

# Giuseppe Savaré - CURRICULUM

## PERSONAL INFORMATION

Savaré Giuseppe                      ORCID: 0000-0002-0104-4158    MatSciNet: 336952  
ResearcherID: H-3651-2014    Scopus: 6602154611                      Google Scholar: w7qH1YYAAA  
Nationality: Italian                      Born in Pavia, 25/08/1966

## EDUCATION

1984–1988: *Laurea (cum laude) in Mathematics*, University of Pavia, Italy

## CURRENT POSITION

2020-: *Full Professor, Mathematical Analysis*. Department of Decision Sciences, Bocconi University

## PREVIOUS POSITIONS

2000–2020: *Full Professor, Mathematical Analysis* Department of Mathematics, University of Pavia

1998–2000: *Associate professor*, University of Pavia

1990–1998: *Researcher*, IMATI-CNR, Pavia

## FELLOWSHIPS AND AWARDS

2019-2023: *Hans Fischer senior fellow*, Institute for Advanced Study (IAS), Technical University of Munich

2019: *Plenary Speaker*, XXI Congresso Unione Matematica Italiana

2016: *Invited Speaker*, 7<sup>th</sup> European Congress of Mathematics, Berlin: section “Analysis and PDEs”

2015: *John von Neumann Visiting Professor*, Technical University of Munich

2014: the results on Gradient Flows has been presented in the *Exposé Bourbaki 1065: Flots de gradient dans les espaces métriques et leurs applications d’après Ambrosio-Gigli-Savaré*, Asterisque 361.

2012: The paper *Chemical reactions as  $\Gamma$ -limit of diffusion* SIAM Rev. 54 (2012) (with M. Peletier, M. Veneroni) has been published as SIAM Review’s SIGEST award.

2011: *Ennio De Giorgi Prize*, awarded by the *Italian Mathematical Union*

1994: *Gioachino Japichino Prize*, awarded by the *Accademia Nazionale dei Lincei*

## SUPERVISION OF GRADUATE STUDENTS

*Bocconi University:*

current: Alessandro Pinzi

*IAS-TUM, Munich:*

2022: [Giacomo Sodini](#) (co-advised with M. Fornasier)

*University of Pavia:*

2019: [Nicolò De Ponti](#)

2017: Luca Minotti

2015: [Giovanni Bonaschi](#) (co-advised with M. Peletier, Eindhoven)

2014: [Dario Mazzoleni](#) (co-advised with A. Pratelli, Erlangen)

2011: Luca Natile

2007: [Marco Veneroni](#)

2006: [Stefano Lisini](#)

2005: [Riccarda Rossi](#)

1998: [Simona Sanfelici](#) (co-advised with P. Colli Franzone)

I also mention the supervision of the Master thesis (subsequently published in international mathematical journals) of [Federico Bassetti](#), [Laura Spinolo](#), [Sara Daneri](#), [Giulia Luise](#)

## POSTDOCTORAL FELLOWS

current: [Luca Tamanini](#)

2017-2019: [Giulia Cavagnari](#)

2016-2017: [Dario Mazzoleni](#)

2015-2016: [Carlo Orrieri](#), [Matteo Muratori](#)

2009-2011: [Edoardo Mainini](#)

2007-2008: [Daniel Matthes](#)

2006-2008: [Antonio Marigonda](#)

## TEACHING ACTIVITIES

*University of Pavia:*

1998-2020: courses of *Mathematical Methods* (Engineering Faculty) and *Mathematical Analysis* (degree in Mathematics), typically two main courses every year, ~140 hours/year

2000-2020: several Ph.D. courses (PDEs, Semigroups, Calculus of Variations, Optimal transport)

*Bocconi University:*

2020-2022: *Mathematical Analysis* (Bachelor), *Introduction to Real Analysis* (Ph.D.)

Several invited courses to *international advanced schools* (as CIME, HIM, SNS, EVEQ)

## ORGANISATION OF SCIENTIFIC MEETINGS

2023: Workshop on *Variational and geometric structures for evolution*, CIRM, Trento (Co-organizers: D. Knees, R. Rossi, M. Thomas)

2023: Workshop on *Optimal Transport, Mean-Field Models, and Machine Learning*, IAS-TUM, Munich (Co-organizers: D. Knees, R. Rossi, M. Thomas)

2022, 2018, 2016, 2014, 2012, 2010, 2008: *Workshops on Optimal Transportation and Applications*, Pisa (Co-organizers: L. Ambrosio, G. Buttazzo, N. Gigli)

2022: *Contemporary Trends in Kinetic Theory and PDEs*, Pavia (Co-organizers: J.A. Carrillo, A. Pulvirenti, M. Zanella)

2018: School on *Optimal transport: numerical methods and applications*, Lake Como School of Advanced Studies (Co-organizer: F. Santambrogio).

2018: Workshop *Optimal Control and Mean Field Games*, Pavia (Co-organizers: G. Cavagnari, S. Lisini, C. Orrieri)

2016: Bimester on *Nonlinear Flows*, Research Centre ESI, Vienna (Co-organizers: E. Feireisl, A. Juengel, A. Mielke, U. Stefanelli)

2014 and 2011: MFO Workshop *Variational Methods for Evolution*, Oberwolfach (Co-organizers: L. Ambrosio, A. Mielke, M. Peletier, F. Otto, U. Stefanelli)

2011: Conference *Analysis and Numerics of PDEs. In memory of Enrico Magenes*, Pavia

2010: BIRS Workshop: *Rate-independent systems: Modeling, Analysis, and Computations*, Banff (Co-organizer: U. Stefanelli)

2008: CIME Course: *Nonlinear Partial Differential Equations and Applications*, Cetraro (Co-organizer: L. Ambrosio)

## INSTITUTIONAL RESPONSIBILITIES

2019: Director of the *Advanced School of Ph.D. Higher Education (SAFD)*, University of Pavia

2014–2016: *University Assessment Commission*, University of Pavia

2001-2008: Director of the *Ph.D. program in Mathematics and Statistics*, University of Pavia

1998–present: IMATI-CNR, research associate

## REVIEWING ACTIVITIES

2023–: *ESAIM: Control, Optimisation and Calculus of Variations*, member of the *editorial board*

2018–: *Unione Matematica Italiana*, member of the *Scientific Advisory Board*

2014–: *C.I.M.E. Foundation*, member of the *Scientific Advisory Board*

2016–: *Applied Mathematics and Optimization*, member of the *editorial board*

2013–: *Potential Analysis*, member of the *editorial board*

## MEMBERSHIPS OF SCIENTIFIC SOCIETIES

2009–: *Membro Corrispondente, Istituto Lombardo, Accademia di Scienze e Lettere*, Milano

## MAJOR COLLABORATIONS

I have collaborated with more than 50 different co-authors. Among the main collaborations (not counting former Ph.D. students and post-docs), I recall

*Luigi Ambrosio* (Scuola Normale Superiore, Pisa, I): *Optimal Transport, Gradient flows, analysis in metric spaces*. Together with N. Gigli, we wrote the monograph *Gradient flows in metric spaces and in spaces of probability measures*, Birkhäuser, 2005 (second edition in 2008)

*Piero Colli Franzone* (Pavia): mathematical models for electrocardiology

*Jean Dolbeault* (Université Paris Dauphine, Paris): Transport distances, functional inequalities

*Massimo Fornasier* (Technical University of Munich): mean-field optimal control

Alexander Mielke (WIAS, Berlin): Rate-independent processes, doubly-nonlinear evolution equations, Optimal Entropy-Transport problems

Nicola Gigli (SISSA, Trieste): analysis in metric-measure spaces

Ricardo H. Nochetto (University of Maryland, USA): optimal error estimates for evolution problems

Mark Peletier (TU Eindhoven, NL): Reaction-diffusion systems, Fokker-Planck equations

Giuseppe Toscani (Pavia, I): nonlinear diffusion equations, entropy methods.

I also collaborated with Yann Brenier, José A. Carrillo, Robert McCann, Wilfrid Gangbo, Alessio Porretta, Dejan Slepcev, Lorenzo Zambotti on various evolutionary models related to Optimal Transport.

## Publications

The [MSC database](#) attributes to me 99 publications, 4790 citations by 2550 authors, H-index 33. Seven of the papers listed below belong to the group of [Highly cited papers](#) according to WOS.

The most relevant contributions of my research activity over the past ten years concern:

◇ **Foundation of the theory of metric-measure spaces with Riemannian Ricci curvature bounded from below (the so-called  $RCD(K, N)$  condition):**

*Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below* (with L. Ambrosio, N. Gigli). *Invent. Math.* 195 (2014), 289–391. (\*)

[Heat flow and Cheeger energy in metric–measure spaces:  $L^2$  and Optimal Transport theory]

*Metric measure spaces with Riemannian Ricci curvature bounded from below* (with L. Ambrosio, N. Gigli). *Duke Math. J.*, 163 (2014):1405–1490. (\*)

[The first analysis and characterization of Riemannian  $RCD(K, \infty)$  spaces with quadratic Cheeger energy]

*Bakry-Émery curvature-dimension condition and Riemannian Ricci curvature bounds* (with L. Ambrosio, N. Gigli). *Annals of Probability*, 43 (2015): 339–404. (\*)

[The full identification between the Bakry-Émery condition and  $RCD(K, \infty)$  spaces]

*Self-improvement of the Bakry-Émery condition and Wasserstein contraction of the heat flow in  $RCD(K, \infty)$  metric measure spaces* *Discrete Contin. Dyn. Syst.* 34 (2014): 1641–1661. (\*)

[Second order calculus, measure-valued  $\Gamma_2$ -tensor, and improved Bakry-Émery condition]

*Convergence of pointed non-compact metric measure spaces and stability of Ricci curvature bounds and heat flows* (with N. Gigli, A. Mondino) *Proc. Lond. Math. Soc.* 111 (2015): 1071–1129. (\*)

*Nonlinear diffusion equations and curvature conditions in metric measure spaces* (with L. Ambrosio, A. Mondino) *Mem. Amer. Math. Soc.* 262 (2019), no. 1270, v+121 pp.

[The full identification between the Bakry-Émery condition and the  $RCD(K, N)$  spaces]

◇ **Metric-Sobolev spaces: identification of the construction by Lipschitz functions and Cheeger energy with the Newtonian approach**

*Density of Lipschitz functions and equivalence of weak gradients in metric measure spaces* (with L. Ambrosio, N. Gigli) *Rev. Mat. Iberoam.* 29 (2013): 969–996. (\*)

◇ **Entropy-Transport formulation for unbalanced optimal transport and the characterization of the new Hellinger-Kantorovich distance**

*Optimal entropy-transport problems and a new Hellinger-Kantorovich distance between positive measures* (with M. Liero, A. Mielke). *Invent. Math.* 211 (2018): 969–1117. (\*)

◇ **Foundation of Balanced Viscosity and Visco-Energetic solutions to rate-independent processes in infinite-dimensional spaces**

*Balanced viscosity (BV) solutions to infinite-dimensional rate-independent systems* (with A. Mielke, R. Rossi). *J. Eur. Math. Soc. (JEMS)* 18 (2016): 2107–2165. [The first contribution to existence, characterization, and properties of Balanced Viscosity solutions in infinite dimension.]

*Viscous corrections of the time incremental minimization scheme and visco-energetic solutions to rate-independent evolution problems.* (with L. Minotti) *Arch. Ration. Mech. Anal.* 227 (2018), no. 2, 477–543.

◇ **The mean-field formulation of spatially inhomogeneous Evolutionary Games**

*Spatially Inhomogeneous Evolutionary Games* (with L. Ambrosio, M. Fornasier, M. Morandotti) *Comm. Pure Appl. Math.* 74 (2021): 1353–1402

In addition to the above papers and topics, in the last ten years I have also been working on *sub-linear diffusion, chemical reactions as  $\Gamma$ -limit of diffusion, Cahn-Hilliard and thin-film equations with nonlinear mobility, viscoplasticity at finite strain, sticky particle dynamics with interactions, mean-field optimal control and mean-field planning, the concavity of Rényi entropy power, variational convergence of gradient flows and rate-independent evolutions, the Weighted Energy-Dissipation principle for gradient flows, reverse approximation of gradient flows, jump processes as generalized gradient flows, the Opial property in Wasserstein spaces, duality properties of metric Sobolev spaces and capacity.*

### Monographs and contributions to volumes

*Sobolev Spaces in Extended Metric-Measure Spaces* in *New Trends on Analysis and Geometry in Metric Spaces*. Lecture Notes in Mathematics, Springer 2022, 117–276.

[The refined theory of metric Sobolev space generated by sub-algebra of Lipschitz functions in extended metric-measure spaces.]

*Gradient flows in metric spaces and in the space of probability measures* (with L. Ambrosio, N. Gigli) Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2005 (second edition 2008).

The two editions have received more than 1500 citations, according to MSC.

*Computational electrocardiology: mathematical and numerical modeling* (with P. Colli Franzone, L. F. Pavarino; contribution). In *Complex systems in biomedicine*, pages 187–241. Springer Italia, Milan, 2006.

### Invited presentations to conferences (selection)

- MFO workshop: *Heat Kernels, Stochastic Processes and Functional Inequalities*, Oberwolfach, 2022
- HCM Conference: *From Dirichlet Forms to Wasserstein Geometry*, Bonn, 2022
- Workshop on *Frontiers in Nonlocal Nonlinear PDEs*, Anacapri, 2022
- Workshop *Geometric Measure Theory and applications*, Cortona, 2021
- MFO workshop: *Applications of Optimal Transportation in the Natural Sciences* (online), Oberwolfach, 2021
- Workshop: *Calculus of Variations and Applications*, SISSA, Trieste, 2020.
- *XXI Congresso Unione Matematica Italiana*, 2019. Plenary Speaker
- Workshop: *Optimal transport and Geometric Analysis*, Venice, 2019.
- ICMS Workshop: *Gradient flows: challenges and new directions*, Edinburgh, 2018.
- BIRS workshop: *Topics in the Calculus of Variations: Recent Advances and New Trends*, Banff, 2018.
- BIRS workshop: *Entropies, the Geometry of Nonlinear Flows, and their Applications*, Banff, 2018.
- MFO workshop: *Variational Methods for Evolution* Oberwolfach, 2017
- MFO workshop: *Applications of Optimal Transportation in the Natural Sciences* Oberwolfach, 2017
- 7<sup>th</sup> European Congress of Mathematics, Berlin: section “Analysis and PDEs”. Invited speaker.
- MFO workshop: *Heat Kernels, Stochastic Processes, and Functional Inequalities*, Oberwolfach, 2016.
- Conference on *New trends in Optimal Transport*, HIM, Bonn, 2015.
- BIRS workshop: *Entropy Methods, PDEs, Functional Inequalities, and Applications*, Banff, 2014.
- International Conference on *Fractal Geometry and Stochastics V*, Tabarz (DE), 2014. Plenary speaker.
- Workshop on *Infinite-Dimensional Geometry*, MSRI, Berkeley, 2013.
- EQUADIFF 2013, Prague. Plenary speaker.
- Conference on *Probability and Geometry*, Poitiers, 2012.
- BIRS Workshop: *Optimal Transportation and Differential Geometry* Banff, 2012.
- MFO workshop: *Interplay of Analysis and Probability in Physics*, Oberwolfach, 2012.
- MFO mini-workshop: *Manifolds with Lower Curvature Bounds*, Oberwolfach, 2012.
- RISM meeting: *Multiphase and Multiphysics problems*, Verbania (IT), 2011.
- BIRS workshop: *Nonlinear Diffusions and Entropy Dissipation: From Geometry to Biology*, Banff, 2010.
- CIRM-HCM Meeting: *Stochastic Analysis, SPDEs, Particle Systems, Optimal Transport* Levico (IT), 2010.

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\*An Highly cited paper according to WOS

- Workshop on *Particle systems, nonlinear diffusions, and equilibration*, HCM, Bonn, 2007.
- Workshop on *Optimal Transportation, and Applications to Geophysics and Geometry*, Edinburgh, 2007.
- Workshop on *Optimal transport: theory and application*, Centro De Giorgi, Pisa, 2006.
- *Nonlinear Diffusion Equations and related PDEs*, UAM, Madrid, 2006
- Workshop on *Modelling and analysis of phase transitions* Centro De Giorgi, Pisa, 2006.
- ICMS Workshop: *Optimal Transportation, Transport Equations and Hydrodynamics*, Edinburgh, 2005.
- *10th Conference on Free Boundary Problems*, Coimbra, June 7-12, 2005. Plenary speaker.

### **Invited courses to international advanced schools**

- CIME course on *New Trends on Analysis and Geometry in Metric Spaces*, Levico Terme, Italy 2017: Sobolev Spaces in Extended Metric-Measure Spaces
- *Gradient flows and entropy methods*, HIM, Bonn, 2015: The Weighted Energy-Dissipation (WED) principle for gradient flows.
- *Analysis and Geometry on Singular Spaces*, Scuola Normale Superiore, Pisa, 2014: Metric measure spaces with Riemannian Ricci curvature bounded from below.
- *Seventh Summer School in Analysis and Applied Mathematics*, Roma, 2013: Gradient flows and rate-independent evolutions: a variational approach.
- CNA Summer School on *"New Vistas in Image Processing and PDEs"* Carnegie Mellon University, Pittsburgh, 2010: Applications of optimal transport to evolutionary PDEs.
- *School on "Optimal transport: Theory and applications"* Institut Fourier, Grenoble, 2009: Gradient flows and optimal transport.
- *EVEQ2008*, Prague, 2008: A variational approach to gradient flows and rate-independent problems.
- *School in Nonlinear Analysis and Calculus of Variations* Scuola Normale Superiore, Pisa, 2006: Gradient flows: a variational approach.

Milano, December 1, 2022

Giuseppe Savaré

## List of publications

- [1] Giuseppe Savaré. “Sobolev spaces in extended metric-measure spaces”. In: *New trends on analysis and geometry in metric spaces*. Vol. 2296. Lecture Notes in Math. Springer, Cham, [2022] ©2022, pp. 117–276. URL: [https://doi.org/10.1007/978-3-030-84141-6\\_4](https://doi.org/10.1007/978-3-030-84141-6_4).
- [2] Giulia Cavagnari et al. “Lagrangian, Eulerian and Kantorovich formulations of multi-agent optimal control problems: equivalence and gamma-convergence”. In: *J. Differential Equations* 322 (2022), pp. 268–364. ISSN: 0022-0396. URL: <https://doi.org/10.1016/j.jde.2022.03.019>.
- [3] Mark A. Peletier et al. “Jump processes as generalized gradient flows”. In: *Calc. Var. Partial Differential Equations* 61.1 (2022), Paper No. 33, 85. ISSN: 0944-2669. URL: <https://doi.org/10.1007/s00526-021-02130-2>.
- [4] Giuseppe Savaré and Giacomo E. Sodini. “A simple relaxation approach to duality for optimal transport problems in completely regular spaces”. In: *J. Convex Anal.* 29.1 (2022), pp. 1–12. ISSN: 0944-6532.
- [5] Luigi Ambrosio and Giuseppe Savaré. “Duality properties of metric Sobolev spaces and capacity”. In: *Math. Eng.* 3.1 (2021), Paper No. 1, 31. URL: <https://doi.org/10.3934/mine.2021001>.
- [6] Luigi Ambrosio et al. “Spatially inhomogeneous evolutionary games”. In: *Comm. Pure Appl. Math.* 74.7 (2021), pp. 1353–1402. ISSN: 0010-3640. URL: <https://doi.org/10.1002/cpa.21995>.
- [7] Giulia Luise and Giuseppe Savaré. “Contraction and regularizing properties of heat flows in metric measure spaces”. In: *Discrete Contin. Dyn. Syst. Ser. S* 14.1 (2021), pp. 273–297. ISSN: 1937-1632. URL: <https://doi.org/10.3934/dcdss.2020327>.
- [8] Emanuele Naldi and Giuseppe Savaré. “Weak topology and Opial property in Wasserstein spaces, with applications to gradient flows and proximal point algorithms of geodesically convex functionals”. In: *Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl.* 32.4 (2021), pp. 725–750. ISSN: 1120-6330. URL: <https://doi.org/10.4171/rlm/955>.
- [9] Florentine Fleissner and Giuseppe Savaré. “Reverse approximation of gradient flows as minimizing movements: a conjecture by De Giorgi”. In: *Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)* 20.2 (2020), pp. 677–720. ISSN: 0391-173X. URL: [https://doi.org/10.2422/2036-2145.201711\\_008](https://doi.org/10.2422/2036-2145.201711_008).
- [10] Matteo Muratori and Giuseppe Savaré. “Gradient flows and evolution variational inequalities in metric spaces. I: Structural properties”. In: *J. Funct. Anal.* 278.4 (2020), pp. 108347, 67. ISSN: 0022-1236. URL: <https://doi.org/10.1016/j.jfa.2019.108347>.
- [11] Luigi Ambrosio, Andrea Mondino, and Giuseppe Savaré. “Nonlinear diffusion equations and curvature conditions in metric measure spaces”. In: *Mem. Amer. Math. Soc.* 262.1270 (2019), pp. v+121. ISSN: 0065-9266. URL: <https://doi.org/10.1090/memo/1270>.
- [12] M. Fornasier et al. “Mean-field optimal control as gamma-limit of finite agent controls”. In: *European J. Appl. Math.* 30.6 (2019), pp. 1153–1186. ISSN: 0956-7925. URL: <https://doi.org/10.1017/s0956792519000044>.
- [13] Carlo Orrieri, Alessio Porretta, and Giuseppe Savaré. “A variational approach to the mean field planning problem”. In: *J. Funct. Anal.* 277.6 (2019), pp. 1868–1957. ISSN: 0022-1236. URL: <https://doi.org/10.1016/j.jfa.2019.04.011>.
- [14] Riccarda Rossi et al. “Weighted energy-dissipation principle for gradient flows in metric spaces”. In: *J. Math. Pures Appl. (9)* 127 (2019), pp. 1–66. ISSN: 0021-7824. URL: <https://doi.org/10.1016/j.matpur.2018.06.022>.
- [15] Matthias Liero, Alexander Mielke, and Giuseppe Savaré. “Optimal entropy-transport problems and a new Hellinger-Kantorovich distance between positive measures”. In: *Invent. Math.* 211.3 (2018), pp. 969–1117. ISSN: 0020-9910. URL: <https://doi.org/10.1007/s00222-017-0759-8>.

- [16] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. “Global existence results for viscoplasticity at finite strain”. In: *Arch. Ration. Mech. Anal.* 227.1 (2018), pp. 423–475. ISSN: 0003-9527. URL: <https://doi.org/10.1007/s00205-017-1164-6>.
- [17] Luca Minotti and Giuseppe Savaré. “Viscous corrections of the time incremental minimization scheme and visco-energetic solutions to rate-independent evolution problems”. In: *Arch. Ration. Mech. Anal.* 227.2 (2018), pp. 477–543. ISSN: 0003-9527. URL: <https://doi.org/10.1007/s00205-017-1165-5>.
- [18] Giuseppe Savaré. “Diffusion, optimal transport and Ricci curvature”. In: *European Congress of Mathematics*. Eur. Math. Soc., Zürich, 2018, pp. 311–331.
- [19] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Diffusion, optimal transport and Ricci curvature for metric measure spaces”. In: *Eur. Math. Soc. Newsl.* 103 (2017), pp. 19–28. ISSN: 1027-488X. URL: <https://doi.org/10.4171/news/103/4>.
- [20] Riccarda Rossi and Giuseppe Savaré. “From visco-energetic to energetic and balanced viscosity solutions of rate-independent systems”. In: *Solvability, regularity, and optimal control of boundary value problems for PDEs*. Vol. 22. Springer INdAM Ser. Springer, Cham, 2017, pp. 489–531.
- [21] Luigi Ambrosio, Matthias Erbar, and Giuseppe Savaré. “Optimal transport, Cheeger energies and contractivity of dynamic transport distances in extended spaces”. In: *Nonlinear Anal.* 137 (2016), pp. 77–134. ISSN: 0362-546X. URL: <https://doi.org/10.1016/j.na.2015.12.006>.
- [22] Luigi Ambrosio, Andrea Mondino, and Giuseppe Savaré. “On the Bakry-Émery condition, the gradient estimates and the local-to-global property of  $RCD^*(K, N)$  metric measure spaces”. In: *J. Geom. Anal.* 26.1 (2016), pp. 24–56. ISSN: 1050-6926. URL: <https://doi.org/10.1007/s12220-014-9537-7>.
- [23] Matthias Liero, Alexander Mielke, and Giuseppe Savaré. “Optimal transport in competition with reaction: the Hellinger-Kantorovich distance and geodesic curves”. In: *SIAM J. Math. Anal.* 48.4 (2016), pp. 2869–2911. ISSN: 0036-1410. URL: <https://doi.org/10.1137/15M1041420>.
- [24] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. “Balanced viscosity (BV) solutions to infinite-dimensional rate-independent systems”. In: *J. Eur. Math. Soc. (JEMS)* 18.9 (2016), pp. 2107–2165. ISSN: 1435-9855. URL: <https://doi.org/10.4171/JEMS/639>.
- [25] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. “Balanced-viscosity solutions for multi-rate systems”. In: *J. Phys. Conf. Ser.* 727 (2016), pp. 012010, 26. ISSN: 1742-6588. URL: <https://doi.org/10.1088/1742-6596/727/1/012010>.
- [26] Virginia Agostiniani, Riccarda Rossi, and Giuseppe Savaré. “On the transversality conditions and their genericity”. In: *Rend. Circ. Mat. Palermo (2)* 64.1 (2015), pp. 101–116. ISSN: 0009-725X. URL: <https://doi.org/10.1007/s12215-014-0184-4>.
- [27] Luigi Ambrosio, Simone Di Marino, and Giuseppe Savaré. “On the duality between  $p$ -modulus and probability measures”. In: *J. Eur. Math. Soc. (JEMS)* 17.8 (2015), pp. 1817–1853. ISSN: 1435-9855. URL: <https://doi.org/10.4171/JEMS/546>.
- [28] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Bakry-Émery curvature-dimension condition and Riemannian Ricci curvature bounds”. In: *Ann. Probab.* 43.1 (2015), pp. 339–404. ISSN: 0091-1798. URL: <https://doi.org/10.1214/14-AOP907>.
- [29] Nicola Gigli, Andrea Mondino, and Giuseppe Savaré. “Convergence of pointed non-compact metric measure spaces and stability of Ricci curvature bounds and heat flows”. In: *Proc. Lond. Math. Soc.* (3) 111.5 (2015), pp. 1071–1129. ISSN: 0024-6115. URL: <https://doi.org/10.1112/plms/pdv047>.
- [30] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below”. In: *Invent. Math.* 195.2 (2014), pp. 289–391. ISSN: 0020-9910. URL: <https://doi.org/10.1007/s00222-013-0456-1>.

- [31] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Metric measure spaces with Riemannian Ricci curvature bounded from below”. In: *Duke Math. J.* 163.7 (2014), pp. 1405–1490. issn: 0012-7094. URL: <https://doi.org/10.1215/00127094-2681605>.
- [32] Sara Daneri and Giuseppe Savaré. “Lecture notes on gradient flows and optimal transport”. In: *Optimal transportation*. Vol. 413. London Math. Soc. Lecture Note Ser. Cambridge Univ. Press, Cambridge, 2014, pp. 100–144.
- [33] Giuseppe Savaré. “Self-improvement of the Bakry-Émery condition and Wasserstein contraction of the heat flow in  $RCD(K, \infty)$  metric measure spaces”. In: *Discrete Contin. Dyn. Syst.* 34.4 (2014), pp. 1641–1661. issn: 1078-0947. URL: <https://doi.org/10.3934/dcds.2014.34.1641>.
- [34] Giuseppe Savaré and Giuseppe Toscani. “The concavity of Rényi entropy power”. In: *IEEE Trans. Inform. Theory* 60.5 (2014), pp. 2687–2693. issn: 0018-9448. URL: <https://doi.org/10.1109/TIT.2014.2309341>.
- [35] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Density of Lipschitz functions and equivalence of weak gradients in metric measure spaces”. In: *Rev. Mat. Iberoam.* 29.3 (2013), pp. 969–996. issn: 0213-2230. URL: <https://doi.org/10.4171/RMI/746>.
- [36] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. “Heat flow and calculus on metric measure spaces with Ricci curvature bounded below—the compact case”. In: *Analysis and numerics of partial differential equations*. Vol. 4. Springer INdAM Ser. Springer, Milan, 2013, pp. 63–115. URL: [https://doi.org/10.1007/978-88-470-2592-9\\_8](https://doi.org/10.1007/978-88-470-2592-9_8).
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