and "listening", resp.). Thus, each player's communication quality is endogenously directed to each of her neighbors. Beyond the above setup's treatment of acquired information as pertaining directly to fundamental payoffs, these papers have a number of model elements that distinguish them from this one. As a closest comparative, Calvó-Armengol et al. (2011) [16]'s first-stage signal qualities are fully observed in the second stage. Thus, strategic values are fully internalized in their model. And as only strategic complements are considered in their formal analysis, the characterization of negative signal responses and corresponding welfare implications discussed above are not considered. The authors find underinvestment in communication, which relates with the under acquisition above when $\sigma_{ij} \geq 0$ for each i and j. In the above, however, states are global rather than local via common state ω, and thus inefficient acquisition under strategic compliments is driven by the correlation in and strategic use of information in the second stage. Further, players facing negative peer effects and with signal responses β_i^* in some interior subset of $(0, e^\dagger)$ over acquire in symmetric networks.

As discussed in the introduction, this paper is closely related to the coordination games with costly information acquisition literature. The above case of symmetric networks provides a generalization to many related results found in the literature, while incorporating strategic substitutes and complements simultaneously through the network. Hellwig and Veldkamp (2009) [34] study costly acquisition of signals chosen from a subset of available signals of various qualities and correlation profiles. The authors Proposition 1 offers a closest analogue to the above Theorems 1 and 2, which establish the feed through of strategic complementarity and substitutability (separately) into first-stage information values.[35] Also reminiscent of their findings, strategic complementarity can imply multiple symmetric equilibria. However, the type of multiplicity of equilibria illustrated above in Example 2 are derived solely from strategic complements, rather than discreteness in the signal technology. In contrast, equilibrium uniqueness under strategic complements and continuous signals is derived in Hellwig and Veldkamp (2009) [34] as well as Myatt and Wallace (2009) [49]. In beauty contest games with a continuum of agents, the extents of complementarities are implicitly bounded. In the above network setting, strategic complements can be more pronounced while the set of convex cost functions considered are less constrained,[36] thus yielding the observed multiplicity under significant positive peer effects.

This literature also offers an exciting research agenda studying the effects of public information on the equilibrium actions and welfare in a general network setting. Morris and Shin (2002) [47] first highlighted the potential adverse effects of public information, showing that players may coordinate on less precise public announcements. In an information acquisition setting, Colombo et al. (2014) [19][37] show that public information crowds out private information[38], while acquired private information is inefficiently low *if and only if* the equilibrium degree of coordination falls short of the efficient benchmark (see Colombo et al. (2014) [19] Corollary 1 and Proposition 5 (ii), resp.). In a network setting under both strategic complements and substitutes, the efficiency of equilibrium coordination depends on each player's informational centrality (e.g. Proposition 3A, above). The effects of public information on both the positive and normative properties of equilibrium information acquisition in these settings are left as open questions.

The above coordination games literature assumes agents to reside on a continuum, and thus the strategic values studied here are not realized. In a network setting, a continuum of players is clearly inapplicable. With the exception of Hauk and Hurkens (2001) for a competitive Cournot

[35] Vives (1988) [61] together with Vives (2008)'s [62] exercise 8.15 similarly establish this feed through for strategic substitutes and compliments, respectively.

[36] To see this, here qualities are chosen from $[0, 1]$ while in these and most of the coordination games literature they are taken from $[0, \infty)$. Appendix A.1 provides a mapping from the accumulated i.i.d. normal draws setup to information qualities. Note that a constant marginal cost to these draws excludes the possibility of initial positive gains as players search for and locate the most efficient sources of informative signals.

[37] They allow for both strategic complements and substitutes (though not simultaneously) in their setting. Their welfare benchmark that corresponds to that taken here involves *not* allowing the planner to enforce the efficient use of information.

[38] Myatt and Wallace (2009) [49] find a similar result, with the publicity of information endogenously determined.

production market, the welfare implication of incentive compatibility in information acquisition are novel. The extent of player *connectedness* in the network as driving the size of her strategic values provides a network characterization. Further, the symmetry in the coordination games with endogenous information that have thus far been studied plays an important part in driving inefficiencies in information acquisition. While symmetric networks offers analogous welfare results to many seen in these papers, the fact that the direction of the utilitarian solution inverts under anti-symmetric networks suggests caution when applying these welfare properties in settings that incorporate anti-symmetric relationships. And as the above application of Section 5 suggests, anti-symmetric relationships may be common in environments with a subset of constrained players.

A number of oligopoly models have studied information acquisition outside of a network setting. Novshek and Sonnenschein (1982) [52] and Vives (1983) [60] study the effects of private information when firms face an uncertain demand function. Taking the extent of information acquisition exogenously, the authors' consider comparative statics of equilibrium production and welfare with respect to signal qualities. Their Lemma 1 establishes a direct dependence in the slope of equilibrium strategies to signal quality, as a function of the extent of complementarity between firms' goods. Similar equilibrium properties obtain under the more general network treatment of Theorems 1 and 2 above, upon homogenizing the size and direction of links. Beyond this close similarity at a positive level, the papers' welfare analyses depart from each other with the consumer side of the market excluded in network games.

Related to transparency, a number of papers have addressed information transmission in a network settings similar to that taken here, but without endogenizing information qualities. Hagenbach and Koessler (2010) [31] and Galeotti et al. (2013) [24] study cheap talk in networks, taking exogenous biases as common information amongst the players. Kondor and Babus (2014) [3] study information diffusion and trade between traders connected through a network.[39] The authors define a "conditional guessing game", which solves for transmission of information in rational expectations, as a function of observed prices and the network structure. And taking an extreme to transparent play, Hagenbach et al. (2014) [32] study full disclosure under certifiable pre-play communication. In the above setting, these authors' acyclicity condition is satisfied.[40]

The above model's exclusion of information transmission provides a first benchmark to information acquisition in a network setting, while maintaining reach in its applications. Studying the incentives to acquire the information that agents carry when also faced with particular transmission mechanisms offers an exciting avenue for research. Both the positive and normative implications under full information disclosure offers a promising starting point.

Finally, Bramoulle et al. (2014) [9] study the set of network games equivalent to potential games. The authors characterize both the presence of multiple equilibria and of equilibria that involve players taking zero action (i.e. a corner solution in their setup) using the size of the lowest eigenvalue for the network's adjacency matrix. In the above, corner solutions in the information-

[39] That is, their network captures the set of feasible trades that can occur.

[40] Precisely, where the worst type in finite set $S_i \subseteq \Theta$ is given by the lowest element *if* $\beta_i^* > 0$, and the highest element if $\beta_i^* < 0$.

response game play an important role when incorporating the possibility of players moving against their signals, as illustrated with Example 2. Bramoulle et al. (2014) [9]'s eigenvalue characterization of corner equilibria provides a valuable tool to designate the presence of players moving against their information. Here, the second-stage game can be characterized as a potential game if the network is symmetric.[41]

5.4 Conclusion

A flexible framework for studying information acquisition in linear peer-effects networks is developed. An intuitive characterization of the equilibrium strategic responsiveness of players to their private signals is derived. Using this characterization, marginal information values are derived in equilibrium. Scaling with the square of this responsiveness, marginal values to information take on a U-shaped dependence on network centrality in the information-response game. Under significant strategic substitutes, the least central players find additional use from information through inferring and moving against the actions of neighbors.

Equilibrium welfare and the strategic motives behind information acquisition are analyzed. The extent of symmetry in pairwise relationships drives the direction of inefficiencies, both when players move in the direction of or against their information. Under moderate and symmetric relationships, players under respond to the network of peer effects. With both strategic complements and substitutes present, the most informed players under acquire information and the least informed players over acquire. Incorporating players moving against their signals, the extend of information acquired by these players is inefficiently low. Thus, the U-shaped non-monotonicity in the incentives to acquire information in networks carries over to welfare.

All of these welfare properties reverse when the presence of anti-symmetric relationships pervade the network structure. As our example of a market in crisis illustrates, anti-symmetric relationships may play an important role when a nontrivial set of traders in the market face liquidity constraints and thus value high market prices. When liquidity becomes scarce through the market, the few unconstrained firms fail to internalize the externalities they impose on the constrained side of the market. When these unconstrained firms move in the direction of their signal realizations, they under acquire information. If they instead move against their signals, their strategic acquisition of information quality exceeds that of the social planner's prescription. The flexibility in peer-effects networks is essential when assessing the welfare implications in these heterogeneous settings, capturing an ray of equilibrium behaviors.

Marginal strategic values take on unambiguous and opposing directions in symmetric and anti-symmetric networks. Players face clear incentives to overstate and understate their informativeness in these respective settings. The size of these incentives are proportional to players' sum-of-squared degrees. Thus, player connectedness characterizes the size of marginal strategic values to information, while symmetry in pairwise relationships continues to capture its direction. The analysis elicits a transparency based policy with a simplistic implementation: certify the information acquired by

[41] The information-response game is no longer equivalent to a potential games when Σ is not symmetric.

the experts: the most central players in the information response game. And when possible, certify the information investments of those moving against their signals: the least central players in the information response game.

In summary, the above network setting offers a flexible framework to both extend and assess the robustness of many results offered by the coordination games with endogenous information literature. While moderate, symmetric networks offer a natural extension to heterogeneous environments, anti-symmetry and the incorporation of players moving against their information offer both positive and normative properties unattained in symmetric settings. The role of observable prices determined in market clearing under rational expectations, as well as to other forms of information transmission are left as important open questions for future work.

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A Appendix

A.1 Linear-in-qualities expectations: examples

Two states. As the most basic example of an information structure embodying Conditions E1-E4, consider the case of two aggregate states $\omega \in \{-1, 1\}$ with $\gamma = 1$ and priors $\Pr(\omega = 1) = \Pr(\omega = -1) = 1/2$. Then, player i's quality e_i gives the probability of the signal being correct, $\Pr(\theta_i = \omega) = \frac{e_i+1}{2}$. The conditional expectation $\mathbb{E}_i[\theta_j|\theta_i, e_i, \mu_i^*] = e_i e_j \theta_i$ for each $j \neq i$ can be derived as the correlation in the players i and j's signals, $e_i e_j$, multiplied by i's signal realization θ_i. In this case, ω is naturally interpreted as giving 'high' ($\omega = 1$) and 'low' ($\omega = -1$) marginal gains to action x_i, for each player i.

Multiple normal draws. Considering the more general definition of $\tilde{\omega}_i$ provided Section 5.2, one can also consider a normally distributed states and signals setup with normal errors. Assume $\omega \sim N(0,1)$, $\omega_i \sim N(0,1)$, and thus $\tilde{\omega}_i \sim N(0, v_0)$ where $v_0 := \gamma_i^2 + \iota_i^2$. Now, consider player i who draws $S_i \in \mathbb{Z}_+$ signals $\{\vartheta_i^s\}_{s=1}^{S_i}$ taking values $\vartheta_i^s = \tilde{\omega}_i + \varepsilon_i^s$ with error $\varepsilon_i^s \sim N(0, v_1)$; that is, each ϑ_i^s has precision v_1^{-1}. Clearly E1 is satisfied. Player i can then use her signals to infer $\tilde{\omega}_i$ by the usual Bayesian updating rule:

$$\mathbb{E}_i\left[\tilde{\omega}_i \left| \{\vartheta_i^s\}_{s=1}^{S_i}\right.\right] = \frac{v_1^{-1}\sum_{s=1}^{S_i}\vartheta_i^s}{v_0^{-1} + S_i v_1^{-1}}.$$

Define the aggregate signal and information quality:

$$\begin{aligned}
\theta_i &:= \frac{1}{\sqrt{v_0 + \frac{v_1}{S_i}}} \frac{1}{S_i} \sum_{s=1}^{S_i} \vartheta_i^s, \\
e_i &:= \frac{1}{\sqrt{v_0}} \frac{v_1^{-1}}{v_0^{-1} + S_i v_1^{-1}} S_i \sqrt{v_0 + \frac{v_1}{S_i}} = \sqrt{\frac{v_0}{v_0 + \frac{v_1}{S_i}}}.
\end{aligned}$$

The average $\frac{1}{S_i}\sum_{s=1}^{S_i}\vartheta_i^s$ will have precision $S_i v_1^{-1}$. It is then straight forward to show that (the extended version of) E2 and E3 are satisfied:

$$\begin{aligned}
\mathbb{E}_i[\tilde{\omega}_i|\theta_i, e_i] &= \sqrt{\gamma_i^2 + \iota_i^2} e_i \theta_i, \\
\mathbb{E}_i\left[\theta_i^2|e_i\right] &= 1.
\end{aligned}$$

Now consider player j who draws $S_j \in \mathbb{Z}_+$ signals $\{\vartheta_j^s\}_{s=1}^{S_i}$ taking values $\vartheta_j^s = \tilde{\omega}_j + \varepsilon_j^s$ with error $\varepsilon_j^s \sim N(0, v_1)$. Then $\mathbb{E}_i[\theta_j|\theta_i, e_i, \mu_i^*]$ is derived from simple linear regression of θ_j on θ_i:

$$\begin{aligned}
\mathbb{E}_i[\theta_j|\theta_i, e_i, \mu_i^*] &= \frac{Cov(\theta_i, \theta_j)}{Sd(\theta_j)}\theta_i \\
&= \frac{\gamma_i\gamma_j v_0}{\sqrt{v_0 + \frac{v_1}{S_i}}\sqrt{v_0 + \frac{v_1}{S_j}}}\theta_i \\
&= \gamma_i\gamma_j e_i e_j \theta_i,
\end{aligned}$$

establishing (the extended version of) condition E4 under sequentially rational μ_i^*.

A.2 Section 3.1 proofs: Equilibrium information acquisition and response

Existence of a second-stage equilibrium is only ensured if the size of peer effects are suitably constrained. This motivates the following assumption, maintained throughout.

Assumption A1. $(\mathbf{I} - [p_{ij}\sigma_{ij}])^{-1}$ *is well defined for every* $\mathbf{p} \in [0,1]^{N(N-1)}$.

Assumption A1 is a strengthening of the condition $p > \lambda\mu_i(\mathbf{G})$ in Bellester et al. (2006) [5] Theorem 1 bounding the spectral radius of the relevant diagonally dominant matrix under complete information. Assumption A1 implies that the relevant diagonally-dominant matrix in the second stage's information-response game remains invertible for all first-stage outcomes. Primarily a technical condition, this suffices for existence and uniqueness of a pure linear Bayesian equilibrium at period $t = 2$.

Proof of Theorem 1. For all purposes, $\mathbf{I}$ will denote the $n \times n$ identity matrix. Linearity of the ex-post best responses allows us to take expectations of (2) and obtain i's first order condition of her information response problem. This gives optimal action:

$$\begin{aligned} X_i(\theta_i, e_i) &= (a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i, e_i]) + \sum_{k\neq i} \sigma_{ik}\mathbb{E}_i[X_k(\theta_k, e_k)|\theta_i, e_i, \mu_i^*] \\ &= (a_i + e_i\theta_i) + \sum_{k\neq i} \sigma_{ik}\mathbb{E}_i[X_k(\theta_k, e_k)|\theta_i, e_i, \mu_i^*]. \end{aligned} \tag{21}$$

Next, we are free to take expectations of (21) over realizations of player i's signal θ_i. Denoting the vector of expected stage two actions $\boldsymbol{\alpha}^*$, this gives:

$$\boldsymbol{\alpha}^* = \mathbf{a} + \Sigma\boldsymbol{\alpha}^*. \tag{22}$$

This can easily be solved to give the following expectations equilibrium:

$$\boldsymbol{\alpha}^* = [\mathbf{I} - \Sigma]^{-1}\mathbf{a}. \tag{23}$$

Note that $\boldsymbol{\alpha}^*$ does not depend on $\mathbf{e}$.[42]

Next, we derive the information responses given in (5). Linearity of this expression is derived by the linearity in best responses (2) and in expectations. Consider the following profile of strategies:

$$\mathbf{X}^*(\boldsymbol{\theta}) = \boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\theta}}\boldsymbol{\beta}^*,$$

with $\beta_i^* \in \mathbb{R}$ denoting each player i's responsiveness to her signal. For each component i taking $\boldsymbol{\beta}_{-i}^*$ as above we verify that i plays a linear strategy. Taking differences of (21) at θ_i and $\theta_i' < \theta_i$ then gives[43]:

[42] Expression (23) is analogous to expression (4) in Ballester et al. (2006) [5], but now in expectations.
[43] One can always find such a signal pair, else signals are never informative.

$$\begin{aligned}
X_i^*\left(\theta_i|e_i\right)-X_i^*\left(\theta_i'|e_i\right) &= \begin{pmatrix} \mathbb{E}_i\left[\tilde{\omega}_i|\theta_i,e_i,\mu_i^*\right]-\mathbb{E}_i\left[\tilde{\omega}_i|\theta_i',e_i,\mu_i^*\right] \\ +\sum_{k\neq i}\sigma_{ik}\left(\mathbb{E}_i\left[X_k^*\left(\theta_k|e_k\right)|\theta_i,e_i,\mu_i^*\right]-\mathbb{E}_i\left[X_k^*\left(\theta_k|e_k\right)|\theta_i',e_i,\mu_i^*\right]\right)\end{pmatrix} \\
&= e_i\theta_i-e_i\theta_i'+\sum_{k\neq i}\sigma_{ik}\left(\mathbb{E}_i\left[\alpha_k^*+\theta_k\beta_k^*|\theta_i,e_i,\mu_i^*\right]-\mathbb{E}_i\left[\alpha_k^*+\theta_k\beta_k^*|\theta_i',e_i,\mu_i^*\right]\right) \\
&= e_i\left(\theta_i-\theta_i'\right)+\sum_{k\neq i}\sigma_{ik}\left(\left(\mathbb{E}_i\left[\theta_k|\theta_i,e_i,\mu_i^*\right]-\mathbb{E}_i\left[\theta_k|\theta_i',e_i,\mu_i^*\right]\right)\beta_k^*\right) \\
&= e_i\left(\theta_i-\theta_i'\right)+\sum_{k\neq i}\sigma_{ij}\left(\left(\gamma^2e_ie_k\theta_i-\gamma^2e_ie_k\theta_i'\right)\beta_k^*\right) \\
&= \left(\theta_i-\theta_i'\right)\left(e_i+\sum_{k\neq i}\sigma_{ik}\gamma^2e_ie_k\beta_k^*\right).
\end{aligned}$$

With $\frac{X_i^*(\theta_i|e_i)-X_i^*\left(\theta_i'|e_i\right)}{\theta_i-\theta_i'}$ independent of the choice of θ_i and θ_i', player i also plays a linear strategy, with (optimal) responsiveness

$$\beta_i^*=\frac{X_i^*\left(\theta_i|e_i\right)-X_i^*\left(\theta_i'|e_i\right)}{\theta_i-\theta_i'}=e_i+\sum_{k\neq i}\sigma_{ik}\gamma^2e_ie_k\beta_k^*. \tag{24}$$

We thus have:

$$\boldsymbol{\beta}^*=\mathbf{e}+\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\boldsymbol{\beta}^*. \tag{25}$$

With $\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}$ well defined by Assumption A1, solving for $\boldsymbol{\beta}^*$ gives the unique linear information response equilibrium:

$$\boldsymbol{\beta}^*=\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{e}.$$

Finally, we can easily write:

$$X_i^*\left(\theta_i|e_i\right)=\alpha_i^*+\theta_i\beta_i^*,$$

for each i, giving the $t=2$ IRE strategy seen in (5).

To establish the stronger uniqueness claim succeeding Theorem 1, the following establishes a similar result to that shown in Dewan and Myatt (2008), adapted to our network setting. The second stage best response function of any i given first stage outcome $\mathbf{e}$ (and correct beliefs μ_i^* regarding $\mathbf{e}_{-i}$) is again:

$$BR_i\left(\mathbf{X}_{-i}|\theta_i,e_i,\mu_i^*\right)=a_i+\mathbb{E}_i\left[\tilde{\omega}_i|\theta_i,e_i,\mu_i^*\right]+\sum_{k\neq i}\sigma_{ik}\mathbb{E}_i\left[X_k^*\left(\theta_k|e_k\right)|\theta_i,e_i,\mu_i^*\right].$$

Suppressing the (e_i,μ_i^*) conditionals, the composition of $BR_i\left(\mathbf{X}_{-i}|\theta_i\right)$ with $BR_j\left(\mathbf{X}_{-j}|\theta_j\right)$ for each $j\neq i$

gives:

$$\begin{aligned}
BR_i^2(\cdot|\theta_i) &= a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i] + \sum_{k\neq i}\sigma_{ik}\mathbb{E}_i\left[a_k + \mathbb{E}_k[\tilde{\omega}_k|\theta_k] + \sum_{k'\neq k}\sigma_{kk'}\mathbb{E}_k[\cdot|\theta_k]\middle|\theta_i\right] \\
&= a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i] + \sum_{k\neq i}\sigma_{ik}\left(a_k + \mathbb{E}_i[\mathbb{E}_k[\tilde{\omega}_k|\theta_k]|\theta_i] + \sum_{k'\neq k}\sigma_{kk'}\mathbb{E}_i[\mathbb{E}_k[\cdot|\theta_k]|\theta_i]\right) \\
&= a_i + \mathbb{E}_i[\tilde{\omega}_i|\theta_i] + \sum_{k\neq i}\sigma_{ik}a_k + \sum_{k\neq i}\sigma_{ik}\mathbb{E}_i[\mathbb{E}_k[\tilde{\omega}_k|\theta_k]|\theta_i] + \sum_{k\neq i}\sum_{k'\neq k}\sigma_{ik}\sigma_{kk'}\mathbb{E}_i[\mathbb{E}_k[\cdot|\theta_k]|\theta_i] \\
&= a_i + \sum_{k\neq i}\sigma_{ik}a_j + e_i\theta_i + \sum_{k\neq i}\sigma_{ik}\gamma^2 e_k^2 e_i\theta_i + \sum_{k\neq i}\sum_{k'\neq j}\sigma_{ik}\sigma_{kk'}\mathbb{E}_i[\mathbb{E}_k[\cdot|\theta_k]|\theta_i].
\end{aligned}$$

In vector form[44]:

$$BR^2(\cdot|\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{a} + \Sigma\mathbf{a} + \mathbf{I}_{\boldsymbol{\theta}}\mathbf{e} + \mathbf{I}_{\boldsymbol{\theta}}\gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\mathbf{e} \\ +\left[\sum_{k\neq i}\sum_{k'\neq k}\sigma_{ik}\sigma_{kk'}\mathbb{E}_i[\mathbb{E}_k[\cdot|\theta_k]|\theta_i]\right] \end{pmatrix}.$$

We can iterate this to yield the τ'th best-response dynamic $BR^\tau(\cdot|\boldsymbol{\theta})$:

$$BR^\tau(\cdot|\boldsymbol{\theta}) = \begin{pmatrix} \left(\mathbf{I} + \sum_{t=1}^{\tau}\Sigma^{t-1}\right)\mathbf{a} + \mathbf{I}_{\boldsymbol{\theta}}\left(\mathbf{I} + \sum_{t=1}^{\tau}\gamma^2(\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}})^{t-1}\right)\mathbf{e} \\ +\left[\sum_{k\neq i}\cdots\sum_{h\neq j}\sigma_{ik}\cdots\sigma_{jh}\mathbb{E}_i[\cdots\mathbb{E}_j[\cdot|\theta_j]|\theta_i]\right] \end{pmatrix}.$$

When each $|\sigma_{ij}| < 1$ the bottom term will converge to zero provided strategies are bounded. More generally, we require the following property to hold.

Definition 3 (non-explosive expectations)**.** *For any sequence of players $(i_1, i_2, \ldots)$ with $i_t \neq i_{t+1}$ and each $i_t \in \{1, \ldots, N\}$ and $t \in \mathbb{N}$, the operator $\overline{\mathbb{E}}_{i_t}[\cdot]$ is defined inductively as $\overline{\mathbb{E}}_{i_t}[\cdot] := \sigma_{i_{t-1}i_t}\overline{\mathbb{E}}_{i_{t-1}}[\mathbb{E}_{i_t}[\cdot]]$, with $\overline{\mathbb{E}}_{i_1}[\cdot] = \mathbb{E}_{i_1}[\cdot]$. Then for any given (potentially non-linear) IRE $\mathbf{X}^*$ and quality profile $\mathbf{e}$, expectations over the network are* non-explosive *if* $\lim_{t\to\infty}\overline{\mathbb{E}}_{i_t}[X_{i_t}(\theta_{i_t}|e_{i_t})] = 0$.

Given expectations are non-explosive, we then obtain:

$$\begin{aligned}
\lim_{\tau\to\infty} BR^\tau(\cdot|\boldsymbol{\theta}) &= \lim_{\tau\to\infty}\left(\mathbf{I} + \sum_{\tau=1}^{k}\Sigma^\tau\right)\mathbf{a} + \mathbf{I}_{\boldsymbol{\theta}}\left(\mathbf{I} + \sum_{\tau=1}^{k}\gamma^2(\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}})^\tau\right)\mathbf{e} \\
&= (\mathbf{I} - \Sigma)^{-1}\mathbf{a} + \mathbf{I}_{\boldsymbol{\theta}}(\mathbf{I} - \mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}})^{-1}\mathbf{e} \\
&= \boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\theta}}\boldsymbol{\beta}^* =: \mathbf{X}^*
\end{aligned}$$

which gives the unique linear information response equilibrium of Theorem 1. Thus, any equilibrium in which expectations are non-explosive must be $\mathbf{X}^*$.

□

[44] $\mathbf{I}_{\boldsymbol{\phi}}$ gives the diagonal matrix with elements from generic vector $\boldsymbol{\phi}$.

Proof of Theorem 2. Writing each player $k \neq i$'s information response strategy as $X_k^*(\theta_k|e_k) = \alpha_k^* + \theta_k\beta_k^*$:

$$\begin{aligned} u_i\left(x_i, \mathbf{X}_{-i}^*|\theta_i, e_i, \mu_i\right) &= \left(a_i + \mathbb{E}_i\left[\tilde{\omega}_i|\theta_i, e_i\right]\right) x_i - \frac{1}{2}x_i^2 + \sum_{k\neq i} \sigma_{ik} x_i \mathbb{E}_i\left[X_k^*\left(\theta_k|e_k\right)|\theta_i, e_i, \mu_i\right] \\ &= \left(a_i + e_i\theta_i\right) x_i - \frac{1}{2}x_i^2 + \sum_{k\neq i} \sigma_{ik} x_i \left(\alpha_k^* + \beta_k^* \mathbb{E}_i\left[\theta_k|\theta_i, e_i, \mu_i\right]\right) \\ &= \left(a_i + e_i\theta_i\right) x_i - \frac{1}{2}x_i^2 + \sum_{k\neq i} \sigma_{ik} x_i \left(\alpha_k^* + \beta_k^*\gamma^2 e_i e_k \theta_i\right). \end{aligned}$$

By the optimality of $\mathbf{X}_i^*$ in stage two, we can apply the envelope theorem:

$$\frac{\partial}{\partial \beta_i^*}\mathbb{E}_i\left[u_i\left(\mathbf{X}^*\left(\boldsymbol{\theta}|\mathbf{e}\right)|\omega, \omega_i\right)|\theta_i, e_i, \mu_i\right] = 0.$$

Further, as information acquisition is unobserved by others in $t = 2$, incentive compatibility of e_i^* requires that the response of β_j^* to shifting e_i be set to zero: $\frac{\partial}{\partial e_i}\beta_j^* = 0$. Thus we obtain:

$$\begin{aligned} & \frac{\partial}{\partial e_i}\mathbb{E}_i\left[u_i\left(\mathbf{X}^*\left(\boldsymbol{\theta}|\mathbf{e}\right)|\omega, \omega_i\right)|e_i, \mu_i\right] \\ = \; & \frac{\partial}{\partial e_i}\mathbb{E}_i\left[\mathbb{E}_i\left[u_i\left(\mathbf{X}^*\left(\boldsymbol{\theta}|\mathbf{e}\right)|\omega, \omega_i\right)|\theta_i, e_i, \mu_i\right]\right] \\ = \; & \frac{\partial}{\partial e_i}\mathbb{E}_i\left[\begin{array}{c} \left(a_i + e_i\theta_i\right)\left(\alpha_i^* + \theta_i\beta_i^*\right) - \frac{1}{2}\left(\alpha_i^* + \theta_i\beta_i^*\right)^2 \\ + \sum_{k\neq i}\sigma_{ik}\left(\alpha_i^* + \theta_i\beta_i^*\right)\left(\alpha_k^* + \beta_k^*\gamma^2 e_i e_k\theta_i\right) \end{array}\right] \\ = \; & \frac{\partial}{\partial e_i}\mathbb{E}_i\left[\begin{array}{c} \left(\beta_i^* e_i - \frac{1}{2}\beta_i^{*2} + \gamma^2\sum_{k\neq i}\sigma_{ik}e_i e_k\beta_i^*\beta_k^*\right)\theta_i^2 \\ + \text{const}_0 + \text{const}_1 \cdot \theta_i \end{array}\right] \\ = \; & \mathbb{E}_i\left[\frac{\partial}{\partial e_i}\left(\begin{array}{c} \left(\beta_i^* e_i - \frac{1}{2}\beta_i^{*2} + \gamma^2\sum_{k\neq i}\sigma_{ik}e_i e_k\beta_i^*\beta_k^*\right)\theta_i^2 \\ + \text{const}_0 \end{array}\right)\right] \\ = \; & \left(\begin{array}{c} \beta_i^*\left(1 + \gamma^2\sum_{k\neq i}\sigma_{ik}\beta_k^* e_k^*\right)\mathbb{E}_{\theta_i}\left[\theta_i^2|e_i\right] \\ + \left(\beta_i^* e_i - \frac{1}{2}\beta_i^{*2} + \gamma^2\sum_{k\neq i}\sigma_{ik}\beta_i^*\beta_k^* e_i e_k\right)\frac{\partial}{\partial e_i}\mathbb{E}_{\theta_i}\left[\theta_i^2|e_i\right] \end{array}\right) \\ = \; & \beta_i^*\left(1 + \gamma^2\sum_{k\neq i}\sigma_{ik}e_k\beta_k^*\right), \end{aligned}$$

with $\mathbb{E}_{\theta_i}\left[\theta_i^2|e_i\right] = 1$ and $\frac{\partial}{\partial e_i}\mathbb{E}_{\theta_i}\left[\theta_i^2|e_i\right] = 0$ by condition E3. This yields i's period $t = 1$ marginal gains to information acquisition:

$$\frac{\partial}{\partial e_i}u_i\left(\mathbf{X}_i^*|e_i, \mathbf{e}_{-i}\right) = \beta_i^*\left(1 + \gamma^2\sum_{k\neq i}\sigma_{ik}e_k\beta_k^*\right) \tag{26}$$

Thus, the period $t = 1$ vector of marginal gains to quality is given by:

$$\left[\frac{\partial}{\partial e_i}u_i\left(\mathbf{X}_i^*|e_i, \mathbf{e}_{-i}\right)\right] = \gamma^2\mathbf{I}_{\boldsymbol{\beta}^*}\Sigma\mathbf{I}_{\mathbf{e}}\boldsymbol{\beta}^* + \boldsymbol{\beta}^*. \tag{27}$$

When $\gamma = 0$ then (25) reduces to $\boldsymbol{\beta}^*(\mathbf{e}) = \mathbf{e}$. Equating marginal gains to marginal costs of quality in IAE gives $\mathbf{e}^* = \kappa'(\mathbf{e}^*)$, which corresponds to expression (6), and yields $e^\dagger$ from (3) for each i so that each player chooses the quality that the isolated player chooses.

When $\gamma > 0$ then (25) can be rearranged as:

$$\gamma^2 \Sigma \mathbf{I_e} \boldsymbol{\beta}^* = \mathbf{I_e}^{-1} \left(\boldsymbol{\beta}^* - \mathbf{e} \right). \tag{28}$$

Substituting this into (27) gives the marginal gains to information:

$$\left[\frac{\partial}{\partial e_i} u_i \left(\mathbf{X}_i^* | e_i, \mathbf{e}_{-i} \right) \right] = \mathbf{I_e}^{-1} \mathbf{I}_{\boldsymbol{\beta}^*} \boldsymbol{\beta}^*.$$

Equating this with the marginal cost of information then gives the first-stage interior IAE condition (6):

$$\mathbf{I}_{\boldsymbol{\beta}^*} \boldsymbol{\beta}^* = \mathbf{I}_{\mathbf{e}^*} \kappa' \left(\mathbf{e}^* \right). \tag{29}$$

□

Proof of Corollary 1. Applying the implicit function theorem to expression (6)[45]:

$$\begin{aligned}
\frac{\partial e_i^*}{\partial \left(\gamma^2\right)} &= -\frac{\frac{\partial\left(\beta_i^{*2}/e_i^*\right)}{\partial(\gamma^2)}}{\frac{\partial\left(\beta_i^{*2}/e_i^* - \kappa'\left(e_i^*\right)\right)}{\partial e_i}} + \sum_{k \neq i} \frac{\partial e_i^*}{\partial \beta_k^*} \frac{\partial \beta_k^*}{\partial \left(\gamma^2\right)} \\
&= -\frac{2\beta_i^*/e_i^* \frac{\partial \beta_i^*}{\partial(\gamma^2)}}{\frac{\partial\left(\beta_i^{*2}/e_i^*\right)}{\partial e_i} - \kappa''\left(e_i^*\right)} + \sum_{k \neq i} \frac{\partial e_i^*}{\partial \beta_k^*} \frac{\partial \beta_k^*}{\partial \left(\gamma^2\right)} \\
&= \frac{2\frac{\beta_i^*}{e_i^*} \frac{\partial \beta_i^*}{\partial(\gamma^2)}}{\kappa''\left(e_i^*\right) - \frac{\left(e_i^* 2\beta_i^* \frac{\partial \beta_i^*}{\partial e_i} - \beta_i^{*2}\right)}{e_i^2}} + \sum_{k \neq i} \frac{\partial e_i^*}{\partial \beta_k^*} \frac{\partial \beta_k^*}{\partial \left(\gamma^2\right)} \\
&= \frac{2\frac{\beta_i^*}{e_i^*} \sum_{k \neq i} e_i^* \sigma_{ik} e_k^* \beta_k^*}{\kappa''\left(e_i^*\right) - \frac{\left(e_i^* 2\beta_i^* \frac{\partial \beta_i^*}{\partial e_i} - \beta_i^{*2}\right)}{e_i^2}} + \sum_{k \neq i} \frac{\partial e_i^*}{\partial \beta_k^*} \frac{\partial \beta_k^*}{\partial \left(\gamma^2\right)}.
\end{aligned}$$

Taking the limit $\gamma \to^+ 0$ of the expression, and noting that $\lim_{\gamma \to +0} \frac{\partial \beta_i^*}{\partial e_i} = 1$ while $\lim_{\gamma \to +0} \frac{\partial e_i^*}{\partial \beta_k^*} = 0$ for each $k \neq i$, yields:

$$\lim_{\gamma \to +0} \frac{\partial e_i^*}{\partial \left(\gamma^2\right)} = \frac{2 e^{\dagger 3} \sum_{k \neq i} \sigma_{ik}}{\kappa''\left(e^\dagger\right) - 1}.$$

Note that $\kappa''\left(e^\dagger\right) - 1 > 0$ by the optimality of $e^\dagger$ at $\gamma = 0$ and Assumption 1.

□

Proof of Lemma 1. The existence of the bound γ^m follows from Assumption 1, by continuity in $\boldsymbol{\beta}^*$ and $\mathbf{e}^*$ for each i at $\gamma = 0$, and by the implicit function theorem. Precisely, $\boldsymbol{\beta}^* = \mathbf{e}^* := \left(e^\dagger, \ldots, e^\dagger\right) > \mathbf{0}$ when $\gamma = 0$, and thus that marginal gains to quality β_i^{*2}/e_i^* are continuous at $\gamma = 0$. Assumption 1 implies a

[45] One could employ the multivariate implicit function theorem, noting that changes in e_i^* will result as second-stage β_k^* for each $k \neq i$ adjust with γ^2. We avoid the multivariate implicit function theorem by employing the chain rule, and summing over partials of e_i^* with respect to β_k^* for each $k \neq i$ (last term).

unique $e^{\dagger}$ solving $\beta_i^{*2}/e_i^* = e^{\dagger} = \kappa'\left(e^{\dagger}\right)$ for each i. Further,

$$\begin{aligned}
\det\left(D_{\mathbf{e}}[\beta_i^{*2}/e_i^* - \kappa'\left(e_i^*\right)]\right)\big|_{(\mathbf{e}=(e^{\dagger},\ldots,e^{\dagger}),\gamma=0)} &= \det\left(\left(1-\kappa'\left(e^{\dagger}\right)\right)\mathbf{I}\right) \\
&= \left(1-\kappa'\left(e^{\dagger}\right)\right)^N \neq 0,
\end{aligned}$$

and thus by the IFT there exists an open neighborhood $U \subseteq [0,1]^N$ of $\left(e^{\dagger},\ldots,e^{\dagger}\right)$ and $W \subseteq [0,1]$ of $\gamma = 0$ such that for every $\gamma \in W$ there is a unique IAE $\mathbf{e}^{*,\gamma} \in U$.

Now, the best response correspondence $\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*;\gamma\right)$ (see proof of Proposition S.1) is upper hemicontinuous in $(\mathbf{e},\gamma)$ by continuity of β_i^{*2}/e_i in $\mathbf{e}$ and γ and of $\kappa(\cdot)$ in e_i at $\mathbf{e} = \left(e^{\dagger},\ldots,e^{\dagger}\right)$ and $\gamma = 0$. There must then also exist some neighborhood $V \subseteq [0,1]^N \times [0,1]$ of $\left(\left(e^{\dagger},\ldots,e^{\dagger}\right),0\right)$ such that $\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*;\gamma\right) \subseteq U$ for any $(\mathbf{e},\gamma) \in V$. This then implies that $[0,1]^N \backslash U$ does not contain any IAE for all $\gamma \in W \cap V \subseteq [0,1]$, and thus that $\mathbf{e}^{*,\gamma}$ gives the unique IAE for each $\gamma \in [0,\gamma^m) \subseteq W \cap V$.

We construct the interval $[0,\gamma^s)$ as follows, which incorporates the potential for multiplie equilibria. $\boldsymbol{\beta}^*$ is continuous in $\mathbf{e}$ with $\boldsymbol{\beta}^* = \left(e^{\dagger},\ldots,e^{\dagger}\right) > \mathbf{0}$ at $\gamma = 0$. Thus for each i, there must exist some $\gamma_i^s > 0$ such that if $\gamma < \gamma_i^s$ then $\beta_i^* > 0$ for any $\mathbf{e} \in [0,1]^N$.[46] Then defining $\gamma^s := \min_i\{\gamma_i^s\}$ and by the existence of IAE given with the proof of Proposition S.1 below, we must have that $\boldsymbol{\beta}^* > \mathbf{0}$ provided $\gamma \in [0,\gamma^s)$ and any IAE $\mathbf{e}^*$.

□

A.3 Section 4 proofs: Equilibrium welfare and the strategic value to information

First we derive equilibrium welfare, expression (8) in the text. Restating player i's expected payoff:

$$u_i\left(x_i,\mathbf{X}_{-i}|\theta_i,\mathbf{e}\right) = \left(a_i + e_i\theta_i\right)x_i - \frac{1}{2}x_i^2 + \sum_{k\neq i}\sigma_{ik}x_i\left(\alpha_{kj}^* + \beta_k^*\gamma^2 e_i e_k\theta_i\right).$$

Subtracting information cost $\kappa(e_i)$ and taking expectations over signals $\boldsymbol{\theta}$ gives her period $t = 1$ value:

$$\begin{aligned}
\nu_i\left(\mathbf{X}_i^*|e_i,\mathbf{e}_{-i}\right) &= \mathbb{E}_i\left[\begin{pmatrix} \left(a_i+e_i\theta_i\right)\left(\alpha_i^*+\theta_i\beta_i^*\right) - \frac{1}{2}\left(\alpha_i^*+\theta_i\beta_i^*\right)^2 \\ +\sum_{k\neq i}\sigma_{ik}\left(\alpha_i^*+\theta_i\beta_i^*\right)\left(\alpha_k^*+\beta_k^*\gamma^2 e_i e_k\theta_i\right)\end{pmatrix}\right] - \kappa\left(e_i\right) \\
&= a_i\alpha_i^* + e_i\beta_i^* - \frac{1}{2}\left(\alpha_i^{*2}+\beta_i^{*2}\right) + \sum_{k\neq i}\sigma_{ik}\left(\alpha_i^*\alpha_k^* + \beta_i^*\beta_k^*\gamma^2 e_i e_k\right) - \kappa\left(e_i\right).
\end{aligned}$$

Writing this in vector form gives:

$$\nu\left(\mathbf{X}^*|\mathbf{e}\right) = \left(\mathbf{I_a}\boldsymbol{\alpha}^* + \mathbf{I_e}\boldsymbol{\beta} - \frac{1}{2}\left(\mathbf{I}_{\boldsymbol{\alpha}^*}\bar{\mathbf{X}}^* + \mathbf{I}_{\boldsymbol{\beta}^*}\boldsymbol{\beta}^*\right) + \mathbf{I}_{\boldsymbol{\alpha}^*}\Sigma\boldsymbol{\alpha}^* + \gamma^2\mathbf{I}_{\boldsymbol{\beta}^*}\mathbf{I_e}\Sigma\mathbf{I_e}\boldsymbol{\beta}^*\right) - \kappa\left(\mathbf{e}\right). \tag{30}$$

Next, left multiplying (22) by $\mathbf{I}_{\boldsymbol{\alpha}^*}$ gives:

$$\mathbf{I}_{\boldsymbol{\alpha}^*}\boldsymbol{\alpha}^* = \mathbf{I_a}\boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\alpha}^*}\Sigma\boldsymbol{\alpha}^*, \tag{31}$$

while rearranging (25) gives:

$$\frac{1}{\gamma^2}\left(\boldsymbol{\beta}^* - \mathbf{e}\right) = \mathbf{I_e}\Sigma\mathbf{I_e}\boldsymbol{\beta}^*. \tag{32}$$

[46]This uses Assumption A1 to maintain that $\boldsymbol{\beta}^*$ is well defined for each $\mathbf{e} \in [0,1]^N$.

Substituting (31) and (32) into (30) then gives:

$$\begin{aligned}\nu\left(\mathbf{X}^*|\mathbf{e}\right) &= \left(\mathbf{I}_{\bar{\mathbf{X}}^*}\boldsymbol{\alpha}^* + \mathbf{I}_{\mathbf{e}}\boldsymbol{\beta}^* - \frac{1}{2}\left(\mathbf{I}_{\boldsymbol{\alpha}^*}\boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\beta}^*}\boldsymbol{\beta}^*\right) + \mathbf{I}_{\boldsymbol{\beta}^*}\left(\boldsymbol{\beta}^* - \mathbf{e}\right)\right) - \kappa\left(\mathbf{e}\right) \\ &= \frac{1}{2}\left(\mathbf{I}_{\boldsymbol{\alpha}^*}\boldsymbol{\alpha}^* + \mathbf{I}_{\boldsymbol{\beta}^*}\boldsymbol{\beta}^*\right) - \kappa\left(\mathbf{e}\right),\end{aligned}$$

giving expression (8).

For the proofs of Lemma 1 and Proposition 3A we next derive expressions for partials $\frac{\partial\boldsymbol{\beta}^*}{\partial e_i}$. This yields expressions for $\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right)$ and $\xi_i^{ex}\left(\mathbf{e},\mathbf{X}^*\right)$ solely in terms of Σ and $\mathbf{e}$

Using u and v for row and column dummies (respectively) the system of equations giving IRE $\boldsymbol{\beta}^*$ can be written as:

$$[u]\text{: } \beta_u^* - e_u\left(1 + \gamma^2\sum_{k\neq u}\sigma_{uk}e_k\beta_k^*\right) = 0,$$

for each $u \in \{1,\ldots,N\}$. Partial differentiating each $[u]$ by β_v^* gives:

$$[f_{uv}]\text{: } \frac{\partial[u]}{\partial\beta_v^*} = \begin{cases} -\gamma^2 e_u\sigma_{uv}e_v & \text{if } u\neq v \\ 1 & \text{if } u = v\end{cases},$$

for each $u, v \in \{1,\ldots,N\}$. In matrix form this is exactly $\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}$. Partial differentiating each $[u]$ by e_i gives:

$$[du]\text{: } \sum_v f_{uv}\frac{\partial\beta_v^*}{\partial e_i} + b_u = 0,$$

for each $du \in \{d1,\ldots,dN\}$, where

$$b_u := \frac{\partial[u]}{\partial e_i} = -\frac{\beta_i^*}{e_i}\cdot\begin{cases}\gamma^2 e_u\sigma_{ui}e_i & \text{if } u\neq i \\ 1 & \text{if } u = i\end{cases}.$$

In vector form $\mathbf{b}$ gives $\frac{\beta_i^*}{e_i}\left(\mathbf{I}-\gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}} - 2\mathbf{I}\right)\mathbf{1}_i$, where $\mathbf{1}_u$ gives the vector of zeros with a one in row u. Solving for $\frac{\partial\beta_u^*}{\partial e_i}$ in matrix form gives the comparative static of $\boldsymbol{\beta}^*$ with respect to e_i:[47]

$$\begin{aligned}\frac{\partial\boldsymbol{\beta}^*}{\partial e_i} &= -F^{-1}\mathbf{b} \\ &= -\left(\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\right)^{-1}\left(\frac{\beta_i^*}{e_i}\left(\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}} - 2\mathbf{I}\right)\mathbf{1}_i\right) \\ &= -\frac{\beta_i^*}{e_i}\left(\mathbf{I} - 2\left(\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\right)^{-1}\right)\mathbf{1}_i \\ &= \frac{\beta_i^*}{e_i}\left(2\left(\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\right)^{-1} - \mathbf{I}\right)\mathbf{1}_i \qquad (33) \\ &= \frac{\beta_i^*}{e_i}\left(\mathbf{I} + \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\right)\left(\mathbf{I} - \gamma^2\mathbf{I}_{\mathbf{e}}\Sigma\mathbf{I}_{\mathbf{e}}\right)^{-1}\mathbf{1}_i. \qquad (34)\end{aligned}$$

$\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right)$ and $\xi_i^{ex}\left(\mathbf{e},\mathbf{X}^*\right)$ can be expressed solely in terms of Σ and $\mathbf{e}$ by substituting (33) into the following expressions:

[47] An equivalent setup of the above is provided in Takayama (1985) [57], pgs. 403-5.

$$
\begin{aligned}
\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right) \quad &: \quad = \beta_i^* \sum_{k\neq i} \gamma^2 e_i e_k \sigma_{ik} \frac{\partial}{\partial e_i} \beta_k^* \\
&= \quad \beta_i^* \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \frac{\partial \boldsymbol{\beta}^*}{\partial e_i} \\
\xi_i^{ex}\left(\mathbf{e},\mathbf{X}^*\right) \quad &: \quad = \sum_{k\neq i} \beta_k^* \frac{\partial}{\partial e_i} \beta_k^* \\
&= \quad \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \frac{\partial \boldsymbol{\beta}^*}{\partial e_i}.
\end{aligned}
$$

For $\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right)$ we have:

$$
\begin{aligned}
\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right) &= \beta_i^* \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \frac{\partial \boldsymbol{\beta}^*}{\partial e_i} \\
&= \frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \left(\mathbf{I} + \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right) \left(\mathbf{I} - \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{-1} \mathbf{1}_i \\
&= \frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \left(\mathbf{I} + \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right) \left(\sum_{\tau=0} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= \frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \left(\mathbf{I} + 2\left(\sum_{\tau=1} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right)\right) \mathbf{1}_i \\
&= \frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I}_{\mathbf{e}^*} + 2\left(\sum_{\tau=2} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right)\right) \mathbf{1}_i \\
&= 2\frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \left(\sum_{\tau=2} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= 2\frac{\beta_i^{*2}}{e_i} \mathbf{1}_i' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \left(\mathbf{I} - \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{-1} \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \mathbf{1}_i.
\end{aligned}
$$

For $\xi_i^{ex}\left(\mathbf{e},\mathbf{X}^*\right)$ we have:

$$
\begin{aligned}
\xi_i^{ex}\left(\mathbf{e},\mathbf{X}^*\right) &= \frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \left(\mathbf{I} + \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right) \left(\mathbf{I} - \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{-1} \mathbf{1}_i \\
&= \frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \left(\mathbf{I} + \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right) \left(\sum_{\tau=0} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= \frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \left(\mathbf{I} + 2\sum_{\tau=1} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= \frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \left(2\sum_{\tau=1} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= 2\frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \left(\sum_{\tau=1} \left(\gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{\tau}\right) \mathbf{1}_i \\
&= 2\frac{\beta_i^*}{e_i} \left(\boldsymbol{\beta}^* - \mathbf{1}_{\beta_i^*}\right)' \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e} \left(\mathbf{I} - \gamma^2 \mathbf{I_e} \Sigma \mathbf{I_e}\right)^{-1} \mathbf{1}_i.
\end{aligned}
$$

Together:

$$\xi_i^{st}(\mathbf{e},\mathbf{X}^*) = \gamma^4 2\frac{\beta_i^{*2}}{e_i^*}\mathbf{1}_i'\mathbf{I_e}\Sigma\mathbf{I_e}\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{I_e}\Sigma\mathbf{I_e}\mathbf{1}_i,$$
$$\xi_i^{ex}(\mathbf{e},\mathbf{X}^*) = \gamma^2 2\frac{\beta_i^{*}}{e_i^*}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\mathbf{I_e}\Sigma\mathbf{I_e}\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{1}_i.$$

One can also use Theorem 1 to substitute in corresponding expressions for $\boldsymbol{\beta}^*$ and β_i^*, respectively, that are solely in terms of Σ and $\mathbf{e}$.

Lemma 1 is established using the leading term of the Taylor expansion:

$$\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1} = \sum_{\tau=0}^{\infty}\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{\tau},$$

which will dominate the sum for small γ. Formal proofs are as follows.

Proof of Lemma 1 and derivations of (18) and (19). We can rewrite the expression for $\xi_i^{st}(\mathbf{X}^*,\mathbf{e})$ by expanding $\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}$ as follows:

$$\xi_i^{st}(\mathbf{e},\mathbf{X}^*) = \gamma^4 2\frac{\beta_i^{*2}}{e_i^*}\mathbf{1}_i'\mathbf{I_e}\Sigma\mathbf{I_e}\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{I_e}\Sigma\mathbf{I_e}\mathbf{1}_i \tag{35}$$
$$= 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\sum_{\tau=2}^{\infty}\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{\tau}\right)\mathbf{1}_i \tag{36}$$
$$= 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2+\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^3\sum_{\tau=0}^{\infty}\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{\tau}\right)\mathbf{1}_i \tag{37}$$
$$= \gamma^4 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2\mathbf{1}_i+\gamma^6 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\mathbf{I}_{\mathbf{e}^*}\Sigma\mathbf{I_e}\right)^3\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{1}_i. \tag{38}$$

For the second term:

$$\frac{\partial}{\partial(\gamma^4)}\left(\gamma^6 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^3\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{1}_i\right)\to 0,$$

for each i, as $\gamma\to 0$. Thus focusing on the first term:

$$\gamma^4 2\frac{\beta_i^{*2}}{e_i}\mathbf{1}_i'\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2\mathbf{1}_i = \gamma^2 2\frac{\beta_i^{*}}{e_i}\mathbf{1}_i'\left[\sum_{k\neq i}e_i\sigma_{ik}e_ke_k\sigma_{kj}e_j\right]\mathbf{1}_i$$
$$= \gamma^4 2\frac{\beta_i^{*2}}{e_i}\sum_{k\neq i}e_i\sigma_{ik}e_ke_k\sigma_{ki}e_i. \tag{39}$$

Taking a partial derivative of (38) with respect to γ^2, and with $e_i^*\to e^\dagger$ as $\gamma\to 0$ for each i, we obtain expression (16):

$$\lim_{\gamma\to^+0}\frac{\partial\xi_i^{st}(\mathbf{e}^*,\mathbf{X}^*)}{\partial(\gamma^4)} = 2e^{\dagger 5}\sum_{k\neq i}\sigma_{ik}\sigma_{ki}.$$

For symmetric Σ (Assumption 2A) with $\sigma_{ki}=\sigma_{ik}$, we can rewrite (41) to give expression (18), as well as the corresponding (negated) expression under network anti-symmetry (Assumption 2B).

Next, we can rewrite the expression for $\xi_i^{ex}(\mathbf{X}^*,\mathbf{e})$, again expanding $\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}$:

$$\begin{aligned}
\xi_i^{ex}(\mathbf{e},\mathbf{X}^*) &= \gamma^2 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\mathbf{I_e}\Sigma\mathbf{I_e}\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\mathbf{1}_i \\
&= 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\left(\sum_{\tau=1}^{\infty}\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{\tau}\right)\mathbf{1}_i \\
&= 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}+\left(\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2\sum_{\tau=0}^{\infty}\gamma^{2\tau}\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{\tau}\right)\mathbf{1}_i \\
&= \gamma^2 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\left(\mathbf{I_e}\Sigma\mathbf{I_e}+\gamma^2\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\right)\mathbf{1}_i. \qquad (40)
\end{aligned}$$

For the second term:

$$\frac{\partial}{\partial(\gamma^2)}\left(\gamma^4 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\left(\left(\mathbf{I_e}\Sigma\mathbf{I_e}\right)^2\left(\mathbf{I}-\gamma^2\mathbf{I_e}\Sigma\mathbf{I_e}\right)^{-1}\right)\mathbf{1}_i\right)\to 0,$$

for each i, as $\gamma\to 0$. Focusing again on the first term:

$$\begin{aligned}
\gamma^2 2\frac{\beta_i^*}{e_i}\left(\boldsymbol{\beta}^*-\beta_i^*\mathbf{1}_i\right)'\mathbf{I_e}\Sigma\mathbf{I_e}\mathbf{1}_i &= \gamma^2 2\frac{\beta_i^*}{e_i}\boldsymbol{\beta}^{*\prime}\mathbf{I_e}\Sigma\mathbf{I_e}\mathbf{1}_i \\
&= \gamma^2 2\frac{\beta_i^*}{e_i}\left(\left[\sum_{k\neq i}e_k\sigma_{kj}e_j\beta_k^*\right]_{j=1}^{N}\right)'\mathbf{1}_i \\
&= \gamma^2 2\frac{\beta_i^*}{e_i}\sum_{k\neq i}e_k\sigma_{ki}e_i\beta_k^*. \qquad (41)
\end{aligned}$$

Then:

$$\lim_{\gamma\to^+0}\frac{\partial\xi_i^{ex}(\mathbf{e}^*,\mathbf{X}^*)}{\partial(\gamma^2)}=2e^{\dagger 3}\sum_{k\neq i}\sigma_{ki},$$

for each i, yielding expression (17).

For symmetric Σ with $\sigma_{ki}=\sigma_{ik}$, we can rewrite (41) to give expression (18):

$$\xi_i^{ex}(\mathbf{e},\mathbf{X}^*)\approx\gamma^2 2\frac{\beta_i^*}{e_i}\sum_{k\neq i}e_k\sigma_{ik}e_i\beta_k^*=\gamma^2 2\frac{\beta_i^*}{e_i}\left(\beta_i^*-e_i\right),$$

with the second equality using Theorem 1. This also yields the corresponding (negated) expression under network anti-symmetry (Assumption 2B).

□

Proof of Propositions 3A and 3B. For part 1 of Proposition 3A, apply the implicit function theorem to the difference $(e_i^{pb}-e_i^*)$ to give[48]:

$$\frac{\partial\left(e_i^{pb}-e_i^*\right)}{\partial(\gamma^4)}=-\frac{\frac{\partial\left(\beta_i^{*2}/e_i^{pb}+\xi_i^{st}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)\right)}{\partial(\gamma^4)}}{\frac{\partial\left(\beta_i^{*2}/e_i^{pb}+\xi_i^{st}(\mathbf{e}^{pb},\mathbf{X}^*)-\kappa'\left(e_i^{pb}\right)\right)}{\partial e_i}}+\frac{\frac{\partial\left(\beta_i^{*2}/e_i^*\right)}{\partial(\gamma^4)}}{\frac{\partial\left(\beta_i^{*2}/e_i^*-\kappa'\left(e_i^*\right)\right)}{\partial e_i}}+\sum_{k\neq i}\left(\frac{\partial e_i^{pb}}{\partial\beta_k^*}-\frac{\partial e_i^*}{\partial\beta_k^*}\right)\frac{\partial\beta_k^*}{\partial(\gamma^2)}.$$

[48] One could employ the multivariate implicit function theorem, noting that changes in e_i^{pb} and e_i^* will result as second-stage β_k^* for each $k\neq i$ adjust with γ^2. We avoid the multivariate implicit function theorem finding the total derivative, summing over partials of e_i^{pb} and e_i^* with respect to β_k^* for each $k\neq i$ (last term).

Taking the limit $\gamma \to^+ 0$ of the expression, $\lim_{\gamma\to+0} \frac{\partial \xi_i^{st}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)}{\partial e_i^{pb}} = 0$, because $\xi_i^{ex}\left(\mathbf{e}^{pb},\mathbf{X}^*\right) = 0$ at $\gamma = 0$, and $\xi_i^{ex}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)$ is $\mathcal{C}^1$ in γ. Thus, the denominators of the first two terms converge to $\kappa''\left(e^\dagger\right) - 1$, as in the proof of Corollary 1. With $e_i^* \to e_i^{pb}$ as $\gamma \to^+ 0$ with both e_i^* and e_i^{pb} $\mathcal{C}^1$ in γ, $\frac{\partial\left(\beta_i^{*2}/e_i^{pb}\right)}{\partial(\gamma^2)} \to \frac{\partial\left(\beta_i^{*2}/e_i^*\right)}{\partial(\gamma^2)}$. Again noting that $\lim_{\gamma\to+0} \frac{\partial \beta_i^*}{\partial e_i^*} = 1$ while $\lim_{\gamma\to+0} \frac{\partial e_i^*}{\partial \beta_k^*} = \lim_{\gamma\to+0} \frac{\partial e_i^{pb}}{\partial \beta_k^*} = 0$ for each $k \neq i$, implying that the second sum converges to zero as $\gamma \to^+ 0$, this leaves:

$$\lim_{\gamma\to^+0} \frac{\partial\left(e_i^{pb} - e_i^*\right)}{\partial\left(\gamma^4\right)} = \frac{\frac{\partial \xi_i^{st}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)}{\partial(\gamma^4)}}{\kappa''\left(e^\dagger\right) - 1} = \frac{e^{\dagger 5} \sum_{k\neq i} \sigma_{ik}^2}{\kappa''\left(e^\dagger\right) - 1} > 0.$$

The second equality following from Lemma 1. By continuity of all functions in γ, this positivity must hold for some neighborhood of $\gamma = 0$.

A similar expression can be derived for j, giving:

$$\lim_{\gamma\to^+0} \frac{\partial\left(e_i^{pb} - e_i^*\right)}{\partial\left(\gamma^4\right)} - \lim_{\gamma\to^+0} \frac{\partial\left(e_j^{pb} - e_j^*\right)}{\partial\left(\gamma^4\right)} = \frac{e^{\dagger 5}\left(\sum_{k\neq i} \sigma_{ik}^2 - \sum_{k\neq j} \sigma_{jk}^2\right)}{\kappa''\left(e^\dagger\right) - 1} > 0,$$

the final inequality following by assumption: $\sum_{k\neq i} \sigma_{ik}^2 > \sum_{k\neq j} \sigma_{jk}^2$. Again, by continuity of all functions in γ, this positivity must hold for some neighborhood of $\gamma = 0$.

For part 2 of Proposition 3A, again apply the implicit function theorem to the difference $(e_i^{pl} - e_i^*)$ to give:

$$\frac{\partial\left(e_i^{pl} - e_i^*\right)}{\partial\left(\gamma^2\right)} = \left(\begin{array}{c} \left(-\frac{\frac{\partial\left(\beta_i^{*2}/e_i^{pl} + \xi_i^{st}\left(\mathbf{e}^{pl},\mathbf{X}^*\right) + \xi_i^{ex}\left(\mathbf{e}^{pl},\mathbf{X}^*\right)\right)}{\partial(\gamma^2)}}{\frac{\partial\left(\beta_i^{*2}/e_i^{pl} + \xi_i^{st}(\mathbf{e}^{pl},\mathbf{X}^*) + \xi_i^{ex}(\mathbf{e}^{pl},\mathbf{X}^*) - \kappa'\left(e_i^{pl}\right)\right)}{\partial e_i}} + \frac{\frac{\partial\left(\beta_i^{*2}/e_i^*\right)}{\partial(\gamma^2)}}{\frac{\partial(\beta_i^{*2}/e_i^* + \xi_i^{st}(\mathbf{e}^{pb},\mathbf{X}^*) - \kappa'(e_i^*))}{\partial e_i}}\right) \\ + \sum_{k\neq i}\left(\frac{\partial e_i^{pl}}{\partial \beta_k^*} - \frac{\partial e_i^*}{\partial \beta_k^*}\right)\frac{\partial \beta_k^*}{\partial(\gamma^2)} \end{array}\right).$$

$$\frac{\partial\left(e_i^{pl} - e_i^*\right)}{\partial\left(\gamma^2\right)} = \left(\begin{array}{c} \left(-\frac{\frac{\partial\left(\beta_i^{*2}/e_i^{pl} + \xi_i^{st}\left(\mathbf{e}^{pl},\mathbf{X}^*\right) + \xi_i^{ex}\left(\mathbf{e}^{pl},\mathbf{X}^*\right)\right)}{\partial(\gamma^2)}}{\frac{\partial\left(\beta_i^{*2}/e_i^{pl} + \xi_i^{st}(\mathbf{e}^{pl},\mathbf{X}^*) + \xi_i^{ex}(\mathbf{e}^{pl},\mathbf{X}^*) - \kappa'\left(e_i^{pl}\right)\right)}{\partial e_i}} + \frac{\frac{\partial\left(\beta_i^{*2}/e_i^* + \xi_i^{st}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)\right)}{\partial(\gamma^2)}}{\frac{\partial(\beta_i^{*2}/e_i^* + \xi_i^{st}(\mathbf{e}^{pb},\mathbf{X}^*) - \kappa'(e_i^*))}{\partial e_i}}\right) \\ + \sum_{k\neq i}\left(\frac{\partial e_i^{pl}}{\partial \beta_k^*} - \frac{\partial e_i^{pb}}{\partial \beta_k^*}\right)\frac{\partial \beta_k^*}{\partial(\gamma^2)} \end{array}\right).$$

Taking the limit $\gamma \to^+ 0$ of the expression, $\lim_{\gamma\to+0} \frac{\partial \xi_i^{ex}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)}{\partial e_i^{pb}} = 0$, $\lim_{\gamma\to+0} \frac{\partial \xi_i^{st}\left(\mathbf{e}^{pl},\mathbf{X}^*\right)}{\partial(\gamma^2)} = 0$ from (38), while $e_i^* \to e_i^{pl}$, along with all of the limits above. This leaves:

$$\lim_{\gamma\to^+0} \frac{\partial\left(e_i^{pl} - e_i^*\right)}{\partial\left(\gamma^2\right)} = \frac{\frac{\partial \xi_i^{ex}\left(\mathbf{e}^{pb},\mathbf{X}^*\right)}{\partial(\gamma^2)}}{\kappa''\left(e^\dagger\right) - 1} = \frac{e^{\dagger 3} \sum_{k\neq i} \sigma_{ik}}{\kappa''\left(e^\dagger\right) - 1} > 0.$$

A similar expression can be derived for j, giving:

$$\lim_{\gamma\to^+0} \frac{\partial\left(e_i^{pl} - e_i^*\right)}{\partial\left(\gamma^2\right)} - \lim_{\gamma\to^+0} \frac{\partial\left(e_j^{pl} - e_j^*\right)}{\partial\left(\gamma^2\right)} = \frac{e^{\dagger 3}\left(\sum_{k\neq i} \sigma_{ik} - \sum_{k\neq j} \sigma_{jk}\right)}{\kappa''\left(e^\dagger\right) - 1} > 0,$$

the final inequality following by assumption: $\sum_{k\neq i} \sigma_{ik} > \sum_{k\neq j} \sigma_{jk}$. By continuity of all functions in γ, this positivity must hold for some neighborhood of $\gamma = 0$.

By Corollary 1 and a similar argument, $e_i^* > e_j^*$ in some neighborhood of $\gamma = 0$. Taking the meet of these two neighborhoods, as well as for each pair i, j with $\sum_{k\neq i} \sigma_{ik} > \sum_{k\neq j} \sigma_{jk}$, gives the result.

The proof of Proposition 3B is analogous to the above.

□

Supplemental Section to:
Information Acquisition and Response in Peer-effects Networks

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this draft: Wednesday 21st January, 2015

S Supplemental Section: Who is more Informed?

Theorem 2 offers an important step toward describing information acquisition under general peer effects. However, the fact that $\boldsymbol{\beta}^*$ is endogenously determined as a function of $\mathbf{e}^*$ limits this result from providing a full description of the incentives to acquire information as a function of player-position in the network. Here we reveal a basic challenge in the task of characterizing exactly who acquires more information than others. In light of this fact, we then develop a class of network structures that robustly order the relative extent of information acquisition across players, for all $\gamma > 0$ and over the set of convex κ.[1] All of the results of this Section will refrain from assumptions on the extent (or lack of) symmetry in pairwise peer-effects. Further, we can modify Assumption 1 requiring only the conditions $\kappa'(0) = 0$ and $\kappa''' \geq 0$. As shown in the proof of Proposition S.1, these will suffice for IAE existence for all $\gamma \in [0, 1]$.

Toward better understanding the players' underlining incentives to acquire information, a useful thought experiment is to walk through the best-response dynamic of the period $t = 1$ game. We allow players to simultaneously choose their preferred e_i taking as given their current sequentially rational belief $\mu^*_{ij}(e_j)$ for each $j \neq i$. Start from the profile $\mathbf{e}^{(1)} := (0, \ldots, 0)$, and for this discussion assume Assumption 1 to hold. Here, signals are neither informative of the state nor informative of the actions of neighboring players. However, each player –mindful of the positive direct effect that the state has on their marginal gains to period $t = 2$ action– will prefer to invest in (unique) quality $e^\dagger$ that solves $e^\dagger = \kappa'\left(e^\dagger\right)$ (see Example 2). Then, given positive quality profile $\mathbf{e}^{(2)} = (e^\dagger, \ldots, e^\dagger)$ and updated beliefs $\mu^*_{ij}\left(e^\dagger\right)$, correlation between players' signals is introduced. That is, players' signals now inform them of what others will see and do. Players with high degree will realize an extra *kick* to their marginal benefit to information in the first stage, as additional quality further informs them of their neighbors' $t = 2$ actions. Players with particularly low degree will also obtain information regarding what their neighbor's will see and do. However, the optimal response to "learning neighbors will likely choose high actions" moves against their private response to learning

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[1] For the former, this is provided the second-stage system yields a finite solution.

that their marginal gains to action are likely high. Thus, the net responsiveness of these players' strategies to their signals decrease. By Theorem 2, this in turn decays the incentives to acquire precise signals in the first stage.

The direction of the best-response dynamic $\left\{\mathbf{e}^{(n)}\right\}_{n=1}^{\infty}$ from $n = 3$ and on will depend on the structure of the network. Whether or not high degree players will continue to invest more in information than low degree players depends on the relative informativeness of neighbors. Thus, though information acquisition can be ordered with respect to informational centrality $\mathbf{b}\left(\Sigma^c, \mathbf{e}\right)$, whether the ordering in this measure ultimately aligns with players' degrees in equilibrium depends on both (i) more delicate properties of the network Σ, (ii) the shape of κ, and (iii) the size of γ. The potential for such sensitivity in $\mathbf{e}^*$ is illustrated with the following example.

Example S.1. *Take the six-player star network with center player* 1 *and periphery players* $i \in \{2, \ldots, 6\}$. *We assume center-periphery peer-effects to be undirected:* $\sigma_{i1} = \sigma_{1i} = p > 0$ *(while* $\sigma_{ij} = 0$ *for each pair* $i, j \in \{1, \ldots, 6\}$*). Here, the center player acquires the most information in a unique equilibrium (see Proposition S.2 below).*

Now, as depicted in Figure 1, consider adding two more players (7 and 8) with links to the center that are weaker than those of the original periphery players: links of size cp *with* $c \in [0, 1)$. *However, these additional players enjoy an added positive link of size* $q > 0$ *between each other, reinforcing their behavior. Now, players 2 through 6 are highly influenced by the most central player (player 1), while players 7 and 8 place less weight on the center but together reinforce each other's actions.*

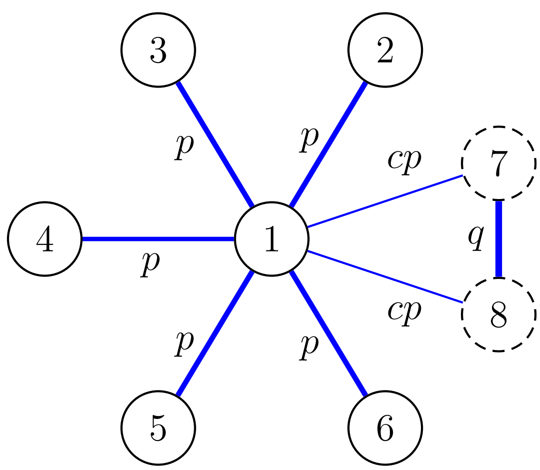


Figure 1: [Example S.1] star with clique

Taking $p = 1/5$ *and* $c = 0$, *for example. For any* $q > 1/5$ *players* $\{7, 8\}$ *have greater degree than players* $\{2, \ldots, 6\}$. *As such, players* $\{7, 8\}$ *acquire more information when* γ *is sufficiently small, by Corollary 1. The ordering in a unique* $\mathbf{e}^*$ *when* γ *is large, however, will also depend on the curvature of the cost function* κ. *Thus, take* $\gamma = 1$ *and* $q = 6.9$ *for example, we borrow again the cost function from Example 2 setting* $\zeta = 2$ *and range* η *from .5 to 2, yielding the black and* *gray cost functions depicted in Figure 2(left).*

When $\eta = .5$ (low convexity) the marginal cost of information varies mildly over a wide range of small e_i values. This results in high dispersion across equilibrium qualities. In this scenario, having access (high influence) to the center player bears heavily on the incentives to acquire a precise signal. As seen in Figure 2(right), players $\{2, \ldots, 6\}$ acquire more information than $\{7, 8\}$. If instead $\eta = 2$ (high convexity) and the marginal cost of information varies quickly over a narrow range of small e_i, equilibrium dispersion is more slight: e_1^* lies only slightly above the equilibrium qualities of the other players. In this scenario, degree centrality again most encourages information acquisition. As under small gamma, $\{7, 8\}$ acquires more information than $\{2, \ldots, 6\}$.

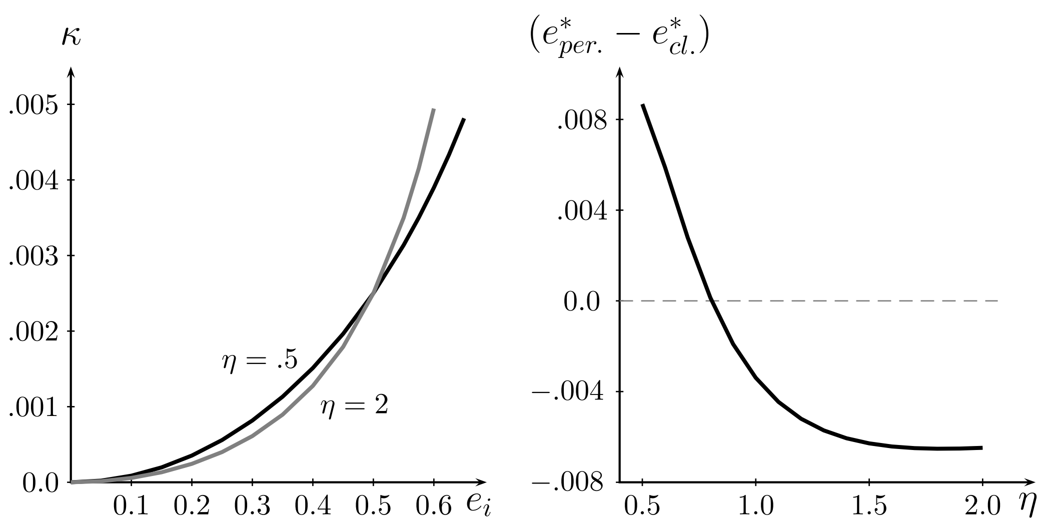


Figure 2: [Example S.1] Sensitivity in $\mathbf{e}^*$ ordering to κ.

Example S.1 illustrates the tautology that the curvature of information costs and the details of the network structure work in tandem to determine the relative extent of acquired qualities across players. This makes the goal of robustly ordering $\mathbf{e}^*$ over players using some fully portable centrality measure, defined solely over the network structure Σ, unreachable. With intercentrality (Ballester et al. (2006) [5]) and Bonacich centrality measures defined solely on Σ, a one-to-one representation of equilibrium information acquisition and the network structure can not exist. This is true even when the network is undirected and non-negative, as Example S.1 shows.

The following begins to constrain the problem of describing information acquisition in our general network setting. We establish network properties that suffice to order equilibrium qualities. This ordering will be independent over γ and hold over the set of convex κ, for at least one IAE. The properties derived will exclude examples such as the star-with-clique above, and align the essential network properties discussed in Example S.1: degree centrality and neighbors' informational centralities.

First, the following definition and equilibrium notion will help to simplify the task of describing the role of network architecture.

Definition S.1. *For given network* Σ *consider a partition* $\mathcal{P} = \{P_c\}_{c=1}^{C}$ *of* $\{1,\ldots,N\}$*, with subsets ("*classes*") indexed by* $c = 1,\ldots,C \leq N$*, where* $C := |\mathcal{P}|$.[2] *Then, player i's* weighting function $w_i : \{1,\ldots,C\} \to \mathbb{R}$ *with respect to* $\mathcal{P}$ *is defined by:*

$$w_i(c) = \sum\nolimits_{k\neq i: k\in P_c} \sigma_{ik}.$$

$\mathcal{P}$ *gives an* equivalence relation *if* $w_i(\cdot) = w_j(\cdot)$ *for each* $i,j \in P_t$ *and for every c.*

Weighting functions aggregate the weights that a given player places on the individual members of each class. We will use $w_c(\cdot)$ to denote the common weighting function of players in equivalence class P_c. Note that an equivalence relation always exists for any network: namely, the *discrete* partition of individual players. One can also find a suitably *coarse* relation that groups all players of equivalent objectives.[3] The goal of partitioning the players in this manner is to discard details of Σ less essential to the problem of information acquisition, while preserving the more germane network properties that drive equilibrium dispersion in $\mathbf{e}^*$. Conducive to this goal, for any equivalence relation $\mathcal{P}$ an equilibrium that is symmetric within classes will always exist.

Proposition S.1. [class-symmetric IAE] *For equivalence relation* $\mathcal{P}$ *and any* $\kappa \in \mathcal{C}^3$ *with* $\kappa'(0) = 0$ *and* $\kappa'', \kappa''' \geq 0$*, there exists a class-symmetric equilibrium in which* $\beta_i^* = \beta_j^*$ *and* $e_i^* = e_j^*$ *if* $i,j \in \mathcal{P}_c$ *for* $c \in \{1,\ldots,C\}$*.*

The second half of Example 1 provides an IAE that violates class symmetry. Precisely, the asymmetric equilibrium violates class symmetry when both players are included within the same class.

Reflecting again on Example S.1, we see that three classes are used to induce sensitivity in the ordering of $\mathbf{e}^*$ to the shape of κ. When players place non-negative aggregate weight on those within their class, this extent of network irregularity (i.e. three classes) is necessary to establish such sensitivity.

Proposition S.2. [two-class networks] *For equivalence relation* $\mathcal{P} = \{r,s\}$ *with* $w_r(r), w_s(s) \geq 0$ *and* $w_r(r) + w_r(s) > w_s(r) + w_s(s)$*, and any* $\kappa \in \mathcal{C}^3$ *with* $\kappa'(0) = 0$ *and* $\kappa'', \kappa''' \geq 0$*, there exists a class-symmetric equilibrium such that* $e_r^* \geq e_s^*$*, and where if* $e_r^*, e_s^* \in (0,1)$ *then* $e_r^* > e_s^*$ *with* $\beta_r^* > \beta_s^*$*.*

Note that given $e_r^* > e_s^*$, $\beta_r^* > \beta_s^*$ in the last statement of the theorem is equivalent to $\beta_r^* > 0$ by Theorem 2. Thus, signal responses are ordered with the highest degree class moving positively with their signal. Allowing for $\beta_s^* < 0$, Proposition S.2 captures a striking equilibrium property. For a class s moving against their information, anticipating the actions of players in P_r, each $j \in P_s$ chooses a quality that is bounded above by e_r^*. With each j's signal used merely to infer the the actions of those in P_r, and with e_r^* intrinsically bounding the extent of this inference, e_r^* provides a

[2] That is, $\bigcup \mathcal{P} = N$ with $P_s \cap P_{s'} = \varnothing$ for distinct $s, s' \in \{1,\ldots,C\}$.

[3] Equivalent in the sense that players within a class set place equivalent weights on other classes. Note that given partitions $\mathcal{P}^1$ and $\mathcal{P}^2$ one can construct coarser partition $\mathcal{P}$ by joining elements $P^1 \in \mathcal{P}^1$ and $P^2 \in \mathcal{P}^2$ to give $P^1 \cup P^2 = P \in \mathcal{P}$ when $P^1 \cap P^2 \neq \emptyset$. That is, a coarsesed set of partitions can be obtained, most often being s single coarsesed set pooling interchangeable players.

natural bound on j's incentives to acquire information.[4] This natural bound can be clearly observed above in Figure 3(right).

Next, the following notions allow for any arbitrary number of classes, and establish alternative conditions on the network structure that suffice for an ordering in $\mathbf{e}^*$, again robust to the relative convexity in κ. This family of *class-ordered* networks will offer a generalization of core-periphery-like structures, incorporating signed, weighted, and directed links. Note that the following ordering in $\mathcal{P}$ is defined solely using properties of the network Σ.

Definition S.2. *We say that class r dominates class s (denoted $r \succsim s$) if the following two conditions hold:*

1. *w_r crosses w_s at most once from below: $w_r(c) \geq w_s(c)$ if $c \geq x$ and $w_r(c) \leq w_s(c)$ if $c \leq x$ for some $x \in \{1, \ldots, C\}$, and*

2. *players in P_r have degree no smaller than players in P_s:*

$$\sum_{c=1}^{C} w_r(c) \geq \sum_{c=1}^{C} w_s(c). \tag{1}$$

r strictly dominates s (denoted $r \succ s$) if the inequality in (1) is strict.

The cumulative ordering (1) with single crossing in condition 1 imply that more central classes are more influenced by others (have higher degree), and that these classes tend to place relatively more weight on the most central players.

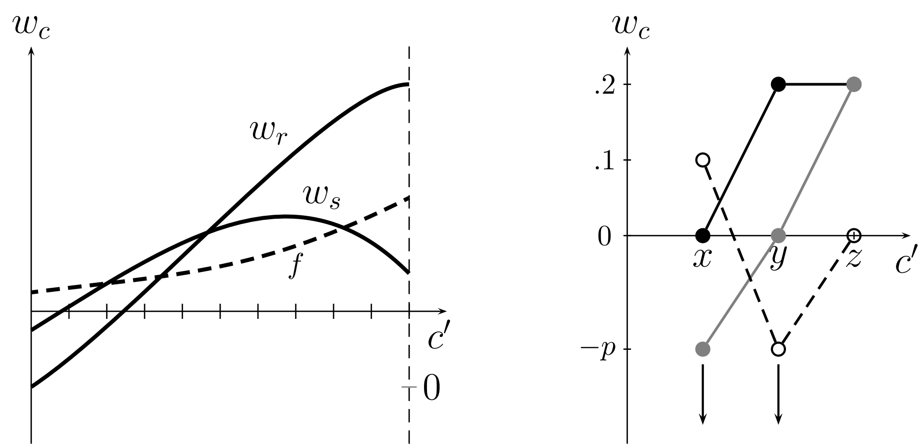


Figure 3: **Left** *Dominance* orders weighting functions to aggregate any non-negative, non-decreasing f in similar order. **Right** The network in Example 3 (Figure 4) is class ordered for all $p > 0$.

[4] The qualification $e_r^* < 1$ is needed to exclude equilibria in which the classes coordinate on simultaneously acquiring perfectly precise signals in order to move against them.

From a technical vantage point, dominance gives an appealingly weak condition that suffices for the relative weighting functions to aggregate any non-negative, non-decreasing function f in similar order. That is, and as illustrated in Figure 3 (left), $r \succsim s$ implies that $w_r(c')$ must lie weakly above $w_s(c')$ for the highest classes c' which give the greatest values $f(c')$. Formally, this gives the following lemma.

Lemma S.1. *If $r \succsim s$ then:*

$$\sum_{c=1}^{C} f(c)\, w_r(c) \geq \sum_{c=1}^{C} f(c)\, w_s(c)\,, \tag{2}$$

for any non-decreasing function f on $\{1, \ldots, C\}$: $f(c') \geq f(c) \geq 0$ for $c' \geq c$. If $r \succ s$ then the inequality in (2) is strict.

The proof of this is simple to obtain and is provided in the appendix. The following class of network structures can now be defined. Note that the ordering in index $\{1, \ldots, C\}$ has thus far been immaterial. Here, however, the ordering in $\mathcal{P}$ plays a more central role.

Definition S.3. *The network Σ is class ordered if there is an equivalence relation $\mathcal{P}$ such that for each $r \in \{2, \ldots, C\}$ we have $r \succsim r-1$. The network Σ is strictly class ordered if $r \succ r-1$ for each $r > 1$.*

The class orderedness of a network establish a definitive ordering amongst its classes. The most connected nodes will place proportionally more of their weight on precisely those classes that are most connected in the network. Above in Example 3, Definition S.3 is satisfied under class ordering $x \succsim y \succsim z$ (see Figure 4, above). Each class's weighting function is plotted in Figure 3 (right). w_c exhibits dominance between adjacent classes: w_y single crossing w_z from below for all $p \geq 0$.[5]

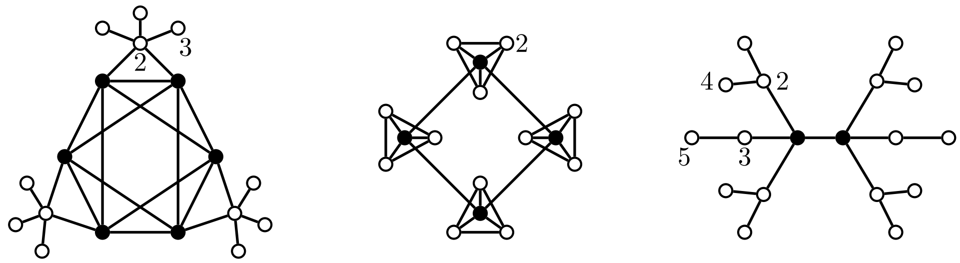
Though examples of networks of two classes may come readily (e.g. star, circle-spoke), the range of class-ordered networks may be less obvious to the reader. The following example lends to the scope of class-ordered structures.

Example S.2. *The binary networks given in Figure 4 where each link designates positive peer effect $\sigma_{ij} = p > 0$ are class ordered. The most central class (i.e. the "core") are given with solid nodes, with the subsequent ordering over classes designated for representative members. Alternatively, all of these examples are also class ordered for $p < 0$ with the ordering over classes reversed.*

We see that class-ordered networks encompass a wide range of structures exhibiting a natural ordering over its players. These networks can be viewed as a generalized family of core-periphery like structures, allowing for weighted links that may be positive, negative, or directed. Many hierarchical[6] social settings will embody these properties. And in network formation environments,

[5] Note that here, $x \succsim z$. Such transitivity need not hold for the network to be class ordered.

[6] I thank Anja Prummer for suggesting the natural application to social hierarchies.



standard core-periphery small-world tree

Figure 4: [Example S.2] class-ordered networks

many related models of investment with endogenous link formation –both under strategic substitutes (Bala and Goyal (2000) [4]) and complements (Hiller (2013) [33])– have been shown to yield core-periphery structures.[7]

We come to the main result of the section. When the network of peer effects takes on the above ordering, the following *class-ordered* equilibria always exist.

Proposition S.3. [class-ordered equilibria] *If Σ is class ordered, taking $r, s \in \{1, \ldots, C\}$ with $r \succsim s$ and constrain $\gamma \in [0, \gamma^s)$. Then, for any $\kappa \in \mathcal{C}^3$ with $\kappa'(0) = 0$, and $\kappa'', \kappa''' \geq 0$, there exists a class-symmetric equilibrium such that $e_r^* \geq e_s^*$.*

Thus, provided players always move in the direction of their signals, player degree robustly orders signal responsiveness independent of the convexity of κ.

The appealing property of class-ordered networks is that highly central players (here, players with the highest degree) proportionally place more of their weight on players that are also of high centrality. Definition S.3 provides an ordering underlining such nested weighting. In class-ordered networks, this ordering captures both value to having high degree with the value to being connected to the most informed players. In a class-ordered equilibria, it is precisely the neighbors with greatest degree who are most informed.

Returning the two-sided market application, if highly connected insiders are also those that enjoy exclusivity in their clientele, informational centrality will likely be ordered according to degree, with the network adopting a class-ordered structure. If instead the more connected insiders tend to compete with each other for workers, as in the case of Figure 11, the ultimate informational centralities realized by each insider will more intimately depend on the shape of κ. Akin to Example 3, when κ displays significant elasticity yielding moderate dispersion in $\mathbf{e}^*$, degree centrality will dominate. If instead κ displays moderate elasticity yielding significant dispersion in $\mathbf{e}^*$, exclusivity will drive information centrality. While all insiders on the sufficiently short side of the market under acquire information relative to the utilitarian benchmark, exactly who most acquires and simultaneously most *under* acquires information will depend on the precise properties of Σ and κ.

[7] Refer to Calvó-Armengol et al. (2011) [13] Section 5.2 for class of "hierarchical" structures that yield properties similar to class orderedness.

B Supplemental Appendix

B.1 Section S proofs: Class-ordered networks

Proof of Proposition S.1. Take the compact subspace of $[0,1]^N$ comprising all class-symmetric vectors $\mathbf{e}$:

$$\mathcal{E}^s := \left\{\mathbf{e} \in [0,1]^N : e_i = e_j \text{ if } i,j \in P \in \mathcal{P}\right\}.$$

Note that $\mathcal{E}^s$ is a closed subset of a compact space, and is thus compact. Now, take the incentive-compatible first-stage best response correspondence for player i:

$$\begin{aligned} BR_i\left(e_i|\mu_i^*,\boldsymbol{\beta}^*\right) &= \arg\max_{e_i\in[0,1]} \mathbb{E}_i\left[u_i\left(\mathbf{X}^*|\omega,\omega_i\right)|e_i,\mu_i^*\right] - \kappa\left(e_i\right), \\ &= \arg\max_{e_i\in[0,1]} \frac{1}{2}\beta_i^{*2} - \kappa\left(e_i\right), \end{aligned} \tag{3}$$

which holds $\boldsymbol{\beta}_{-i}^*$ and μ_i^* fixed but allows β_i^* to optimally adjust to e_i. The second equality uses expression (8) derived in Section A.3. First, by the compactness of $[0,1]$ and continuity of β_i^* and of $\kappa(e_i)$ in e_i,[8] $BR_i\left(e_i|\mu_i^*,\boldsymbol{\beta}^*\right)$ is non-empty by the Weierstrass extreme-value theorem.

By construction, the set:

$$\left[BR_i\left(e_i|\mu_i^*,\boldsymbol{\beta}^*\right)\right] \cap \mathcal{E}^s$$

is non-empty, and thus the restriction:

$$\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right) := \left[BR_i\left(e_i|\mu_i^*,\boldsymbol{\beta}^*\right)\right] \cap \mathcal{E}^s, \tag{4}$$

is a well defined vector-valued mapping from $\mathcal{E}^s \to \mathcal{E}^s$. By continuity of $\boldsymbol{\beta}^*$ and κ in $\mathbf{e} \in [0,1]$ a compact set, and applying the Maximum theorem, $\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right)$ is upper hemicontinuous. Marginal gains to information are given by:

$$\beta_i^{*2}/e_i = e_i\left(1 + \sum_{k\neq 1}\sigma_{ik}e_k\beta_k^*\right)^2$$

by Theorem 1, which is linear in e_i by incentive compatability (μ_k^* and β_k^* are held fixed) and obtains $\beta_i^{*2}/e_i = 0$ at $e_i = 0$. When $\kappa'(0) = 0$ and $\kappa''' \geq 0$, each $\overline{BR}_i\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right)$ is convex valued: if $\beta_i^{*2}/e_i > \kappa'(e_i)$ for some e_i then $\beta_i^{*2}/e_i' > \kappa'(e_i')$ for each $e_i' > e_i$, and if $\beta_i^{*2}/e_i < \kappa'(e_i)$ then $\beta_i^{*2}/e_i' < \kappa'(e_i')$ for each $0 < e_i' < e_i$ (excluding $e_i = 0$ which gives a minimum).[9] $\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right)$ then gives a convex polyhedron in $[0,1]^N$. Then, by Kakutani's fixed point theorem, $\overline{BR}\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right)$ yields a fixed point in $\mathcal{E}^s$. By construction of $\mathcal{E}^s$, the properties of the fixed point satisfy those of the theorem.

□

Proof of Proposition S.2. Assuming quality profile $e_r \geq e_r$ we show that there exists a first-stage best response for class r weakly above every best response for class s. Write the system giving the IRE as a function of (e_r, e_s):

[8] Continuity follows from Assumptions A1 and $\kappa \in \mathcal{C}$.

[9] With $\kappa'(0) = 0$ and $\kappa''' \geq 0$, each $\overline{BR}_i\left(\mathbf{e},\boldsymbol{\beta}^*,\boldsymbol{\mu}^*\right)$ will either (i) give a unique value in $[0,1]$ if $\kappa''' > 0$ or (i) give a corner or the entire unit interval if $\kappa''' = 0$ (quadratic κ).

$$\begin{array}{l}[1]\ \beta_r^* - \left(e_r + \gamma^2 e_r \left(w_r(r) e_r \beta_r^* + w_r(s) e_s \beta_s^*\right)\right) = 0 \\ [2]\ \beta_s^* - \left(e_s + \gamma^2 e_s \left(w_s(s) e_s \beta_s^* + w_s(r) e_r \beta_r^*\right)\right) = 0\end{array}.$$

Together these imply:

$$\beta_r^* = \frac{e_r\left(1+\gamma^2 e_s^2\left(w_r(s)-w_s(s)\right)\right)}{\left(1-\gamma^2 w_r(r) e_r\right)\left(1-\gamma^2 w_s(s) e_s\right)-\gamma^4 w_r(s) w_s(r) e_r^2 e_s^2}, \quad (5)$$

$$\beta_s^* = \frac{e_s\left(1+\gamma^2 e_r^2\left(w_s(r)-w_r(r)\right)\right)}{\left(1-\gamma^2 w_r(r) e_r\right)\left(1-\gamma^2 w_s(s) e_s\right)-\gamma^4 w_r(s) w_s(r) e_r^2 e_s^2}. \quad (6)$$

Multiplying by e_r and e_s, respectively:

$$\begin{aligned} e_r\beta_r^* &= \frac{e_r^2\left(1+\gamma^2 e_s^2\left(w_r(s)-w_s(s)\right)\right)}{\left(1-\gamma^2 w_r(r) e_r\right)\left(1-\gamma^2 w_s(s) e_s\right)-\gamma^4 w_r(s) w_s(r) e_r^2 e_s^2}, \\ e_s\beta_s^* &= \frac{e_s^2\left(1+\gamma^2 e_r^2\left(w_s(r)-w_r(r)\right)\right)}{\left(1-\gamma^2 w_r(r) e_r\right)\left(1-\gamma^2 w_s(s) e_s\right)-\gamma^4 w_r(s) w_s(r) e_r^2 e_s^2}. \end{aligned}$$

With $e_r \geq e_s$, then $e_r\beta_r^* \geq e_s\beta_s^*$ is implied by:

$$\begin{aligned} w_r(s) - w_s(s) &> w_s(r) - w_r(r) \\ \Leftrightarrow w_r(s) + w_r(r) &> w_s(r) + w_s(s), \end{aligned}$$

which is assumed.

Now, rewriting the system as:

$$\begin{array}{l}[1]\ \beta_r^{*2} = e_r\beta_r^*\left(1+\gamma^2\left(w_r(r) e_r\beta_r^* + w_r(s) e_s\beta_s^*\right)\right) \\ [2]\ \beta_s^{*2} = e_s\beta_s^*\left(1+\gamma^2\left(w_s(s) e_s\beta_s^* + w_s(r) e_r\beta_r^*\right)\right)\end{array}.$$

$$\begin{aligned} \beta_r^{*2} - \beta_s^{*2} &= e_r\beta_r^* - e_s\beta_s^* + \gamma^2\left(e_r\beta_r^*\left(w_r(r) e_r\beta_r^* + w_r(s) e_s\beta_s^*\right) - e_s\beta_s^*\left(w_s(s) e_s\beta_s^* + w_s(r) e_r\beta_r^*\right)\right) \\ &= e_r\beta_r^* - e_s\beta_s^* + \gamma^2 e_s\beta_s^* e_r\beta_r^*\left(\left(w_r(r)\frac{e_r\beta_r^*}{e_s\beta_s^*} + w_r(s)\right) - \left(w_s(s)\frac{e_s\beta_s^*}{e_r\beta_r^*} + w_s(r)\right)\right) \\ &= e_r\beta_r^* - e_s\beta_s^* + \gamma^2 e_s\beta_s^* e_r\beta_r^*\left(\begin{array}{c} w_r(r)\left(\frac{e_r\beta_r^*}{e_s\beta_s^*} - 1\right) - w_s(s)\left(\frac{e_s\beta_s^*}{e_r\beta_r^*} - 1\right) \\ + \left(w_r(r) + w_r(s)\right) - \left(w_s(s) + w_s(r)\right)\end{array}\right) \\ &= e_r\beta_r^* - e_s\beta_s^* + \gamma^2 e_s\beta_s^* e_r\beta_r^*\left(\begin{array}{c} \left(e_r\beta_r^* - e_s\beta_s^*\right)\left(\frac{w_r(r)}{e_s\beta_s^*} + \frac{w_s(s)}{e_r\beta_r^*}\right) \\ + \left(w_r(r) + w_r(s)\right) - \left(w_s(s) + w_s(r)\right)\end{array}\right) \\ &= \left(\begin{array}{c} \left(e_r\beta_r^* - e_s\beta_s^*\right)\left(1+\gamma^2\left(w_r(r) e_r\beta_r^* + w_s(s) e_s\beta_s^*\right)\right) \\ +\gamma^2 e_s\beta_s^* e_r\beta_r^*\left(w_r(r) + w_r(s)\right) - \left(w_s(s) + w_s(r)\right)\end{array}\right). \end{aligned}$$

If $w_r(r)$ and $w_s(s)$ are positive, then $\beta_r^{*2} - \beta_s^{*2} \geq 0$ is implied by $\left(w_r(r) + w_r(s)\right) - \left(w_s(s) + w_s(r)\right) > 0$ (with strict inequality when $e_s, e_r > 0$), which are all assumed. Take any class-symmetric $\mathbf{e}$ and $\boldsymbol{\beta}$ that satisfy $e_r \geq e_s$. Again denote $BR(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*) := [BR_i(e_i|\mu_i^*, \boldsymbol{\beta}^*)]$ from the proof of Proposition S.1. For each $j \in P_s$ and any $e_j \in BR_j(e_j|\mu_j^*, \boldsymbol{\beta}^*)$, by Theorem 2 we must have either $e_j = 1$ with $e_j\kappa'(e_j) < \beta_j^{*2}$ or $e_j < 1$ with $e_j\kappa'(e_j) = \beta_j^{*2}$ for e_j to be a best response. Thus, in either case by $\beta_r^{*2} \geq \beta_s^{*2}$, the marginal gain $\beta_r^{*2}/e_j \geq \beta_s^{*2}/e_j$ when e_i is set to e_j, implying that any $i \in P_r$ would have a profitable deviation up away from e_j. This then implies existence of some $e_i \in BR_i\left(\mathbf{e}|\mu_i^*, \boldsymbol{\beta}^*\right) \geq e_j$.

Now take the compact subspace of $[0,1]^2$ that includes all weakly increasing class-symmetric vectors $\mathbf{e}$: $\mathcal{E}^+ := \left\{\mathbf{e} \in [0,1]^2 : e_i \geq e_j, \, i \in P_r, j \in P_s\right\}$. Note that $\mathcal{E}^+$ is a closed subset of a compact space, and is thus compact. By the above, $BR\left(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\right) \cap \mathcal{E}^s \cap \mathcal{E}^+$ is non-empty, and thus the restriction:

$$\overline{\overline{BR}}\left(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\right) := BR\left(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\right) \cap \mathcal{E}^s \cap \mathcal{E}^+,$$

where $\mathcal{E}^s$ is given by (B.1), is a well defined mapping from $\mathcal{E}^s \cap \mathcal{E}^+ \to \mathcal{E}^s \cap \mathcal{E}^+$. By continuity of $\boldsymbol{\beta}^*$ and κ in $\mathbf{e} \in [0,1]$ a compact set, and applying the Maximum theorem, $\overline{\overline{BR}}\left(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\right)$ is upper-hemicontinuous. $\kappa'(0) = 0$ and $\kappa''' \geq 0$ again suffice for $\overline{BR}\left(\mathbf{e}, \boldsymbol{\beta}^*, \boldsymbol{\mu}^*\right)$ to be convex valued (see proof of Theorem S.1). By Kakutani's fixed point theorem, $\overline{\overline{BR}}\left(\mathbf{e}|\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\right)$ yields a fixed point in $\mathcal{E}^s \cap \mathcal{E}^+$.

Finally, we show that $e_r^* > e_s^*$ and $\beta_r^* > 0$ when $e_r^*, e_s^* \in (0,1)$. Rewriting (5) and (6) evaluated at IAE with $e_r^* \geq e_s^*$:

$$\begin{aligned}
\frac{\beta_r^*}{e_r^*} &= \frac{\left(1 + \gamma^2 e_s^{*2}\left(w_r(s) - w_s(s)\right)\right)}{\left(1 - \gamma^2 w_r(r) e_r^*\right)\left(1 - \gamma^2 w_s(s) e_s^*\right) - \gamma^4 w_r(s) w_s(r) e_r^{*2} e_s^{*2}}, \\
\frac{\beta_s^*}{e_s^*} &= \frac{\left(1 + \gamma^2 e_r^{*2}\left(w_s(r) - w_r(r)\right)\right)}{\left(1 - \gamma^2 w_r(r) e_r^*\right)\left(1 - \gamma^2 w_s(s) e_s^*\right) - \gamma^4 w_r(s) w_s(r) e_r^{*2} e_s^{*2}}.
\end{aligned}$$

If $\beta_r^* < 0$, this impies that $\gamma^2 e_s^{*2}\left(w_r(s) - w_s(s)\right) < 1$, which implies also that $\gamma^2 e_r^{*2}\left(w_s(r) - w_r(r)\right) < 1$ and $\gamma^2 e_r^{*2}\left(w_s(r) - w_r(r)\right) < \gamma^2 e_s^{*2}\left(w_r(s) - w_s(s)\right)$ by $\left(w_s(r) - w_r(r)\right) < \left(w_r(s) - w_s(s)\right)$ and $e_r^{*2} \geq e_s^{*2}$. Thus, $\beta_s^*/e_s^* < \beta_r^*/e_r^*$, implying that $\beta_s^{2*}/e_s^* > \beta_r^{*2}/e_r^*$, and thus by Theorem 2 that $e_s^* > e_r^*$ as $e_r^* < 1$, yielding a contradiction. Thus $\beta_r^* > 0$ with $\beta_r^{*2} > \beta_s^{*2}$ by the above, implying that $e_r^* > e_s^*$.

□

Proof of Lemma S.1. Let x be defined as in Definition S.2. Rearranging the second part of Definition S.2 gives:

$$\sum\nolimits_{c \geq x} \left(w_r(c) - w_s(c)\right) \geq \sum\nolimits_{c < x} \left(w_s(c) - w_r(c)\right). \tag{7}$$

Then, rearranging the result:

$$\begin{aligned}
\sum_{c=1}^{C} f(c) w_r(c) - \sum_{c=1}^{C} f(c) w_s(c) &= \begin{pmatrix} \sum_{c \geq x} f(c)\left(w_r(c) - w_s(c)\right) \\ -\sum_{c<x} f(c)\left(w_s(c) - w_r(c)\right) \end{pmatrix} \\
&\geq \begin{pmatrix} \sum_{c \geq x} f(x)\left(w_r(c) - w_s(c)\right) \\ -\sum_{c<x} f(x)\left(w_s(c) - w_r(c)\right) \end{pmatrix} \\
&= f(x) \begin{pmatrix} \sum_{c \geq x} \left(w_r(c) - w_s(c)\right) \\ -\sum_{c<x} \left(w_s(c) - w_r(c)\right) \end{pmatrix} \\
&\geq 0.
\end{aligned}$$

The first inequality follows from $f(\cdot)$ non-decreasing, while the second inequality follows from (7) and $f(x) \geq 0$. The final inequality is strict if $f(c) > 0$ for each c and $r \succ s$.

□

Proof of Proposition S.3. We use class indices for all strategies and weighting functions, when convenient. First, we will need the following definitions and Lemma. Take $i \in P_r \in \mathcal{P}$ and $j \in P_s \in \mathcal{P} \setminus \{P_r\}$. Take any class-symmetric $\mathbf{e}$ that satisfies the conditions of the theorem. The set of class-ordered profiles:

$$\mathcal{E}^+ := \left\{\mathbf{e} \in [0,1]^N : \ e_i \geq e_j \text{ or each } i \in P_r, j \in P_s \text{ with } r \geq s\right\}$$

is a closed, compact subset of $\mathbb{R}^{2N}$. By i's first order condition of the IRE:

$$\beta_i^* = \beta_r^* = e_r \left(1 + \gamma^2 \sum_c w_r(c)\, e_c \beta_c\right). \tag{8}$$

Denote as a function of $\mathbf{e}$ and $\boldsymbol{\beta}$:

$$\Lambda_r := \sum_c w_r(c)\, e_c \beta_c.$$

Λ_r captures the size of the aggregate peer effect on i in $\boldsymbol{\beta}^*$. Analogous expressions can be derived for class s. By our choice of $(\mathbf{e}, \boldsymbol{\beta})$ and with $\gamma \in [0, \gamma^s)$, $e_c\beta_c$ is non-negative and non-decreasing across classes. By the class orderedness of Σ and Lemma S.1, the factor $(1 + \gamma^2 \sum_c w_r(c)\, e_c\beta_c)$ must also be increasing across classes, and by our choice of $\mathbf{e}$ the vector of optimal responses $\boldsymbol{\beta}^*$ must also respect the ordering $\beta_r^* \geq \beta_s^*$ if and only if $r \geq s$. We can now establish the following Lemma.

Lemma S.B.1. *If $\gamma \leq \gamma^s$, Σ is class ordered and $\mathbf{e}$ is class ordered (i.e. weakly increasing across classes), then for $i \in P_r$, $j \in P_{r-1}$, and for every $e_j \in BR_j\left(e_j|\mu_j^*, \boldsymbol{\beta}^*\right)$, there exists $e_i \in BR_i\left(e_i|\mu_i^*, \boldsymbol{\beta}^*\right)$ with $e_i \geq e_j$.*

Proof of Lemma S.B.1. Again use $BR_i\left(e_i|\mu_i^*, \boldsymbol{\beta}^*\right)$ to denote i's first-stage incentive-compatible best response. If $\beta_s^* \geq 0$ then $\beta_r^{*2} \geq \beta_s^{*2}$. For any $e_j \in BR_j\left(e_j|\mu_j^*, \boldsymbol{\beta}^*\right)$, by Theorem 2 we must have either $e_j = 1$ with $e_j\kappa'(e_j) < \beta_j^{*2}$ or $e_j < 1$ with $e_j\kappa'(e_j) = \beta_j^{*2}$ for e_j to be a best response. Thus, in either case by $\beta_r^{*2} \geq \beta_s^{*2}$, the marginal gain $\beta_r^{*2}/e_j \geq \beta_s^{*2}/e_j$ when e_i is set to e_j, implying that any $i \in P_r$ would have a (weak) profitable deviation up away from e_j. This then implies existence of some $e_i \in BR_i\left(\mathbf{e}|\mu_i^*, \boldsymbol{\beta}^*\right) \geq e_j$.

□

The proof proceeds analogous to that of Proposition S.1. Take $BR_i\left(e_i|\mu_i^*, \boldsymbol{\beta}^*\right)$ the incentive-compatible best-response correspondence for i in her first-stage problem, holding $\boldsymbol{\mu}_{-i}^*$ and $\boldsymbol{\beta}_{-i}^*$ fixed. The set:

$$\left[BR_i\left(e_i|\mu_i^*, \boldsymbol{\beta}^*\right)\right] \cap \mathcal{E}^s \cap \mathcal{E}^+$$

is non-empty by construction,[10] and thus the restriction:

$$\overline{\overline{BR}}\left(\mathbf{e}, \boldsymbol{\beta}^*, \boldsymbol{\mu}^*\right) := \left[BR_i\left(e_i|\mu_i^*, \boldsymbol{\beta}^*\right)\right] \cap \mathcal{E}^s \cap \mathcal{E}^+$$

is a well defined vector-valued mapping from $\mathcal{E}^s \cap \mathcal{E}^+ \to \mathcal{E}^s \cap \mathcal{E}^+$. By continuity of $\boldsymbol{\beta}^*$ and κ in $\mathbf{e} \in [0,1]$ a compact set, and applying the Maximum theorem, $\overline{\overline{BR}}\left(\mathbf{e}, \boldsymbol{\beta}^*, \boldsymbol{\mu}^*\right)$ is upper-hemicontinuous. $\kappa'(0) = 0$ and $\kappa''' \geq 0$ again suffice for $\overline{BR}\left(\mathbf{e}, \boldsymbol{\beta}^*, \boldsymbol{\mu}^*\right)$ to be convex valued (see proof of Theorem S.1). By Kakutani's fixed point theorem, $\overline{\overline{BR}}\left(\mathbf{e}, \boldsymbol{\beta}^*, \boldsymbol{\mu}^*\right)$ yields a fixed point in $\mathcal{E}^s \cap \mathcal{E}^+$. By construction of $\mathcal{E}^s$ and $\mathcal{E}^+$, the properties of the fixed point satisfy those of the theorem.

□

[10] To include $r > s+1$, Lemma S.B.1 is used here $r - s$ times.