ELIA BRUE'

elia.brue@unibocconi.it
Bocconi University, Department of Decision Sciences
Via Sarfatti 25 20136 Milano (MI) – Italy

PERSONAL DATA

Place and date of birth: 30th April 1993, Macerata (MC), Italy

Citizenship: Italian citizen

Websites:

- Google Scholar: `https://scholar.google.it/citations?user=6QVC4J0AAAAJ&hl=it`
- CVGMT: `https://cvgmt.sns.it/person/2687/`
- Research Gate: `https://www.researchgate.net/profile/Elia-Brue`

APPOINTMENTS

- **University Bocconi, Milan, Italy** Starting from September 2023
  Associate Professor
- **University Bocconi, Milan, Italy** February 2023-September 2023
  Assistant Professor
- **Institute for Advanced Study, Princeton, USA** November 2020-December 2022
  Postdoctoral member
  Supervisor: Prof. Camillo De Lellis

EDUCATION

- **Scuola Normale Superiore, Pisa, Italy** November 2017-October 2020
  Ph.D. in Mathematics, cum laude.
  Title of the thesis (English): *Structure of non-smooth spaces with Ricci curvature bounded below.*
  Advisor: Prof. Luigi Ambrosio
- **Scuola Normale Superiore, Pisa, Italy** October 2015-May 2017
  Master degree in Mathematics, grade: 110/110 cum laude
  Title of the thesis: *Sulla buona positura di equazioni differenziali associate a campi debolmente differenziabili.*
  Advisor: Prof. Luigi Ambrosio
- **University of Pisa, Pisa, Italy** October 2012-May 2015
  Bachelor degree in Mathematics, grade: 110/110 cum laude
  Title of the thesis (Italian): *Trasporto ottimale e disuguaglianza isoperimetrica anisotropa.*
  Advisor: Prof. Luigi Ambrosio

AWARDS

- Frontiers of Science Award in Mathematics 2024.

- Pitagora Prize 2023 of the Department of Mathematics and Computer Science at UNICAL, sponsored by UMI (Italian Mathematical Society).
- Guido Stampacchia Prize 2022 of the Accademia di Scienze Fisiche e Matematiche della Società Nazionale di Scienze, Lettere e Arti in Napoli.
- Iapichino Prize 2021 of the Accademia Nazionale dei Lincei. For young researchers in Mathematical Analysis.
- Postdoctoral fellowship (2 years) at the Institute for Advanced Study in Princeton.

RESEARCH

My research interests are in the fields of Geometric Analysis and Partial Differential Equations, including work related to Ricci curvature, metric geometry, incompressible fluid mechanics, and passive scalars with rough velocity fields.

Publications

(1) L. Ambrosio, E. Brué, D. Trevisan: *Lusin-type approximation of Sobolev by Lipschitz functions, in Gaussian and* $\mathrm{RCD}(K,\infty)$ *spaces.* Adv. Math. 339 (2018), 426–452.

(2) E. Brué, D. Semola: *Regularity of Lagrangian flows over* $\mathrm{RCD}^*(K,N)$ *spaces.* J. Reine Angew. Math. 765 (2020), 171–203.

(3) E. Brué, D. Semola: *Constancy of the dimension for* $\mathrm{RCD}(K,N)$ *spaces via regularity of Lagrangian flows.* Comm. Pure Appl. Math. 73 (2020), no. 6, 1141–1204.

(4) E. Brué, Q.-H. Nguyen: *On the Sobolev space of functions with derivative of logarithmic order.* Adv. Nonlinear Anal. 9 (2020), no. 1, 836–849.

(5) E. Brué, Q.-H. Nguyen, G. Stefani: *A maximal function characterization of absolutely continuous measures and Sobolev functions.* Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl. 30 (2019), no. 3, 599–614.

(6) L. Ambrosio, E. Brué, D. Semola: *Rigidity of the 1-Bakry–Émery inequality and sets of finite perimeter in* RCD *spaces.* Geom. Funct. Anal. 29 (2019), no. 4, 949–1001.

(7) G. Antonelli, E. Brué, D. Semola *Volume bounds for the quantitative singular strata of non collapsed* RCD *metric measure spaces.* Anal. Geom. Metr. Spaces 7 (2019), no. 1, 158–178.

(8) E. Brué, Q.-H. Nguyen: *Sobolev estimates for solutions of the transport equation and ODE flows associated to non-Lipschitz drifts.* Math. Ann. 380 (2021), no. 1-2, 855–883.

(9) E. Brué, E. Pasqualetto, D. Semola: *Rectifiability of* $\mathrm{RCD}(K,N)$ *spaces via* δ*-splitting maps.* Ann. Fenn. Math. 46 (2021), no. 1, 465–482.

(10) E. Brué, Q.-H. Nguyen: *Sharp regularity estimates for solutions of the continuity equation drifted by Sobolev vector fields.* Anal. PDE 14 (2021), no. 8, 2539–2559

(11) E. Brué, S. Di Marino, F. Strá: *Linear Lipschitz and* C^1 *extension operators through random projection.* J. Funct. Anal. 280 (2021), no. 4, 108868.

(12) E. Brué, Q.-H. Nguyen: *Advection diffusion equations with Sobolev velocity field.* Comm. Math. Phys. 383 (2021), no. 1, 465–487.

(13) E. Brué, M. Colombo, C. De Lellis: *Positive solutions of transport equations and classical nonuniqueness of characteristic curves.* Arch. Ration. Mech. Anal. 240 (2021), no. 2, 1055–1090.

(14) E. Brué, Q. Deng, D. Semola: *Improved regularity of Lagrangian flows on* $\mathrm{RCD}(K,N)$ *spaces.* Nonlinear Anal. 214 (2022), Paper No. 112609.

(15) E. Brué, M. Calzi, G. Comi, G. Stefani: *A distributional approach to fractional Sobolev spaces and fractional variation: asymptotics II.* C. R. Math. Acad. Sci. Paris 360 (2022), 589–626.

(16) E. Brué, A. Naber, D. Semola: *Boundary regularity and stability for spaces with Ricci bounded below.* Invent. Math. 228 (2022), no. 2, 777–891.

(17) E. Brué, E. Pasqualetto, D. Semola: *Rectifiability of the reduced boundary for sets of finite perimeter over* $\mathrm{RCD}(K,N)$ *spaces.* J. Eur. Math. Soc. (JEMS) 25 (2023), no. 2, 413–465.

(18) G. Antonelli, E. Brué, M. Fogagnolo, M. Pozzetta: *On the existence of isoperimetric regions in manifolds with nonnegative Ricci curvature and Euclidean volume growth.* Calc. Var. Partial Differential Equations 61 (2022), no. 2, Paper No. 77.

(19) D. Albritton, E. Brué, M. Colombo: *Non-uniqueness of Leray solutions of the forced Navier-Stokes equations.* Ann. of Math. (2) 196 (2022), no. 1, 415–455.

(20) E. Brué, E. Pasqualetto, D. Semola: *Constancy of the dimension in codimension one and locality of the unit normal on* $RCD(K,N)$ *spaces.* Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)24(2023), no.3, 1765–1816.

(21) E. Brué, A. Mondino, D. Semola: *The metric measure boundary of spaces with Ricci curvature bounded below.* Geom. Funct. Anal. 33 (2023), no. 3, 593–636.

(22) E. Brué, C. De Lellis: *Anomalous dissipation for the forced 3D Navier-Stokes equations.* Comm. Math. Phys. 400 (2023), no. 3, 1507–1533.

(23) E. Brué, M. Colombo, G. Crippa, C. De Lellis, M. Sorella: *Onsager critical solutions of the forced Navier-Stokes equations.* Comm. Math. Phys. 403 (2023), no. 2, 1171–1192.

(24) E. Brué, M. Colombo: *Nonuniqueness of solutions to the Euler equations with vorticity in a Lorentz space.* Comm. Math. Phys. 403 (2023), no. 2, 1171–1192.

(25) D. Albritton, E. Brué, M. Colombo: *Gluing non-unique Navier-Stokes solutions.* Ann. PDE 9 (2023), no. 2, Paper No. 17.

(26) E. Brué, A. Naber, D. Semola: *Stability of tori under lower sectional curvature.* Accepted in Geometry and Topology.

(27) E. Brué, A. Naber, D. Semola: *Fundamental groups and the Milnor conjecture.* Accepted in Ann. of Math. (2).

Submitted papers

(28) E. Brué, K. Suzuki: *BV Functions and Sets of Finite Perimeter on Configuration Spaces.* Submitted.

(29) E. Brué, M. Coti Zelati, E. Marconi: *Enhanced dissipation for two-dimensional Hamiltonian flows.* Submitted.

(30) E. Brué, R. Jin, Y. Li, D. Zhang: *Non-uniqueness in law of leray solutions to 3d forced stochastic Navier-Stokes equations.* Submitted.

(31) E. Brué, A. Naber, D. Semola: *Six dimensional counterexample to the Milnor Conjecture.* Submitted.

(32) E. Brué, M. Colombo, A. Kumar: *Sharp Nonuniqueness in the Transport Equation with Sobolev Velocity Field.* Submitted.

(33) E. Brué, A. Pigati, D. Semola: *Topological regularity and stability of noncollapsed spaces with Ricci curvature bounded below.* Submitted.

Books and Lecture notes

(34) L. Ambrosio, E. Brué, D. Semola: *Lectures on Optimal Transport.* Springer International Publishing, 130 (2021).

(35) D. Albritton, E. Brué, M. Colombo, C. De Lellis, V. Giri, M. Janisch, H. Kwon: *Instability and nonuniqueness for the* 2*d Euler equations in vorticity form, after M. Vishik.* To appear in Annals of Mathematics Studies.

INVITED LECTURES

· National Conference on "Calcolo delle Variazioni", Levico Terme (Trento), February 2018.

· Conference on "Vector Fields Surfaces and Perimeters in Singular Geometries", University of Ferrara, February 2018.

· Seminar at SISSA (Trieste), April 2018.

· Seminar at the Hausdorff Center for Mathematics (Bonn), July 2018.

· Seminar at University of Basel, November 2018.

· 1147 AMS meeting in Honolulu. Special Session on Analysis of Nonlinear Geometric Equations, organized by Aaron Naber and Richard Bamler, March 2019.

· Conference "Optimal Transport and Geometric Analysis" (Venice), April 2019. Invited speaker.

· Seminar at the Hausdorff Center for Mathematics (Bonn), June 2019.

· Conference "GMT and PDEs in Basel – A young researchers meeting" (Basel), July 2019. Invited speaker.

· Oberwolfach conference "Partial differential equations", July 2019. Invited participant.

· Seminar at the Department of Mathematics and Statistics, University of Jyväskylä, September 2019.

· Colloquium talk at Shanghaitech University (Shanghai), November 2019.

· Seminar at Oberwolfach conference "Heat Kernels, Stochastic Processes and Functional Inequalities", November 2019.

· Seminar at "Seminari d'analisi de Barcelona", December 2019.

· National Conference on "Calcolo delle Variazioni", Levico Terme (Trento), February 2020.

· Seminar of Analysis at EPFL (Lausanne), February 2020.

· Seminar of Geometry and Geometric Analysis Working Group at Courant Institute of Mathematics, March 2020.

· Online seminar PDE, Shanghaitech University, May 2020.

· Online seminar at Universität of Hamburg, June 2020.

· Online Analysis seminar at UCSD, San Diego, October 2020.

· Analysis seminar at IAS (Princeton), November 2020.

· Online seminar of analysis working group at Warwick University, November 2020.

· Online conference "Nonlinear Differential Problems via Variational, Topological and Set-valued Methods", March 2021. Invited speaker.

· Online conference "Coronaseminar", May 2021. Invited speaker.

· Columbia Geometry and Analysis Seminar, October 2021.

· Oxford Analysis Seminar, October 2021.

· Differential Geometry Seminar at UC Berkeley, November 2021.

· Seminar Analysis and Mathematical Physics at University of Basel, December 2021.

· Online Workshop on "Transport, Fluids and Mixing", Pisa, January 2022. Invited speaker.

· Online conference "mms and convergence seminar", January 2022. Invited speaker.

· JHU Analysis and PDE seminar. Baltimore, John Hopkins University, March 2022.

· SIAM online PDE meeting, March 2022. Invited speaker.

· Princeton Differential Geometry and Geometric Analysis Seminar, March 2022.

· Seminario di equazioni differenziali e applicazioni, Università di Padova, May 2022.

· "Geometrie", organized by Aaron Naber, André Neves, Burkhard Wilking. Oberwolfach, June 2022.

· Conference "Isoperimetrics problems". University of Pisa, June 2022. Invited speaker.

· Conference "International PDE Conference 2022". Oxford, July 2022. Plenary speaker.

· Short talk at the IAS, September 2022.

· Princeton fluid seminar, October 2022.

· Analysis Seminar at Bielefeld University, November 2022.

· Colloquium at Caltech (Pasadena), December 2022.

· Giornata di Dipartimento Tor Vergata, Roma 2022.

· Trilateral meeting Australia-Italy-Taiwan, Perth (Australia), January 2023.

· Analysis seminar, University of Pavia. April 2023.

· Geometric analysis seminar University Bicocca, Milan. May 2023.

· National Conference on "Calcolo delle Variazioni", Levico Terme (Trento), May 2023.

· "Meeting on Nonlinear Evolution PDEs, Fluid Dynamics and Transport Equations" in Erice, May 2023.

· "Stability and dynamics in fluid mechanics and kinetic theory", Imperial College (London), July 2023.

· Geometry and Topology seminar, Tohoku University (Senday), August 2023.

· Conference "Geometry and Probability" (Saitama), August 2023.

· Conference "Three days between Analysis and Geometry in Trento" (Trento), August 2023.

· Convegno UMI 2023, speaker at the sessions "Analisi reale e disuguaglianze geometrice", "Equazioni alle derivate parziali" (Pisa), September 2023.

· Conference "Enjoying Probability and Fluids in Lausanne" (Lausanne), September 2023.

· Conference "XI Giornata di Studio Politecnico di Milano - Università di Pavia" (Milan), October 2023.

· Colloquium at Chicago University, November 2023.

· Conference "Christmas workshop on fluid mechanics" (Milan), December 2023.

· Analysis seminar, Università degli studi di Milano, April 2024.

· Analysis seminar ETH Zurich, May 2024.

MINICOURSES

- During the academic year 2021-22, I participated in the Special Year on *h-Principle and Flexibility in Geometry and PDEs* organized by Camillo De Lellis and László Székelyhidi Jr. at the Institute for Advanced Study in Princeton. In this context, I delivered two minicourses:
  - "Non-uniqueness and instability in fluid flows", in collaboration with D. Albritton.
  - "Positive Lyapunov exponents and mixing in stochastic fluid flow", in collaboration with S. Punson-Smith.
- "Instability and non-uniqueness of Leray solutions to the forced Navier-Stokes equations" at the event "Online PDE Lecture series" organized by Quoc-Hung Nguyen.
- "Instability and non-uniqueness in fluid dynamics" at the event "Meeting on Nonlinear Evolution PDEs, Fluid Dynamics and Transport Equations" in Erice, May 2023.

- "The fundamental groups of manifolds with nonnegative Ricci curvature" at the conference "Three days between Analysis and Geometry in Trento" in Trento, August 2023.

SCIENTIFIC VISITS

- Hausdorff Center for Mathematics (Bonn), May 19th to June 1st, 2019.
- University of Jyväskylä (Jyväskylä), September 30th to October 5th, 2019.
- Shanghaitech University (Shanghai), October 29th to November 12th, 2019.
- EPFL (Lausanne), February 17th to February 22nd, 2020.
- Institute for Advanced Study (Princeton), March 1st to March 31st, 2020.
- Oxford University (Oxford), July 18th to July 29th, 2022.
- Northwestern University (Evanston), October 9th to October 16th, 2022.
- EPFL (Lausanne), January 22nd to January 28th, 2023.
- Tohoku University (Sendai), August 5th to August 12th, 2023.
- ETH Zurich, May 12th to May 17th, 2023.

TEACHING

- "Elements of Real and Complex analysis" at University Bocconi (Milan), spring 2023.
- "Mathematical methods in computer science" at University Bocconi (Milan), fall 2023.
- "Instability and nonuniqueness in fluid dynamics" at SISSA (Trieste), spring 2024.
- "Elements of Real and Fourier analysis" at University Bocconi (Milan), spring 2024.

CONFERENCE AND SEMINAR ORGANIZATION

- "RISM Summer School: Exotic solutions and well-posedness in PDEs and ODEs", 10-14 July 2023. In collaboration with Maria Colombo and Luigi Ambrosio.
- "Analysis and Mathematical Physics seminar" at the IAS (Princeton), second term 2020. In collaboration with C. De Lellis and H. Kwon.
- "Analysis and Mathematical Physics seminar" at the IAS (Princeton), first term 2022. In collaboration with B. Bjoern and C. De Lellis.

EDITORIAL WORK

Currently, I am an editor of the following journals:

- Mathematics in Engineering — January 2023-Present
- Nonlinear Analysis — October 2023-Present

May 19, 2024